ESTIMATES FOR THE DISTRIBUTION OF THE SUPRENUM OF SQUARE-GAUSSIAN STOCHASTIC PROCESSES DEFINED ON NONCOMPACT SETS

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YU. V. KOZACHENKO AND T. V. FEDORYANICH

Abstract. Estimates for the distribution of the supremum of square-Gaussian stochastic processes defined on $\mathbb{R}^+$ are found in the paper. Using these results, we find estimates for the deviation in the uniform metric between the correlogram and the correlation function of a real stationary Gaussian stochastic process. A criterion for testing a hypothesis concerning the correlation function is also constructed.

1. Introduction

Throughout the paper we use properties of square-Gaussian stochastic processes. Square-Gaussian processes have been studied by many authors. For example, Kozachenko and Oleshko [1] obtained some results concerning the distribution of the supremum of such processes. Using the metric entropy, Kozachenko and Stadnik [2] found estimates for the distribution of the supremum for a wide class of stochastic processes including square-Gaussian processes. Similar estimates are obtained by Kurchenko [3] and Ponomarenko [4].

In the current paper we consider the problem of estimating the distribution of the supremum of square-Gaussian stochastic processes defined on noncompact sets. Using these results, we find estimates for the distribution of the deviation in the uniform metric in $(0, +\infty)$ between the correlogram and the correlation function for a real-valued Gaussian stationary stochastic process and construct a criterion for testing a hypothesis concerning the correlation function of the process on the interval $(a, b)$.

2. Estimates for the distribution of the supremum of square-Gaussian stochastic processes defined on a separable metric space

Let $\{\Omega, \mathcal{B}, P\}$ be a standard probability space.

Definition 2.1 [5]. Let $T$ be some set of parameters and $\Xi = \{\xi_t, t \in T\}$ a family of jointly Gaussian random variables with $E\xi_t = 0$ (for example, $\xi_t$ can be a Gaussian stochastic process). The family $SG_{\Xi}(\Omega)$ of random variables $\zeta \in SG_{\Xi}(\Omega)$ that are either of the form

$$\zeta = \xi^T A \xi - E\xi^T A \xi$$

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or are mean square limits of sequences of random variables \( \zeta_n \) of the form (1):

\[
\zeta_n = \xi_n^T A_n \xi_n - A_n \xi_n^T \xi_n, \quad n \geq 1,
\]

is called the space of square-Gaussian random variables, where \( \xi = (\xi_1, \ldots, \xi_N)^T \) is a Gaussian random vector for all \( N \geq 1 \), \( \mathbb{E} \xi = 0 \), the random variables \( \xi_i \), \( i = 1, \ldots, N \), belong to \( \Xi \), and \( A \) is a symmetric matrix.

**Definition 2.2** ([5]). A stochastic process \( \zeta = \{\zeta(t), t \in T\} \) is called square-Gaussian with respect to \( \Xi \) if the random variable \( \zeta(t) \), \( t \in T \), belongs to the space \( SG_{\Xi}(\Omega) \) and \( \sup_{t \in T} \mathbb{E} \zeta^2(t) < \infty \).

Let \( (T, m) \) be a compact metric space equipped with the metric \( m \) and let \( X = \{X(t), t \in T\} \) be a separable square-Gaussian stochastic process.

Assume that there exists a continuous increasing function \( \sigma = \{\sigma(h), h > 0\} \) such that \( \sigma(h) \to 0 \) as \( h \to 0 \) and

\[
\sup_{m(t,s)<h,t,s\in T} (\mathbb{E}(X(t) - X(s))^2)^{1/2} \leq \sigma(h).
\]

**Remark 2.1.** If a process \( X(t) \) is continuous in the norm of the space \( L_2 \), then the function

\[
\sigma(h) = \sup_{m(t,s)<h,t,s\in T} (\mathbb{E}(X(t) - X(s))^2)^{1/2}
\]

satisfies the above property if \( \sigma \) is continuous and increasing.

