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RATIONALLY ISOTROPIC QUADRATIC SPACES ARE LOCALLY ISOTROPIC. III

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Dedicated to Professor S. V. Vostokov with great respect

ABSTRACT. Let R be a regular semilocal domain containing a field such that all the residue fields are infinite. Let K be the fraction field of R. Let $(R^n, q\colon R^n \to R)$ be a quadratic space over R such that the quadric $\{q = 0\}$ is smooth over R. If the quadratic space $(R^n, q\colon R^n \to R)$ over R is isotropic over K, then there is a unimodular vector $v \in R^n$ such that $q(v) = 0$. If $\operatorname{char}(R) = 2$, then in the case of even n the assumption on q is equivalent to the fact that q is a nonsingular quadratic space and in the case of odd $n > 2$ this assumption on q is equivalent to the fact that q is a semiregular quadratic space.

§1. INTRODUCTION

This paper extends the main results of [Pa, PaP] to the case of characteristic two. The new proof relies upon a new idea not involving resolution of singularities in any form.

Let k be an infinite field, possibly with $\operatorname{char}(k) = 2$, let X be a k-smooth irreducible affine scheme, and let $x_1, x_2, \ldots, x_s \in X$ be closed points. Let P be a free $k[X]$-module of rank $n > 0$. If n is odd, then let $(P, q\colon P \to k[X])$ be a semiregular quadratic module over $k[X]$ in the sense of [Kn, Chapter IV, §3]. If n is even, then let $(P, q\colon P \to k[X])$ be a quadratic space in the sense of [Kn, Chapter I, (5.3.5)].

Remark 1. In both cases, this is equivalent to saying that the X-scheme $Q := \{q = 0\} \subset \mathbf{P}^{n-1}_X$ is smooth over X.

Proof. For quadratic forms over a field, being regular or semiregular in the sense of [Kn, Ch. I, (5.3.5)] is equivalent to being nondegenerate in the sense of [EKM, §7A]. In this case, smoothness is equivalent to nondegeneracy by [EKM, Proposition 22.1]. To extend the assertion to quadratic forms over rings, one can use [Kn, Chapter IV, (3.1.5)]. □

Let $p\colon Q \to X$ be the projection. For a nonzero element $f \in k[X]$, let $Q_f = p^{-1}(X_f)$, where X_f is the principal open subset. Let $U = \operatorname{Spec}(\mathcal{O}_{X,\{x_1,x_2,\ldots,x_s\}})$. Set ${}_UQ = U \times_X Q$. For a k-scheme D equipped with k-morphisms $U \leftarrow D$ and $D \to X_f$, set ${}_DQ = {}_UQ \times_U D$ and $Q_{D,f} = D \times_{X_f} Q_f$.

Proposition 2. *If $n > 1$, then there exists a finite surjective étale k-morphism $U \leftarrow D$ of odd degree, a morphism $D \to X_f$, and an isomorphism of D-schemes ${}_DQ \xleftarrow{\bar\Phi_D} Q_{D,f}$.*

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Given this proposition, we may prove the following theorem.

Theorem 3. *Assume that $f \in k[X]$ is a nonzero element such that there is a section $s\colon X_f \to Q_f$ of the projection $Q_f \to X_f$. Then there is a section $s_U\colon U \to {}_UQ$ of the projection ${}_UQ \to U$.*

Proof of the Main Theorem. We give a proof of the theorem only in the local case and leave the semilocal case to the reader. So, $s = 1$, and we shall write x for x_1 and $\mathcal{O}_{X,x}$ for $\mathcal{O}_{X,\{x_1\}}$. If $f \in k[X] - m_x$, then there is nothing to prove. Now, let $f \in m_x$, then by Proposition 2 there is a finite surjective *étale* k-morphism $U \leftarrow D$ of odd degree, a morphism $D \to X_f$, and an isomorphism of the D-schemes ${}_DQ \xleftarrow{\bar{\Phi}_D} Q_{D,f}$.

