A VARIANT OF A THEOREM BY SPRINGER

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To the memory of D. K. Faddeev

Abstract. The theorem in question gives a sufficient condition for a quadratic space over a local ring \( R \) to contain a hyperbolic plane over \( R \).

Our main aim in this paper is to prove the following result, which is a variant of the Springer theorem \([3]\) for quadratic spaces over local rings.

**Theorem.** Let \( R \) be a local Noetherian domain that has an infinite residue field of characteristic different from 2. Let \( S = R[T]/(F(T)) \) be an integral étale extension over \( R \). Let \((V, q)\) be a quadratic space over \( R \) such that the \( S \)-quadratic space \((V \otimes_R S, q \otimes_R S)\) contains a hyperbolic plane \( \mathbb{H}_S \). If the degree of the polynomial \( F(T) \) is odd, then the space \((V, q)\) already contains a hyperbolic plane over \( R \).

This theorem is a main ingredient in the proof of the following result established in \([2]\).

**0.1. Theorem.** Let \( R \) be a regular local ring, \( K \) its field of fractions, and \((V, \varphi)\) a quadratic space over \( R \). Suppose \( R \) contains a field of characteristic zero. If \((V, \varphi) \otimes_R K\) is isotropic over \( K \), then \((V, \varphi)\) is isotropic over \( R \); i.e., there exists a unimodular isotropic vector \( v \in V \) with \( \varphi(v) = 0 \).

It is well known that any finite étale extension \( S \) of \( R \) has the form \( S = R[T]/(F(T)) \), where \( F(T) \) is a monic separable polynomial. If \( A \) is a semilocal ring and \((W, \phi)\) is a quadratic space over \( A \), then \( W \) contains a hyperbolic plane if and only if \( W \) contains a unimodular isotropic vector \( w \). A vector \( w \) is said to be unimodular if \( w \) can be taken as the first vector \( w_1 \) of a free \( A \)-base \( w_1, \ldots, w_n \) of the \( A \)-module \( W \).

§1. Preliminaries

In this section we formulate two results to be used in the proof of the main theorem. These two results will be proved in \( \S 3 \). We need to fix some notation:

- \( k \) is an infinite field \((\text{char}(k) \neq 2)\);
- \( f(t) \) is a monic separable polynomial of degree \( n \) over \( k \);
- \( l = k[t]/(f(t)) \) is a separable \( k \)-algebra;
- \( \theta = t \mod f(t) \) is an element of \( l \);
- \((W, \phi)\) is a quadratic space over \( k \) of rank \( \geq 3 \);
- \((W_i, \phi_i)\) is the quadratic space \((W \otimes_k l, \phi \otimes_k l)\) over \( l \);
- \( W[t]^{(m)} = W \cdot 1 + W \cdot t + \cdots + W \cdot t^{m-1} \subset W[t] = W \otimes_k k[t] \);
- \( k[t]^{(m)} = k \cdot 1 + k \cdot t + \cdots + k \cdot t^{m-1} \subset k[t] \);

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is the required isotropic vector. Indeed, 

1.2. Let \( W \) be a direct summand. Then there exists a unimodular isotropic vector \( w \in W \), and for any such vector \( w \) there exists an element \( v(t) \in W[t]^{(n)} \) satisfying the following conditions:

1.3. Lemma. Let \( u \in U \) be a unimodular isotropic vector. Then for any isotropic vector \( v \in U \) with \( \langle v, u \rangle \in A^* \) there exists a vector \( v \in U \) such that

1.4. Lemma. Let \( k[t] \) be the polynomial ring over the field \( k \), and let \((W, \phi)\) be the quadratic space over \( k \). Let \( w(t) = w_0 + w_1 \cdot t + \cdots + w_{n-1} \cdot t^{n-1} \) be an element of \( W[t] \) with \( w_i \in W \). Suppose that the polynomial \( \phi^{(n)}(w(t)) \) \( \in k[t] \) is separable, and let \( g(t) \in k[t] \) be an irreducible polynomial dividing \( \phi^{(n)}(w(t)) \). Then \( w(t) \) does not vanish modulo \( g(t) \).