In what follows we use the following notation:

\[
\varepsilon_0 = \inf_{t \in T} \sup_{s \in T} m(t,s),
\]

\[
\delta_0 = \sup_{t \in T} (\mathbb{E}|X(t)|^2)^{1/2},
\]

\( \sigma^{-1}(h) \) is the inverse function to \( \sigma(u) \), \( t_0 = \sigma(\varepsilon_0) \), \( N(\varepsilon) \) is the minimal number of closed balls of radius \( \varepsilon \) that cover \( (T, m) \), and let \( r(u) > 0 \), \( u \geq 1 \), be an increasing function such that \( r(u) \to \infty \) as \( u \to \infty \) and \( r(u^t) \) is convex for \( t \geq 0 \).

Lemma 4.1 in [5] implies the following result.

**Theorem 2.1.** If

\[
\int_0^{t_0} r \left( \frac{u}{\sigma^{-1}(v)} \right) dv < \infty,
\]

then

\[
\mathbb{E} \exp \left\{ u \sup_{t \in T} |X(t)| \right\} \leq 2 \left( R \left( \frac{u \sqrt{2} \delta_0}{1 - p} \right) \right)^{1-p} \left( R \left( \frac{u \sqrt{2} t_0}{1 - p} \right) \right)^p \times r^{-1} \left( \frac{1}{t_0 p} \right) \int_0^{t_0} r \left( \frac{u}{\sigma^{-1}(v)} \right) dv
\]

for all \( p \) and \( u \) such that \( 0 < p < 1 \) and

\[
0 < u < \frac{1 - p}{\sqrt{2}} \min \left\{ \frac{1}{\delta_0}, \frac{1}{t_0} \right\},
\]

where

\[
R(z) = (1 - z)^{-1/2} \exp \left\{ \frac{-z}{2} \right\}, \quad 0 \leq z < 1.
\]
Corollary 2.1. Let the assumptions of Theorem 2.1 hold and \( z_0 = \max(\delta_0, t_0) \). Then

\[
\mathbb{E} \exp \left\{ u \sup_{t \in \mathbf{T}} |X(t)| \right\} \leq 2 R \left( \frac{\sqrt{2} z_0}{1 - p} \right)^{p-1} \left( \frac{1}{t_0 p} \right) r( N(\sigma^{(-1)}(v)) ) dv
\]

for

\[
0 < u \leq \frac{1 - p}{z_0 \sqrt{2}}.
\]

Proof. Inequality (4) follows from Theorem 2.1 since the function \( R(z) \) increases for \( 0 < z < 1 \).

Let \((\mathbf{T}, m)\) be a separable finite-dimensional metric space.

Assume that the space \((\mathbf{T}, m)\) can be represented as a countable union of compact sets \(B_k, k = 1, 2, \ldots\), that is,

\[
\mathbf{T} = \bigcup_{k=1}^{\infty} B_k.
\]

Consider a separable square-Gaussian stochastic process \(X = \{X(t), t \in \mathbf{T}\}\). Assume that there are increasing functions \(\sigma_k = \{\sigma_k(h), h > 0\}\) such that \(\sigma_k(h) \to 0\) as \(h \to 0\) and

\[
\sup_{m(t, s) < h, t, s \in B_k} (\mathbb{E}(X(t) - X(s))^2)^{1/2} \leq \sigma_k(h).
\]

Let

\[
\varepsilon_{0k} = \inf_{t \in B_k, s \in B_k} m(t, s),
\]

\[
\delta_{0k} = \sup_{t \in B_k} \left( \mathbb{E}|X(t)|^2 \right)^{1/2},
\]

\(\sigma_k^{(-1)}\) be the inverse functions to \(\sigma_k\), \(\tau_{0k} = \sigma_k(\varepsilon_{0k})\), \(z_{0k} = \max(\delta_{0k}, \tau_{0k})\). Let \(N_k(u)\) be the minimal number of closed balls of radius \(u\) that cover \(B_k\), and let \(r(u) > 0, u \geq 1\), be an increasing function such that \(r(u) \to \infty\) as \(u \to \infty\) and \(r(e^t)\) is convex for \(t \geq 0\).

Theorem 2.1 implies the following result.