The section s defines a section $s_D = (id, s)\colon D \to Q_{D,f}$ of the projection $Q_{D,f} \to D$. Next, $\bar{\Phi}_D \circ s_D\colon D \to {}_DQ$ is a section of the projection ${}_DQ \to D$. Finally, if $p_1\colon {}_DQ \to {}_UQ$ is the projection, then $p_1 \circ \bar{\Phi}_D \circ s_D\colon D \to {}_UQ$ is a U-morphism of U-schemes. Recall that $U \leftarrow D$ is a finite surjective *étale* k-morphism of odd degree and U is local with an infinite residue field. Therefore, by a version of Springer's theorem proved in [PR], there is a section $s_U\colon U \to {}_UQ$ of the projection ${}_UQ \to U$. (If $\operatorname{char}(k) = 2$, the proof of the version of Springer's theorem given in [PR] works well with a very mild modification). The theorem is proved. □

The Main Theorem has the following immediate consequences.

Corollary 4. *Let $\mathcal{O} = \mathcal{O}_{X,\{x_1,x_2,\dots,x_s\}}$ be a semilocal ring as above, and let $k(X)$ be the rational function field on X. Let P be a free $\mathcal{O}$-module of rank $n > 1$ and $q\colon P \to \mathcal{O}$ a form over a ring $\mathcal{O}$ as above, that is, the $\mathcal{O}$-scheme $Q := \{q = 0\} \subset \mathbf{P}^{n-1}_{\mathcal{O}}$ is smooth over $\mathcal{O}$. If the equation $q = 0$ has a nontrivial solution over $k(X)$, then it has a unimodular solution over $\mathcal{O}$.*

Corollary 5. *Let R be a semilocal regular domain containing a field, and let R be such that all the residue fields are infinite. Let K be the fraction field of R. Let P be a free R-module of rank $n > 1$ and $q\colon P \to R$ a quadratic form over R such that the R-scheme $Q := \{q = 0\} \subset \mathbf{P}^{n-1}_R$ is smooth over R. If the equation $q = 0$ has a nontrivial solution over K, then it has a unimodular solution over R.*

Corollary 6. *Let R be a semilocal regular domain containing a field, and let R be such that all the residue fields are infinite. Let K be the fraction field of R. Let P be a free R-module of even rank $n > 0$ and $q\colon P \to R$ a quadratic form over R such that the R-scheme $Q := \{q = 0\} \subset \mathbf{P}^{n-1}_R$ is smooth over R. Let $u \in R^\times$ be a unit. If u is represented by q over K, then u is represented by q already over R.*

If $1/2 \in R$, then the same holds true for a quadratic space of an arbitrary rank.

Proof. The proof is literally the same as the proof of the same result in the case of characteristic not equal to 2, given in [PaP, Corollary 3.3]. The even rank condition provides that in the course of the proof after adding a summand of rank 1 one obtains a semiregular quadratic form of odd rank. □

§2. Proof of Proposition 2

Proof. Since the conclusion of the proposition only depends on the affine scheme X locally in the Zariski topology, we may replace X by its appropriate open affine subscheme containing x. The following lemma is a corollary to Lemma 2.2.1 and Proposition 3.1.7. in [Kn, Chapter IV].

Lemma 7. *For $n > 1$, there exists an affine open subset X^0 containing x and a Galois étale cover $\overline{X^0} \xrightarrow{\rho} X^0$ such that the $k[X^0]$-module $P \otimes_{k[X]} k[X^0]$ is isomorphic to $k[X^0]^n$*

and $\rho^*(q)$ *is proportional to the quadratic form* $\perp_{i=1}^m T_iT_{i+m}$ *in the case of* $n = 2m$ *and is proportional to the semiregular quadratic form* $\perp_{i=1}^m T_iT_{i+m} \perp T_n^2$ *in the case of* $n = 2m + 1$.

Taking this lemma into account, we may assume the quadric Q to be locally split for the *étale* topology.