1.5. Lemma. Let \( A \) be a semilocal ring, and let \((U, \psi)\) be a quadratic space over \( A \). Let \( v \in U \) be a unimodular isotropic vector. Then a hyperbolic plane can be split out of \((U, \psi)\), i.e.,

\[
(U, \psi) \cong (U', \psi') \perp \mathbb{H}.
\]

Proof. This is easy.
2.1. Theorem. Let $R$ be a local Noetherian domain that has an infinite residue field of characteristic different from 2. Let $S = R[T]/(F(T))$ be an integral étale extension over $R$. Let $(V, q)$ be a quadratic space over $R$ such that the $S$-quadratic space $(V \otimes_R S, q \otimes_R S)$ contains a hyperbolic plane $\mathbb{H}_S$. If $\deg F(T)$ is odd, then the space $(V, q)$ contains a hyperbolic plane $\mathbb{H}_R$.

Proof. The case of rank $(q) = 2$ is obvious. So we may assume rank $(q) \geq 3$. Let $m$ be the maximal ideal of $R$, and let $k$ be the residue field $R/m$ of $R$. Let $l = S/mS = k[t]/(f(t))$, where $f(t) = F[T] \mod m$. Since $S$ is étale over $R$, so is the $k$-algebra $l$. In particular, the $k$-algebra $l$ is separable.

We shall write $(V_R, q_R)$ for $(V, q)$ and $(\bar{V}, \bar{q})$ for the reduction modulo $m$ of the $R$-quadratic space $(V_R, q_R)$. Let $(V_S, q_S)$ be the scalar extension of $(V_R, q_R)$ up to $S$, and let $(\bar{V}_l, \bar{q}_l)$ be the reduction modulo $mS$ of the $S$-quadratic space $(V_S, q_S)$. Clearly, $(\bar{V}_l, \bar{q}_l) = (\bar{V}, \bar{q}) \otimes_k l$. Now, set $(W, \phi) = (V, q)$. Then $(W \otimes_k l, \phi \otimes_k l) = (\bar{V}_l, \bar{q}_l)$, and we shall write $(W_l, \phi_l)$ for $(\bar{V}_l, \bar{q}_l)$.

By assumption, the space $(W_l, \phi_l)$ contains a hyperbolic plane $\mathbb{H}_l$ as a direct summand. Thus, we can apply Proposition 1.1. So, using the notation of Proposition 1.1, we can find a vector $w \in W_l$ (unimodular and isotropic) and an element $v(t) = v_0 \cdot 1 + v_1 \cdot t + \cdots + v_{n-1} \cdot t^{n-1} \in W[t]^{(n)}$ satisfying the following conditions:

1. $\phi_l(v(t)) = k[t]$ has degree $2n - 2$;
2. $\phi_l(v(t))$ is a separable polynomial over $k$;
3. $\langle w, v(t) \rangle$ is an invertible element of $l$;
4. $\phi_l(v(\theta)) = 0 \in l$, where $\theta = t \mod f(t)$ is the element of $l$.

Recall that the quadratic space $(W_l, \phi_l)$ is the reduction modulo $mS$ of the quadratic space $(V_S, q_S)$. By Lemma 1.3, the element $v = v(\theta) \in W_l$ can be lifted up to a unimodular isotropic vector $v \in V_S$.

Since $S = R[T]/(F(T))$, we have $V_S = V_R \otimes_R R[T]/(F(T))$. Set $\Theta = T \mod F(T)$. There are elements $v_0, v_1, \ldots, v_{n-1} \in V_R$ such that $v = v_0 \cdot 1 + v_1 \cdot \Theta + \cdots + v_{n-1} \cdot \Theta^{n-1}$. Consider the element $v(T) = v_0 \cdot 1 + v_1 \cdot T + \cdots + v_{n-1} \cdot T^{n-1} \in V_R \otimes_R R[T]$, and consider the diagram

$$
\begin{array}{c}
W_l \quad \xrightarrow{\phi_l} \quad k \times k \quad \xrightarrow{\text{Disc}} \quad k \\
| \hspace{1cm} | \hspace{1cm} | \\
V_l \quad \xrightarrow{\bar{q}_l} \quad R \times R \quad \xrightarrow{\text{Disc}} \quad R \\
| \hspace{1cm} | \hspace{1cm} | \\
W_S \quad \xrightarrow{q_R} \quad V_R \times V_R \quad \xrightarrow{\text{Disc}} \quad R \\
| \hspace{1cm} | \hspace{1cm} | \\
V_S \quad \xrightarrow{E(T)} \quad V \times \cdots \times V \\
\end{array}
$$