Theorem 2.2. If

\[
\int_0^{\tau_{0k}} r \left( N_k(\sigma_k^{(-1)}(v)) \right) dv < \infty
\]

for all \(k\), then

\[
\mathbb{E} \exp \left\{ u \sup_{t \in B_k} |X(t)| \right\} \leq 2 R \left( \frac{\sqrt{2} \delta_{0k}}{1 - p} \right)^{p-1} \left( \frac{1}{\tau_{0k} p} \right) r( N_k(\sigma_k^{(-1)}(v)) ) dv
\]

for all \(p\) and \(u\) such that \(0 < p < 1\) and

\[
0 < u < \frac{1 - p}{\sqrt{2}} \min \left\{ \frac{1}{\delta_{0k}}, \frac{1}{\tau_{0k}} \right\},
\]

where \(R(z)\) is defined in (3) for \(0 \leq z < 1\).

Theorem 2.3. Let \(c(t), t \in \mathbf{T}\), be a continuous function such that \(c(t) > 0\) for all \(t\). Put

\[
\gamma_k = \sup_{t \in B_k} |c(t)|.
\]
If
\[ d = \sum_{k=1}^{\infty} \gamma_k z_{0k} < \infty, \]
\[ \int_0^{p_{t_{0k}}} r(N_k(\sigma_k^{(-1)}(v))) \, dv < \infty, \]
(3) for some \( 0 < p < 1, \)
\[ \prod_{k=1}^{\infty} \left( \frac{1}{p_{t_{0k}}} \int_0^{p_{t_{0k}}} r(N_k(\sigma_k^{(-1)}(v))) \, dv \right)^{\gamma_k z_{0k}/d} < \infty, \]
then
\[ E \exp \left\{ u \sup_{t \in T} |c(t)X(t)| \right\} \leq 2R \left( \frac{ud\sqrt{2}}{1 - p} \prod_{k=1}^{\infty} \left( \frac{1}{p_{t_{0k}}} \int_0^{t_{0k}} r(N_k(\sigma_k^{(-1)}(v))) \, dv \right)^{\gamma_k z_{0k}/d} \right) \]
for all \( 0 < u < \frac{1 - p}{d\sqrt{2}}. \)

**Proof.** It is obvious that
\[ \sup_{t \in T} |c(t)X(t)| \leq \sup_{k \in B_k} |c(t)| \cdot |X(t)| = \sum_{k=1}^{\infty} \gamma_k \sup_{t \in B_k} |X(t)|. \]

Then
\[ E \exp \left\{ u \sup_{t \in T} |c(t)X(t)| \right\} \leq E \exp \left\{ \sum_{k=1}^{\infty} \gamma_k \sup_{t \in B_k} |X(t)| \right\} \]
for \( u > 0. \) Let \( z_{0k} = \max(\delta_{0k}, t_{0k}). \) Since the function \( R(z) \) increases for \( 0 < z < 1, \) we have
\[ E \exp \left\{ u \sup_{t \in B_k} |X(t)| \right\} \leq 2R \left( \frac{u\sqrt{2}}{1 - p} \right) \left( \frac{1}{p_{t_{0k}}} \int_0^{t_{0k}} r(N_k(\sigma_k^{(-1)}(v))) \, dv \right) \]
for all \( k \) and
\[ 0 < u < \frac{1 - p}{\sqrt{2}} \frac{1}{z_{0k}} \]
by Theorem 2.2 and Corollary 2.1.