Further, by [PSV, Propositions 2.3 and 2.4] one can find an open X' in X containing x and an open affine $S \subset \mathbf{P}^{d-1}$ ($d = \dim(X)$) and a smooth morphism $r' \colon X' \to S$ turning X' into a smooth relative curve over S with geometrically irreducible fibers. Moreover, the morphism r' could be chosen so that

- its restriction $r'|_{X'\cap Z} \colon Z' = X' \cap Z \to S$ is a finite morphism, where Z is the vanishing locus of $f \in k[X]$;
- it factors as the composition $X' \xrightarrow{\Pi'} \mathbf{A}^1 \times S \xrightarrow{\mathrm{pr}_S} S$, where $\Pi' \colon X' \to \mathbf{A}^1 \times S$ is a finite surjective morphism.

Below we write X for X', Z for Z', $r \colon X \to S$ for $r' \colon X' \to S$, and $\Pi_S \colon X \to \mathbf{A}^1 \times S$ for $\Pi' \colon X' \to \mathbf{A}^1 \times S$.

Denote by $\mathcal{X} = U \times_S X \xrightarrow{r_U} U$ the base change of the morphism r under the composition $U \xrightarrow{\mathrm{can}} X \xrightarrow{r} S$, where *can*: $U \to X$ is a canonical inclusion. Obviously, this morphism r_U has a section $\Delta \colon U \to \mathcal{X}$ induced by *can*. Denote by $\Pi \colon \mathcal{X} \to \mathbf{A}^1 \times U$ the base change of the morphism Π_S under the map $U \to S$. Finally, denote by $f_U \in k[\mathcal{X}]$ the element $\mathrm{pr}_X^*(f)$, where $\mathrm{pr}_X \colon \mathcal{X} = U \times_S X \to X$ is the projection to the second factor.

One can easily check that $\big(r_U \colon \mathcal{X} \to U, f_U, \Delta\big)$ is a nice triple as defined in [PSV, Definition 3.1]. Actially, we merely recalled the construction of a *basic nice triple* from [PSV, §6]. Denote by ${}_U\mathcal{Q}$ the quadric over $\mathcal{X}$ that is the pullback of the quadric ${}_UQ$ under the morphism r_U. Denote by $\mathcal{Q}_X$ the quadric over $\mathcal{X}$ that is a pullback of a quadric Q under the morphism $\mathrm{pr}_X \colon \mathcal{X} \to X$.

Proposition 8. *There exist a finite étale covering* $\theta \colon \widetilde{\mathcal{X}} \to \mathcal{X}$ *turning* $\widetilde{\mathcal{X}}$ *into a relative curve with geometrically connected fibers over* U *such that*

- *there is a section* $\widetilde{\Delta} \colon U \to \widetilde{\mathcal{X}}$ *with* $\theta \circ \widetilde{\Delta} = \Delta$;
- *there is an isomorphism of* $\widetilde{\mathcal{X}}$*-schemes* $\Phi \colon \theta^*(\mathcal{Q}_X) \to \theta^*({}_U\mathcal{Q})$;
- *there is a finite surjective morphism* $\pi \colon \widetilde{\mathcal{X}} \to \mathbf{A}^1 \times U$ *over* U.

Proof. Since our X has been chosen to be sufficiently small, by Lemma 7 an isomorphism class of a quadric Q is attached to a certain cohomological class in $H^1_{et}(X, \mathbf{PGO}_n)$. Therefore, the quadrics $\mathcal{Q}_X$ and ${}_U\mathcal{Q}$ correspond to their classes $\alpha_X, \alpha_U \in H^1_{et}(\mathcal{X}, \mathbf{PGO}_n)$. In order to find an isomorphism between two $\widetilde{\mathcal{X}}$-schemes $\theta^*(\mathcal{Q}_X)$ and $\theta^*({}_U\mathcal{Q})$, it suffices to check that $\theta^*(\alpha_X) = \theta^*(\alpha_U)$ in $H^1_{et}(\widetilde{\mathcal{X}}, \mathbf{PGO}_n)$.