where $E(v_0, \ldots, v_{n-1}) = u_0 \cdot 1 + u_1 \cdot \Theta + \cdots + u_{n-1} \cdot \Theta^{n-1} \in V_S$,

$$
q_R(u_0, \ldots, u_{n-1}) = q_R(u_0) + 2 \cdot (u_0, u_1) \cdot T + q_R(u_1) \cdot T^2 + \cdots + q_R(u_n) \cdot T^{2n-2},
$$

and $\text{Disc}(a_0, a_1, \ldots, a_{2n-2})$ denotes the discriminant of the polynomial $a_0 + a_1 T + \cdots + a_{2n-2} T^{2n-2}$. The vertical arrows are canonical maps.

Clearly, this diagram commutes, and the maps $E, \phi$ are isomorphisms. In particular, $v = v_1 \mod m$ for the components $v_0, \ldots, v_{n-1}$ (respectively, $v_0, \ldots, v_{n-1}$) of the element $v(t) \in W[t]$ (respectively, $v(T) \in V_S[T]$). Thus, the reduction modulo $m$ of the polynomial $q_R^{(n)}(v(T))$ coincides with the polynomial $\phi_l^{(n)}(v(t))$. The latter polynomial is separable and has degree $2n - 2$ by the choice of $v(t)$. 


Since \(v(\Theta) = v\) by the very choice of the element \(v(T)\), and since \(v\) is \(q_S\)-isotropic, we have \(q_R(v(\Theta)) = 0\). Thus, \(q_R^{(n)}(v(T))\) vanishes modulo \(F(T)\) in the ring \(R[T]\); i.e., there exists a polynomial \(H(T)\) in \(R[T]\) such that

\[
q_R^{(n)}(v(T)) = F(T) \cdot H(T).
\]

After reduction modulo \(m\), we get the relation

\[
\phi^{(n)}(v(t)) = f(t) \cdot h(t),
\]

where \(h(t)\) is the reduction of \(H(t)\) modulo \(m\). Since \(\deg \phi^{(n)}(v(t)) = 2n - 2\) and \(\deg f(t) = n\), we see that \(\deg h(t) = n - 2\). Since \(\deg H(T) \leq n - 2\) and \(H(T)\) is monic (the highest coefficient is invertible). Next, \(\phi^{(n)}(v(t))\) is separable. Thus, \(h(t)\) is also separable. This shows that the \(R\)-algebra \(S' = R[T]/(H(T))\) is an étale \(R\)-algebra.

Let \(A\) denote the class of \(T\) modulo \(H(T)\) in the ring \(S'\). Then the vector \(v(A) = v_0 \cdot 1 + v_1 \cdot A + \cdots + v_n \cdot A^{n-1}\) of the quadratic \(S'\)-space \((V_{S'},q_{S'})\) is isotropic. In fact, \(q_{S'}(v(A)) = F(A) \cdot H(A) = 0\) in \(S'\). If the vector \(v(A)\) is unimodular, then a hyperbolic plane \(\mathbb{H}_{S'}\) can be split out of the quadratic space \((V_{S'},q_{S'})\) over \(S'\) (see Lemma 1.4). So, in this case we have constructed a finite étale extension \(S' = R[T]/(H(T))\) such that

\[
\deg H(T) = \deg F(T) - 2
\]

and the space \((V_{S'},q_{S'})\) contains a hyperbolic plane \(\mathbb{H}_{S'}\) as a direct summand. Repeating this procedure several times, finally we get a direct hyperbolic summand of the quadratic space \((V_R,q_R)\) itself. Thus, to complete the proof of the theorem it remains to check that the vector \(v(A) \in V_{S'}\) is unimodular.