Let \( \{q_k\} \) be a sequence of real numbers such that \( q_k > 1 \) for all \( k \geq 1 \) and
\[ \sum_{k=1}^{\infty} q_k^{-1} = 1. \]
Bounds (4) and (5) together with the Hölder inequality imply that
\[
E \exp \left\{ u \sup_{t \in T} |c(t)X(t)| \right\} \leq E \prod_{k=1}^{\infty} \exp \left\{ u^{\gamma_k} \sup_{t \in B_k} |X(t)| \right\}
\]
\[
\leq \prod_{k=1}^{\infty} \left( E \exp \left\{ u^{\gamma_k} \sup_{t \in B_k} |X(t)| \right\} \right)^{1/q_k}
\]
\[
\leq \prod_{k=1}^{\infty} \left( 2R \left( \frac{u^{\gamma_k} q_k \sqrt{2} \sigma_{20k}}{1 - p} \right) r(-1) \left( \frac{1}{t_0 k^p} \int_0^{t_0 k^p} r \left( N_k (\sigma_k^{-1}(v)) \right) dv \right) \right)^{1/q_k}
\]
\[
= \prod_{k=1}^{\infty} 2^{1/q_k} \prod_{k=1}^{\infty} \left( R \left( \frac{u^{\gamma_k} q_k \sqrt{2} \sigma_{20k}}{1 - p} \right) \right)^{1/q_k}
\]
\[
\times \prod_{k=1}^{\infty} \left( r(-1) \left( \frac{1}{t_0 k^p} \int_0^{t_0 k^p} r \left( N_k (\sigma_k^{-1}(v)) \right) dv \right) \right)^{1/q_k}
\]
for \( u > 0 \) such that \( 0 < u^{\gamma_k} q_k < (1 - p)/(\sqrt{2} \sigma_{20k}) \), \( k = 1, 2, \ldots \). The latter bound holds for any \( u \) such that
\[
0 < u < \frac{1 - p}{\sqrt{2} \sigma_{20k} q_k}
\]
for all \( k = 1, 2, \ldots \). Put
\[
d = \sum_{k=1}^{\infty} \gamma_k z_{20k}
\]
and choose
\[
q_k = \frac{d}{\gamma_k z_{20k}} \quad \text{such that} \quad q_k > 1 \quad \text{and} \quad \sum_{k=1}^{\infty} q_k^{-1} = 1.
\]
Then
\[
R \left( u \frac{q_k \gamma_k z_{20k} \sqrt{2}}{1 - p} \right) = R \left( u \frac{d \sqrt{2}}{1 - p} \right)
\]
and
\[
\prod_{k=1}^{\infty} \left( R \left( u \frac{q_k \gamma_k z_{20k} \sqrt{2}}{1 - p} \right) \right)^{1/q_k} = \left( R \left( u \frac{d \sqrt{2}}{1 - p} \right) \right)^{\sum_{k=1}^{\infty} 1/q_k} = R \left( u \frac{d \sqrt{2}}{1 - p} \right)
\]
for any \( u \) such that
\[
0 < u < \frac{1 - p}{d \sqrt{2}}
\]
Thus
\[
E \exp \left\{ u \sup_{t \in T} |c(t)X(t)| \right\} \leq 2R \left( u \frac{d \sqrt{2}}{1 - p} \right) \prod_{k=1}^{\infty} \left( r(-1) \left( \frac{1}{t_0 k^p} \int_0^{t_0 k^p} r \left( N_k (\sigma_k^{-1}(v)) \right) dv \right) \right)^{\gamma_k z_{20k}/d}
\]
for
\[
0 < u < \frac{1 - p}{d \sqrt{2}}. \quad \square
\]
Theorem 2.4. If the assumptions of Theorem 2.3 hold, then
\[
P \left\{ \sup_{t \in \mathcal{T}} |c(t)X(t)| > x \right\} \leq 2 \exp \left\{ -x(1-p) \frac{d\sqrt{2}}{d\sqrt{2}} \right\} \left( 1 + \frac{\sqrt{2}x(1-p)}{d\sqrt{2}} \right)^{1/2} \tilde{\Phi}(p)
\]
for arbitrary \( x > 0 \) and for the same \( 0 < p < 1 \) as in Theorem 2.3, where
\[
\tilde{\Phi}(p) = \prod_{k=1}^{\infty} r^{(-1)} \left( \frac{1}{t_{0k}p} \right) \int_{0}^{t_{0k}p} r \left( N_k \left( \sigma_k(-1)(v) \right) \right) dv \gamma_{az0k/d}.
\]