By [PSV, Theorem 3.6], for given classes $\alpha_X, \alpha_U \in H^1_{et}(\mathcal{X}, \mathbf{PGO}_n)$ there exists a morphism of nice triples $\big(\widetilde{r} \colon \widetilde{\mathcal{X}} \to U, \widetilde{f}, \widetilde{\Delta}\big) \xrightarrow{\theta} \big(r_U \colon \mathcal{X} \to U, f_U, \Delta\big)$ providing the required equality $\theta^*(\alpha_X) = \theta^*(\alpha_U)$. □

Taking into account the conclusion of the last proposition, we are under the hypotheses of Lemma 10 from Appendix A for the relative U-curve $\widetilde{\mathcal{X}}$ and its closed subset $\mathcal{Z}$ defined by an equation $f_U \circ \theta = 0$. (More precisely, then one should take the connected component $\widetilde{\mathcal{X}}^c$ of $\widetilde{\mathcal{X}}$ containing $\widetilde{\Delta}(U)$ and the closed subset $\mathcal{Z} \cap \widetilde{\mathcal{X}}^c$ of $\widetilde{\mathcal{X}}^c$.)

By Lemma 10, there exists an open subscheme $\mathcal{X}^0 \hookrightarrow \widetilde{\mathcal{X}}$ and a finite surjective morphism $\alpha \colon \mathcal{X}^0 \to \mathbf{A}^1 \times U$ such that α is *étale* over $0 \times U$ and $1 \times U$ and $\alpha^{-1}(0 \times U) = \widetilde{\Delta}(U) \coprod D_0$. Moreover, if we define D_1 as $\alpha^{-1}(1 \times U)$, then $D_1 \cap \mathcal{Z} = \varnothing$ and $D_0 \cap \mathcal{Z} = \varnothing$. One has $[D_1 \colon U] = [D_0 : U] + 1$. Thus, either $[D_1 : U]$ is odd or $[D_0 : U]$ is odd.

Without loss of generality we assume that $[D_1 : U]$ is odd, otherwise taking D_0 for D_1. Since $D_1 \cap \mathcal{Z} = \varnothing$, the image of the composition $D_1 \hookrightarrow \tilde{\mathcal{X}} \xrightarrow{\theta} \mathcal{X} \xrightarrow{\mathrm{pr}_X} X$ does not intersect Z. Therefore, we obtaine the morphism $D_1 \to X_f = X - Z$, the morphism $1 \times U \xleftarrow{\alpha|_{D_1}} D_1$ and the isomorphism $\bar{\Phi} := \Phi|_{D_1}$ satisfying the conclusion of Proposition 2 (here Φ is from Proposition 8). The proposition is proved.

Remark 9. At the end of the proof of Proposition 2, one can use the isomorpism Φ constructed in Proposition 11 instead of that from Proposition 8. The former proposition does not rely on [PSV, Theorem 3.6] and has an elementary proof which is valid, however, only when the characteristic of the ground field k is not equal to 2. □

§3. Appendix A: a version of the geometric lemma

Let k be an infinite field, X a k-smooth algebraic variety, $x \in X$ a point, $\mathcal{O} = \mathcal{O}_{X,x}$ the local ring, and let $U = \mathrm{Spec}(\mathcal{O})$. Let $\mathcal{X}/U$ be a U-smooth relative curve with geometrically connected fibres equipped with a finite surjective morphism $\pi\colon \mathcal{X} \to \mathbf{A}^1 \times U$ and with a section $\Delta\colon U \to \mathcal{X}$ of the projection $p\colon \mathcal{X} \to U$. Let $\mathcal{Z} \subset \mathcal{X}$ be a closed subset finite over U. The following lemma is a version of Lemma 5.1 in [OP].

Lemma 10. *There exists an open subscheme $\mathcal{X}^0 \hookrightarrow \mathcal{X}$ and a finite surjective morphism $\alpha\colon \mathcal{X}^0 \to \mathbf{A}^1 \times U$ such that α is étale over $0 \times U$ and $1 \times U$ and $\alpha^{-1}(0 \times U) = \Delta(U) \coprod D_0$. Moreover, such $\mathcal{X}^0$ and α can be chosen so that if we define D_1 as $\alpha^{-1}(1 \times U)$, then $D_1 \cap \mathcal{Z} = \varnothing$ and $D_0 \cap \mathcal{Z} = \varnothing$.*