For this, we denote the ring \(S'/mS'\) by \(k'\) and observe that \(k' = k[t]/(h(t))\), where, as above, \(h(t)\) is the reduction modulo \(m\) of the polynomial \(H(t)\). We denote by \(V_{k'}\) the \(k'\)-module \(V \otimes_k k'\) and consider the commutative diagram (with the same elements \(v(T)\) and \(v(t)\) as above in this proof)

\[
\begin{array}{ccc}
v(\alpha) & \in & V_{k'} \\
v(A) & \in & V_{S'} \\
\end{array}
\]

Here \(Ev(u_0 + u_1 T + \cdots + u_{n-1} T^{n-1}) = u_0 + u_1 \cdot A + \cdots + u_{n-1} \cdot A^{n-1}\) and \(Ev(w_0 + w_1 \cdot t + \cdots + w_{n-1} \cdot t^{n-1}) = w_0 + w_1 \cdot \alpha + \cdots + w_{n-1} \cdot \alpha^{n-1}\) and \(\alpha = t \mod h(t) \in k'\).

To check that \(v(A)\) is unimodular, it suffices to verify that \(v(\alpha)\) is unimodular. Observe that \(V_{k'} = \mathbb{V} \otimes_k k[t]/(h(t))\). Let \(h(t) = h_1(t) \cdot \cdots \cdot h_r(t)\) be the factorization of \(h(t)\) into irreducible polynomials. Since \(h(t)\) is separable, we have \(h_i(t) \neq h_j(t)\) for \(i \neq j\). Then \(V_{k'} = \prod_{i=1}^r \mathbb{V} \otimes_k (k[t]/(h_i(t)))\), so that \(v(\alpha)\) is unimodular in \(V_{k'}\) if and only if the elements \(v(t) \in \mathbb{V}[t]\) do not vanish modulo any of the \(h_i(t)\) \((i = 1,\ldots,r)\).

The polynomial \(h_i(t)\) divides \(h(t)\), and the polynomial \(h(t)\) divides \(h(\phi^{(n)}(v(t)))\) (see (**)). Therefore, \(h_i(t)\) divides \(\phi^{(n)}(v(t))\). Since \(h_i(t)\) is irreducible and \(\phi^{(n)}(v(t))\) is separable, Lemma 1.4 shows that \(v(t)\) does not vanish modulo \(h_i(t)\). Thus, the element \(v(\alpha)\) is indeed a unimodular vector in \(V_{k'}\), and the element \(v(A)\) is a unimodular vector in \(V_{S'}\).

This completes the proof of the theorem. \(\square\)

\section*{3. Proof of Proposition 1.1}

\subsection*{3.1.} Since the quadratic space \((W_1,\phi_1)\) contains a hyperbolic plane \(\mathbb{H}_1\) as a direct summand, we can find a unimodular isotropic vector \(w \in W_1\). We choose and fix such a vector.
Now, set \( X(l) = \{ v \in W \mid \phi(v) = 0 \} \) and consider the map \( \rho_W : X(l) \to l \) taking \( v \) to the scalar product \( \langle w, v \rangle \in l \). Then, consider the following diagram of sets and their polynomial maps:

\[
\begin{array}{c}
X(l) \xleftarrow{\rho_W} l \\
\downarrow i \quad \downarrow j \\
W \xleftarrow{ev} W \times \cdots \times W = W[t]^{(n)} \xrightarrow{\phi(n)} k[t]^{(2n-1)} \xrightarrow{\text{Disc}} k,
\end{array}
\]

where \( ev \) is the map defined at the beginning of \( \mathbb{I} \) (clearly, this map is an isomorphism), and where \( Y = ev^{-1}(X(l)) \), the maps \( i, j \) are inclusions, and \( \phi(n) \) is defined at the beginning of \( \mathbb{I} \). The map Disc takes a polynomial \( g(t) \in k[t]^{(2n-1)} \) to its discriminant. It is well known that Disc\((g(t))\) has a polynomial expression in terms of the coefficients of \( g(t) \).

