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The editorial board congratulates the bright leader of the Leningrad/St. Petersburg school of partial differential equations Nina Nikolaevna Ural′tseva on her anniversary.

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*The American Mathematical Society scheme for transliteration of Cyrillic may be found at the end of index issues of *Mathematical Reviews*.

FOREWORD

This and the next issue of the *St. Petersburg Mathematical Journal* are dedicated to the anniversary of the glorious Petersburg mathematician Nina Nikolaevna Uraltseva, who is an outstanding expert in the fields of Partial Differential Equations and Calculus of Variations.

N. N. Uraltseva was a student of Olga Alexandrovna Ladyzhenskaya. Their joint monographs "Linear and quasilinear equations of elliptic type" (Moscow, 1964) and "Linear and quasilinear equations of parabolic type" (Moscow, 1967, in collaboration with V. A. Solonnikov) have been translated into several languages and are well known all over the world. Other works of Nina Nikolaevna have also received international recognition. Among them we indicate the results on the regularity of solutions to variational inequalities and degenerate quasilinear equations.

Special mention should be addressed to a series of papers by N. N. Uraltseva on the free boundary problems for elliptic and parabolic equations, awarded the P. L. Chebyshev Prize by the Government of St. Petersburg in 2017. Some of the results from this cycle are summarized in the monograph"Regularity of Free Boundaries in Obstacle-Type Problems" (AMS, Providence RI, 2012, jointly with A. Petrosyan and H. Shahgholian), which is also widely known.

Nina Nikolaevna plays a huge role in the development of the St. Petersburg school of Partial Differential Equations. Four of her students became Sci. Doctors, and nine became PhDs.

N. N. Uraltseva is Honorary Doctor of the St. Petersburg University and of the Royal Institute of Technology (Kungliga Tekniska Högskolan, Sweden), Honored Scientist of the Russian Federation, laureate of the P. L. Chebyshev Prize by Academy of Sciences of the USSR and of the USSR State Prize (together with O. A. Ladyzhenskaya), laureate of the Humboldt Research Award (Germany).

The personal charm and the benevolence of Nina Nikolaevna are noted by everyone who at least once fell into the field of her attraction.

The authors of the jubilee issues join the numerous congratulations of friends, colleagues and students, and wish Nina Nikolaevna good health and new interesting problems.

D. E. Apushkinskaya, A. I. Nazarov

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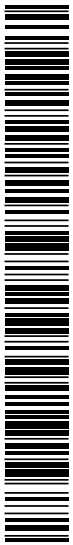
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