Proof. We obtain from Theorem 2.3 and the Chebyshev inequality that
\[
P \left\{ \sup_{t \in \mathcal{T}} |c(t)X(t)| > x \right\} \leq \frac{E \exp \left\{ \frac{u \sup_{t \in \mathcal{T}} |c(t)X(t)|}{u x} \right\}}{\exp \left\{ \frac{u x}{2(1-p)} \right\}} \exp \left\{ -u x \tilde{\Phi}(p) \right\}
\]
for all \( x > 0 \) and
\[
0 < u < \frac{1-p}{d\sqrt{2}}.
\]
Put
\[
D = \frac{d\sqrt{2}}{1-p}.
\]
Then
\[
P \left\{ \sup_{t \in \mathcal{T}} |c(t)X(t)| > x \right\} \leq Z(u, x) \tilde{\Phi}(p)
\]
for \( 0 < u < D^{-1} \), where
\[
Z(u, x) = 2(1-uD)^{-1/2} \exp \left\{ \frac{-u}{2}(D + 2x) \right\}.
\]
It is easy to check that this function attains its minimum in the interval \( 0 < u < D^{-1} \) at the point
\[
u = \frac{1}{D} - \frac{1}{D + 2x} < \frac{1}{D}.
\]
Therefore
\[
\inf_{0 < u < D^{-1}} Z(u, x) = 2 \exp \left\{ -\frac{x}{D} \right\} \left( \frac{D}{D + 2x} \right)^{-1/2}.
\]
This relation completes the proof of the theorem. \(\square\)

3. Estimates for the uniform deviation between the correlogram and correlation function of a Gaussian stationary stochastic process

Let \( \xi = \{ \xi(t), t \geq 0 \} \) be a real-valued, mean-square continuous, stationary, Gaussian stochastic process with \( E \xi(t) = 0 \) and the correlation function \( \rho(\tau) = E \xi(t + \tau)\xi(t) \). To estimate \( \rho(\tau) \) we use the correlogram
\[
\hat{\rho}_T(\tau) = \frac{1}{T} \int_{0}^{T} \xi(t + \tau)\xi(t) \, dt.
\]
This estimator is unbiased:
\[
E \hat{\rho}_T(\tau) = \rho(\tau).
\]
Assume that the spectral density of the stochastic process \( \xi(t) \) exists and denote it by
\[
f = \{ f(\lambda), \lambda \in \mathbb{R} \}.
\]
Assume further that \( f \in L_2(\mathbb{R}) \). In other words, \( f \) is square integrable, that is,
\[
\int_{-\infty}^{+\infty} f^2(\lambda) \, d\lambda < \infty.
\]
This implies that the correlation function \( \rho(\tau) \) of the stochastic process \( \xi \) is also square-integrable, that is,
\[
\| \rho \|_2^2 = \int_{-\infty}^{+\infty} \rho^2(\tau) \, d\tau < \infty.
\]

Let
\[
X(T, \tau) = \hat{\rho}_T(\tau) - \rho(\tau).
\]
Note that \( X(T, \tau) \) is a square-Gaussian stochastic process. We estimate the moments \( E(X(T, \tau))^2 \) and \( E(X(T, \tau) - X(T_1, \tau_1))^2 \).

Define the space \( (T, m) \) as follows:
\[
T = \{ (T, \tau) : A < T < \infty, a \leq \tau \leq b, 0 \leq a < b, A > 0 \},
\]
\[
m((T', \tau'), (T'', \tau'')) = \max\{|T' - T''|, |\tau' - \tau''|\}.
\]

**Lemma 3.1.** Assume that
\[
(9) \quad \int_{-\infty}^{+\infty} f^2(\lambda) \, d\lambda < \infty.
\]

Then
\[
\sup_{(T, \tau) \in T} E(X(T, \tau))^2 = \frac{C_1}{T},
\]

where
\[
C_1 = \left(1 + \sqrt{2}\right) \| \rho \|_2^2.
\]