This diagram is the diagram of \( k \)-rational points and their maps induced by the following diagram:

\[
\begin{array}{c}
R_{l/k}(A^1_k) \xleftarrow{ev} R_{l/k}(X_l) \xrightarrow{\sim} Y \\
\downarrow i \quad \downarrow j \\
R_{l/k}(W_l) \xleftarrow{ev} W \times \cdots \times W \xrightarrow{\phi(n)} A^1_k \times \cdots \times A^1_k \xrightarrow{\text{Disc}} A^1_k
\end{array}
\]

of algebraic varieties. Here \( R_{l/k} \) is the Weil restriction functor. Next, \( W \) (respectively, \( W_l \)) is the \( k \)-vector space (respectively, the \( l \)-vector space) viewed as a \( k \)-variety (respectively, as an \( l \)-variety), \( X_l \) is the affine quadric in \( \mathbb{W} \) given by the equation \( \phi_l = 0 \), \( \mathbb{E}_w \) is the Weil restriction of the morphism \( \rho_w : X_l \to A^1_k \) that takes \( v \) to \( \langle w, v \rangle \), the variety \( Y \) is the preimage of \( R_{l/k}(X_l) \) under the evaluation isomorphism \( ev : \mathbb{W} \times \cdots \times \mathbb{W} \to R_{l/k}(W_l) \), and the morphisms \( i, j \) are closed embeddings. The product \( \mathbb{W} \times \cdots \times \mathbb{W} \) consists of \( n \) factors and the product \( A^1_k \times \cdots \times A^1_k \) consists of \( 2n - 1 \) factors.

We define open subsets of the variety \( Y \) as follows:

\[
\begin{align*}
U_1 &= \text{def} \; \phi_l^{-1}(\mathbb{A}^1_k \times \cdots \times \mathbb{A}^1_k \cap (\mathbb{A}^1_k - \{0\})), \\
U_2 &= \text{def} \; \phi_l^{-1}(\text{Disc}^{-1}(\mathbb{A}^1_k - \{0\})), \\
U_3 &= \text{def} \; ev^{-1}(\mathbb{E}_w^{-1}(R_{l/k}(A^1_k - \{0\}))).
\end{align*}
\]

We need the following lemmas.

**3.2. Lemma.** \( Y \) is a \( k \)-rational variety.

**3.3. Lemma.** \( U_1 \cap U_2 \neq \emptyset \).

**3.4. Lemma.** \( U_3 \neq \emptyset \).

With these three lemmas at hand, we can prove Proposition \( \mathbb{I.1} \) as follows. Since \( Y \) is a \( k \)-rational variety, any one of its nonempty open subsets has a \( k \)-rational point (even infinitely many points). Thus, by Lemmas 3.2, 3.3, and 3.4 we can find a point \( v \in (U_1 \cap U_2 \cap U_3)(k) \subset Y(k) \subset (\mathbb{W} \times \cdots \times \mathbb{W})(k) \).

Set \( v(t) = j(v) \). Then the element \( v(t) \) satisfies conditions (1), (2), and (3) of Proposition \( \mathbb{I.1} \) by the very definition of the open sets \( U_1, U_2, \) and \( U_3 \). Since \( v(\theta) \stackrel{\text{def}}{=} ev(v(t)) \), we see that \( v(\theta) \) is a \( k \)-rational point of the variety \( R_{l/k}(X_l) \), i.e., \( v(\theta) \in X(l) \). Thus, \( \phi_l(v(\theta)) = 0 \), and condition (4) is also satisfied. To complete the proof of Proposition \( \mathbb{I.1} \) it remains to prove Lemmas 3.2, 3.3, and 3.4.
3.5. Proof of Lemma 3.2. Since \( Y \) is isomorphic to the variety \( R_{l/k}(X_l) \), it suffices to check the \( k \)-rationality of the latter. The quadric \( X_l \) has an \( l \)-rational point, so that \( X_l \) is an \( l \)-rational variety. This implies the existence of nonempty open subvarieties \( V_1 \) in \( X_l \) and \( V_2 \) in \( \mathbb{A}_{l/k}^{r-1} \) \((r = \dim_k W)\) and an isomorphism \( \alpha : V_1 \cong V_2 \) of \( l \)-varieties. Consider the diagram of \( l \)-varieties

\[
X_l \supset V_1 \cong V_2 \subset \mathbb{A}_{l/k}^{r-1}
\]

and apply the Weil restriction functor to this diagram. This leads to a diagram

\[
R_{l/k}(X_l) \supset R_{l/k}(V_1) \cong R_{l/k}(V_2) \subset R_{l/k}(\mathbb{A}_{l/k}^{r-1})
\]

of \( k \)-varieties. Since the left-hand and the right-hand side inclusions are open embeddings and since the variety \( R_{l/k}(\mathbb{A}_{l/k}^{r-1}) \) is an affine space over \( k \), we conclude that \( R_{l/k}(X_l) \) is \( k \)-rational. The lemma is proved.