Proof. Let $\bar{\mathcal{X}}$ be the normalization of the scheme $\mathbf{P}^1 \times U$ in the function field $k(\mathcal{X})$ of $\mathcal{X}$. Let $\bar{\pi}\colon \bar{\mathcal{X}} \to \mathbf{P}^1 \times U$ be the morphism. Let $\mathcal{X}_\infty = \bar{\pi}^{-1}(\infty \times U)$ be the set theoretic preimage of $\infty \times U$. Let $\bar{p}\colon \bar{\mathcal{X}} \to U$ be the structure map. Let $u \in U$ be the closed point, and let $\bar{X}_u = \bar{\mathcal{X}} \times_U u$.

Let $L' = \bar{\pi}^*(\mathcal{O}_{\mathbf{P}^1 \times U}(1))$, $L'' = \mathcal{O}_{\bar{\mathcal{X}}}(\Delta(U))$. Let $D_\infty = (\bar{\pi}^*)(\infty \times U)$ be the pull-back of the Cartier divisor $\infty \times U \subset \mathbf{P}^1 \times U$. Choose and fix a closed embedding $i\colon \bar{\mathcal{X}} \hookrightarrow \mathbf{P}^n \times U$ of U-schemes. Set $L = i^*(\mathcal{O}_{\mathbf{P}^n \times U}(1))$.

The sheaf L is very ample. Thus, the sheaf $L'' \otimes L$ is also very ample. So, there exists a closed embedding $i''\colon \bar{\mathcal{X}} \hookrightarrow \mathbf{P}^{n''} \times U$ of U-schemes such that $L'' \otimes L = (i'')^*(\mathcal{O}_{\mathbf{P}^{n''} \times U}(1))$. Using the Bertini theorem, we can choose a hyperplane $H'' \subset \mathbf{P}^{n''} \times U$ such that

(a'') $H'' \cap \Delta(U) = \varnothing$, $H'' \cap \mathcal{Z} = \varnothing$, $H'' \cap D_\infty = \varnothing$.

Define a Cartier divisor D'' on $\bar{\mathcal{X}}$ as the closed subscheme $H'' \cap \bar{\mathcal{X}}$ of $\bar{\mathcal{X}}$. Regard $D''_1 := D'' \coprod D_\infty$ as a Cartier divisor on $\bar{\mathcal{X}}$. Clearly, $\mathcal{O}_{\bar{\mathcal{X}}}(D''_1) = L'' \otimes L \otimes L'$.

The sheaf L is very ample. Thus, the sheaf $L' \otimes L$ is also very ample. So, there exists a closed embedding $i'\colon \bar{\mathcal{X}} \hookrightarrow \mathbf{P}^{n'} \times U$ of U-schemes such that $L' \otimes L = (i')^*(\mathcal{O}_{\mathbf{P}^{n'} \times U}(1))$. Using the Bertini theorem, we can choose a hyperplane $H' \subset \mathbf{P}^{n'} \times U$ such that

(a') $H' \cap \Delta(U) = \varnothing$, $H' \cap \mathcal{Z} = \varnothing$, $H' \cap D''_1 = \varnothing$;

(b') the scheme theoretic intersection $H' \cap \bar{X}_u$ is a $k(u)$-smooth scheme.

Define a Cartier divisor D' on $\bar{\mathcal{X}}$ as the closed subscheme $D' = H' \cap \bar{\mathcal{X}}$ of $\bar{\mathcal{X}}$.

Regard $D'_1 := D' \coprod \Delta(U)$ as a Cartier divisor on $\bar{\mathcal{X}}$. Clearly, $\mathcal{O}_{\bar{\mathcal{X}}}(D'_1) = L' \otimes L \otimes L''$.