**Proof.** Since
\[
\int_0^T \int_0^T f(t-s) \, dt \, ds = 2 \int_0^T (T-u)f(u) \, du
\]
for every even function \( f \), we obtain from the Isserlis formula \([7]\) that
\[
E(X(T, \tau))^2 = E\hat{\rho}^2(\tau) - (E\hat{\rho}(\tau))^2
\]
\[
= \frac{2}{T^2} \int_0^T (T-u)(\rho^2(u) + \rho(u-\tau)\rho(u+\tau)) \, du.
\]

It follows from \( \rho \in L_2(\mathbb{R}) \) that
\[
\int_0^{+\infty} \rho^2(u) \, du = \frac{1}{2} \int_{-\infty}^{+\infty} \rho^2(u) \, du < \infty
\]
as \( \rho(\tau) \) is even. Then
\[
\int_0^{+\infty} \rho(u-\tau)\rho(u+\tau) \, du \leq \sqrt{2} \int_0^{+\infty} \rho^2(v) \, dv < \infty
\]
for \( \tau > 0 \), whence
\[
E(X(T, \tau))^2 \leq \frac{2 + 2\sqrt{2}}{T} \int_0^{+\infty} \rho^2(u) \, du = \frac{(1 + \sqrt{2}) \| \rho \|_2^2}{T}.
\]
\( \square \)
Lemma 3.2. Let

\[ T = \bigcup_{k=1}^{\infty} B_k, \]

where

\[ B_k = \{(T, \tau) : T_k \leq T \leq T_{k+1}, a \leq \tau \leq b\} \]

is such that \( T_k < T_{k+1}, T_{k+1} - T_k \to \text{const} > 1, \) and \( T_k \to \infty \) as \( k \to \infty. \)

Let \( (T, \tau) \in B_k \) be such that \( T \leq T' \). Then

\[ \sup_{(T, \tau), (T', \tau') \in B_k, m((T, \tau), (T', \tau')) < h} \left( \frac{\sigma}{\mu} \int \left( X(T, \tau) - X(T', \tau') \right)^2 \right)^{1/2} \leq \sigma_k(h), \]

where

\[ \sigma_k(h) = \frac{C_1^{1/2}}{T_k^{1/2} \left( \ln \left( e^{C/2} + C/h \right) \right)^{\alpha/2}}, \]

\( C > 0 \) is an arbitrary constant,

\[ \bar{f} = \int_{-\infty}^{+\infty} f_2(\lambda) \left( \ln(\alpha + C|\lambda|/2) \right)^{2\alpha} d\lambda, \]

\[ C_2 = 8\pi \left[ \ln \left( \frac{\alpha}{b-a} \right) \right]^{-\alpha} + \|f\|_2^{1/2} \]

\[ + 2\|\rho\|_2^2 \left( 1 + \frac{6T_k + \frac{T_k^2}{T_k}}{T_k^2} \right) \frac{T_k - T_{k+1}}{T_k} \left( \ln \left( e^{\alpha + \frac{1}{T_k}} \right) \right)^{\alpha}. \]

Proof. Let \( (T, \tau) \in B_k \) and \( (T', \tau') \in B_k \) be such that \( T \leq T' \). Then

\[ \int \left( X(T, \tau) - X(T', \tau') \right)^2 \]

\[ \leq \left[ \frac{1}{T^2} \int_0^T \int_0^T \rho^2(t-s) + \rho(t-s+\tau) \rho(t-s-\tau) \right] dt ds \]

\[ - \frac{2}{T^2} \int_0^T \int_0^T \rho(t-s) \rho(t-s+\tau+\tau') + \rho(t-s+\tau) \rho(t-s-\tau') \right] dt ds \]

\[ + \frac{1}{T^2} \int_0^T \int_0^T \rho^2(t-s) + \rho(t-s+\tau') \rho(t-s-\tau') \right] dt ds \]

\[ + \frac{2}{T^2} \int_0^T \int_0^T \rho(t-s) \rho(t-s+\tau+\tau') + \rho(t-s+\tau) \rho(t-s-\tau') \right] dt ds \]

\[ - \frac{2}{TT'} \int_0^T \int_0^{T'} \rho(t-s) \rho(t-s+\tau+\tau') + \rho(t-s+\tau) \rho(t-s-\tau') \right] dt ds \]