3.6. Proof of Lemma 3.3. The morphism \( ev : Y \rightarrow R_{l/k}(X_l) \) is an isomorphism by the very definition of the variety \( Y \). Therefore, it suffices to check that the variety

\[
\mathcal{E}_{w}^{-1}(R_{l/k}(\mathbb{A}_{l/k}^{1} - \{0\}_l))
\]

is nonempty. We show that this variety has \( k \)-rational points. For this, we recall that the morphism

\[
\mathcal{E}_{w} : R_{l/k}(X_l) \rightarrow R_{l/k}(\mathbb{A}_{l/k}^{1})
\]

induces a map of \( k \)-rational points, which coincides with

\[
\rho_w : X(l) \rightarrow l \quad (v \mapsto (v, w)).
\]

Take any \( v \in X(l) \) with \( \langle w, v \rangle \in l^\times \) and observe that \( \rho_w(v) \in l^\times \) for this element \( v \) and moreover, the group \( l^\times \) coincides with the set of \( k \)-rational points of the variety \( R_{l/k}(\mathbb{A}_{l/k}^{1} - \{0\}_l) \). Thus, the element \( v \) is a \( k \)-rational point of the variety \( \mathcal{E}_{w}^{-1}(R_{l/k}(\mathbb{A}_{l/k}^{1} - \{0\}_l)) \). The lemma is proved.

3.7. Proof of Lemma 3.4. We show that already \( (U_1 \cap U_2)(l) \neq \emptyset \). For this, we consider an \( l \)-basis \( e_1, e_2, \ldots, e_r \) of the free \( l \)-module \( W_l \) such that \( e_1^2 = 0, e_2^2 = 0, (e_1, e_2) = 1 \), and \( (e_1, e_i) = (e_2, e_i) = 0 \) for \( i \geq 3 \). For any polynomial \( h(t) \in l[t]^{(n)} \) and any element \( e \in W_l \), we set

\[
h(t) \cdot e = (a_0 \cdot e) \cdot 1 + (a_1 \cdot e) \cdot t + \cdots + (a_{n-1} \cdot e) \cdot t^{n-1} \in l \otimes_k W_l[t]^{(n)} = W_l[t]^{(n)}.
\]

Since \( l = k[t]/(f(t)) \) and \( \theta = t \mod f(t) \) (see §1 for the definition of \( \theta \)), we have a unique decomposition

\[
f(t) = f^{(1)}(t)(t - \theta) \in l[t].
\]

Let \( g(t) \in k[t] \) be a separable polynomial of degree \( n - 2 \) that is coprime to \( f(t) \). Consider the element

\[
v(t) = f^{(1)}(t) \cdot e_1 + (t - \theta)g(t) \cdot e_2 \in l \otimes_k W_l[t]^{(n)} = W_l[t]^{(n)}.
\]

3.8. Claim. \( v(t) \in (U_1 \cap U_2)(l) \).

To check this, first we observe that

\[
(\ast) \quad \phi^{(n)}(v(t)) = 2 \cdot f^{(1)}(t)(t - \theta) \cdot g(t) \cdot (e_1, e_2) = 2f(t) \cdot g(t).
\]

Therefore, \( \phi(v(\theta)) = 2 \cdot f(\theta) \cdot g(\theta) = 0 \)
in \( l \), and therefore \( v(t) \in \mathbb{Y}(l) \).

Next, \( (\ast) \) shows that \( \phi^{(n)}(v(t)) \in k[t] \subset l[t] \) and it has degree \( 2n - 2 \). So, \( v(t) \in U_1(l) \).

Finally, \( \phi^{(n)}(v(t)) \) is separable, because \( f(t) \) and \( g(t) \) are separable and coprime polynomials in \( k[t] \). Thus, \( \text{Disc}(\phi^{(n)}(v(t))) \in k^\times \hookrightarrow l^\times \), whence \( v(t) \in U_2(l) \). So, \( (U_1 \cap U_2)(l) \ni v(t) \), and consequently, \( (U_1 \cap U_2)(l) \neq \emptyset \). The lemma is proved.
References


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