Observe that D' is an essentially k-smooth scheme finite and *étale* over U. Let s' and s'' be global sections of $L' \otimes L \otimes L''$ such that the vanishing locus of s' is the Cartier divisor D'_1 and the vanishing locus of s'' is the Cartier divisor D''_1. Clearly, $D'_1 \cap D''_1 = \varnothing$. Thus, the morphism $f = [s' : s'']\colon \bar{\mathcal{X}} \to \mathbf{P}^1$ is regular elsewhere. Set

$$\bar{\alpha} = (f, \bar{p})\colon \bar{\mathcal{X}} \to \mathbf{P}^1 \times U.$$

Clearly, $\bar{\alpha}$ is a finite surjective morphism. Set $\mathcal{X}^0 = \bar{\alpha}^{-1}(\mathbf{A}^1 \times U)$ and

$$\alpha = \bar{\alpha}|_{\mathcal{X}^0}\colon \mathcal{X}^0 \to \mathbf{A}^1 \times U.$$

Clearly, α is a finite surjective morphism and $\mathcal{X}^0$ is an open subscheme of $\mathcal{X}$. Since α is a finite surjective morphism and $\mathcal{X}^0$ and $\mathbf{A}^1 \times U$ are regular schemes, the morphism α is flat by a theorem of Grothendieck. Since D_1' is finite *étale* over U, the morphism α is *étale* over $0 \times U$. So, we may choose a point $1 \in \mathbf{P}^1$ such that α is *étale* over $1 \times U$ and $(\alpha)^{-1}(1 \times U) \cap \mathcal{Z} = \varnothing$. If we set $D_0 = D_1'$, then $\alpha^{-1}(0 \times U) = \Delta(U) \amalg D_0$ and $D_0 \cap \mathcal{Z} = \varnothing$. The lemma is proved. □

§4. Appendix B

In this section we give an "elementary" proof of a version of Proposition 8 that is valid in characteristic not equal to 2 but does not require a reference to a general equating theorem, see [PSV, Theorem 3.6].

In the case of characteristic not equal to 2, a regular quadratic space is diagonalizable locally in the Zariski topology. Therefore, we may assume that

$$q(T_1e_1 + \cdots + T_ne_n) = a_1T_1^2 + \cdots + a_nT_n^2,$$

where $a_i \in k[X]^*$. Consider two quadratic spaces q_1 and q_2 over $\mathcal{X} = X \times_S X$ induced by the quadratic space q over X by projections on the first and the second factor, respectively.

Proposition 11. *There exist an étale covering $\theta\colon \widetilde{\mathcal{X}} \to \mathcal{X}$ such that*

- *quadratic spaces θ^*q_1 and θ^*q_2 over $\widetilde{\mathcal{X}}$ are isomorphic;*
- *there is a morphism $\widetilde{\Delta}\colon X \to \widetilde{\mathcal{X}}$ such that the composition $\theta \circ \widetilde{\Delta}$ coincides with the diagonal embedding $\Delta\colon X \to X \times_S X = \mathcal{X}$.*

Proof. We take $\widetilde{\mathcal{X}}$ to be an affine scheme such that its coordinate ring is equal to $k[X] \otimes_{k[S]} k[X]\big[\sqrt{a_1} \otimes \sqrt{a_1}, \dots, \sqrt{a_n} \otimes \sqrt{a_n}\big]$. Since $a_i \in k[X]^*$, it follows that $k[\widetilde{\mathcal{X}}]$ is an *étale* extension of $k[\mathcal{X}] = k[X] \otimes_{k[S]} k[X]$. The quadratic space $\theta^*(q_1)$ is written as $\perp_{i=1}^n (a_i \otimes 1)T_i^2$ in the chosen coordinates, and the quadratic space $\theta^*(q_2)$ is written as $\perp_{i=1}^n (1 \otimes a_i)T_i^2$. Since $(a_i \otimes 1) \cdot (1 \otimes a_i)$ is a square in $k[\widetilde{\mathcal{X}}]$, these quadratic spaces are isomorphic. Consider a fiber of the morphism θ over $\Delta(X)$:

$$k[\Delta] \otimes_{k[\tilde{\mathcal{X}}]} \big(k[X] \otimes_{k[S]} k[X]\big) = k[X][t_1, \dots, t_n]/\big(t_1^2 - a_1^2, \dots, t_1^2 - a_n^2\big) = \bigoplus_{r=1}^{2^n} k[X].$$

Choosing a summand, one constructs a suitable section $\widetilde{\Delta}$. □

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