\[ + \frac{1}{T^2} \int_0^T \int_0^{T'} \rho^2(t-s) + \rho(t-s+\tau') \rho(t-s-\tau') \right] dt ds \]

\[ - \frac{1}{T^2} \int_0^T \int_0^{T'} \rho^2(t-s) + \rho(t-s+\tau') \rho(t-s-\tau') \right] dt ds \]

\[ = I + A_1 + A_2. \]
Now we estimate $A_1$, $A_2$, and $I$. We have

$$A_1 = \left| \frac{2}{T^2} \int_0^T \int_0^T \left[ \rho(t-s) \rho(t-s+\tau-\tau') + \rho(t-s+\tau) \rho(t-s-\tau') \right] \, dt \, ds \right| - \frac{2}{TT'} \int_0^T \int_0^{T'} \left[ \rho(t-s) \rho(t-s+\tau-\tau') + \rho(t-s+\tau) \rho(t-s-\tau') \right] \, dt \, ds$$

$$\leq \left| \frac{2}{T^2} \int_T^{T'} \left( \int_0^T \rho^2(t-s) \, dt \int_0^T \rho^2(t-s+\tau-\tau') \, dt \right)^{1/2} \, ds \right|$$

$$+ \int_T^{T'} \left( \int_0^T \rho^2(t-s+\tau) \, dt \int_0^T \rho^2(t-s-\tau') \, dt \right)^{1/2} \, ds$$

$$+ \left( \frac{2}{T^2} - \frac{2}{TT'} \right) \left[ \int_0^T \left( \int_0^{T'} \rho^2(t-s) \, dt \int_0^{T'} \rho^2(t-s+\tau-\tau') \, dt \right)^{1/2} \, ds \right]$$

$$+ \int_0^T \left( \int_0^{T'} \rho^2(t-s+\tau) \, dt \int_0^{T'} \rho^2(t-s-\tau') \, dt \right)^{1/2} \, ds \right|$$

$$\leq \frac{4}{T^2} [T'-T] \int_{-\infty}^{+\infty} \rho^2(u) \, du + \left| \frac{2}{T^2} - \frac{2}{TT'} \right| 2T \int_{-\infty}^{+\infty} \rho^2(u) \, du$$

$$= \frac{4(T'+T) [T'-T]}{T^2 T'} \left\| \rho \right\|_2^2$$

$$\leq 4 \left\| \rho \right\|_2^2 \frac{2T_{k+1}}{T_k} \frac{|T'-T|}{T^2}.$$

Similarly

$$A_2 = \left| \frac{1}{T^2} \int_0^T \int_0^{T'} \left[ \rho^2(t-s) + \rho(t-s+\tau') \rho(t-s-\tau') \right] \, dt \, ds \right| \left| \frac{1}{T^2} \int_0^{T'} \int_0^{T} \left[ \rho^2(t-s) + \rho(t-s+\tau') \rho(t-s-\tau') \right] \, dt \, ds \right|$$

$$\leq \left| \frac{2}{T^2} \int_T^{T'} \left( \int_0^T \rho^2(t-s) \, dt \int_0^{T'} \rho^2(t-s+\tau') \, dt \right)^{1/2} \, ds \right|$$

$$\leq \left| \frac{2}{T^2} \int_T^{T'} \left( \int_0^T \rho^2(t-s) \, dt \int_0^{T'} \rho^2(t-s+\tau') \, dt \right)^{1/2} \, ds \right|$$

$$\leq \left| \frac{2}{T^2} \int_T^{T'} \left( \int_0^T \rho^2(t-s) \, dt \int_0^{T'} \rho^2(t-s+\tau') \, dt \right)^{1/2} \, ds \right|$$

$$\leq \left| \frac{2}{T^2} \int_T^{T'} \left( \int_0^T \rho^2(t-s) \, dt \int_0^{T'} \rho^2(t-s+\tau') \, dt \right)^{1/2} \, ds \right|$$

$$\leq \left| \frac{2}{T^2} \int_T^{T'} \left( \int_0^T \rho^2(t-s) \, dt \int_0^{T'} \rho^2(t-s+\tau') \, dt \right)^{1/2} \, ds \right|$$
Using a lemma in [7] we get

\[ E(X(T, \tau) - X(T', \tau'))^2 \leq \frac{8\pi}{T} \left( \left[ \int_{-\infty}^{+\infty} f^2(\lambda) \sin^2 \left( \frac{\lambda(\tau - \tau')}{2} \right) d\lambda \right] + \|f\|_2 \left[ \int_{-\infty}^{+\infty} f^2(\lambda) \sin^2 \left( \frac{\lambda(\tau - \tau')}{2} \right) d\lambda \right]^{1/2} \right), \]

where

\[ \|f\|_2 = \int_{-\infty}^{+\infty} f^2(\lambda) d\lambda < \infty. \]

Since

\[ \left| \sin \frac{\lambda(\tau - \tau')}{2} \right| \leq \left( \frac{\ln(e^\alpha + u/2)}{\ln(e^\alpha + v/2)} \right)^\alpha \]

for all \( u \geq 0, v > 0, \) and \( \alpha > 0 \) (see [8]), we have

\[ \left| \sin \frac{\lambda(\tau - \tau')}{2} \right| \leq \left( \frac{\ln(e^\alpha + C/|\tau - \tau'|)}{\ln(e^\alpha + C/|\tau - \tau'|)^2} \right)^\alpha, \]

where \( C > 0 \) is an arbitrary constant. Then we rewrite (12) in the following form:

\[ I \leq \frac{8\pi}{T} \left( \int_{-\infty}^{+\infty} f^2(\lambda) \left( \ln \left( e^\alpha + \frac{C|\lambda|}{2} \right) \right)^{2\alpha} d\lambda \left( \frac{1}{\ln(e^\alpha + C/|\tau - \tau'|)^2} \right)^\alpha \right) + \|f\|_2 \left( \int_{-\infty}^{+\infty} f^2(\lambda) \left( \ln \left( e^\alpha + \frac{C|\lambda|}{2} \right) \right)^{2\alpha} d\lambda \right)^{1/2} \left( \frac{1}{\ln(e^\alpha + C/|\tau - \tau'|)^2} \right)^{1/2}. \]

Inequality (10) implies that

\[ \tilde{f} = \int_{-\infty}^{+\infty} f^2(\lambda) \left( \ln \left( e^\alpha + \frac{C|\lambda|}{2} \right) \right)^{2\alpha} d\lambda < \infty. \]

Thus

\[ I \leq \frac{\tilde{C}_2}{T \left( \ln \left( e^\alpha + C/|\tau - \tau'| \right) \right)^\alpha}, \]

where

\[ \tilde{C}_2 = 8\pi \left[ \tilde{f} \left( \ln \left( e^\alpha + \frac{C}{b - a} \right) \right)^{-\alpha} + \|f\|_2 \tilde{f}^{1/2} \right]. \]

Hence

\[ E(X(T, \tau) - X(T', \tau'))^2 \leq \frac{\tilde{C}_2}{T \left( \ln \left( e^\alpha + C/|\tau - \tau'| \right) \right)^\alpha} + \frac{C^* |T' - T|}{T^2} \]

for

\[ C^* = 2 \|\rho\|_2 \left( 1 + \frac{6T_{k+1}T_k}{T_k^2} + \frac{T_{k+1}^2}{T_k^2} \right). \]

Since

\[ \sup_{h < T_{k+1} - T_k} \left( \ln \left( e^\alpha + \frac{1}{h} \right) \right)^{\alpha} = \left( T_{k+1} - T_k \right) \left( \ln \left( e^\alpha + \frac{1}{T_{k+1} - T_k} \right) \right)^{\alpha}, \]

we obtain

\[ \sup_{(T, \tau), (T', \tau') \in B_k, m((T, \tau), (T', \tau')) < h} \left( E(X(T, \tau) - X(T', \tau'))^2 \right)^{1/2} \leq \sigma_k(h), \]
where
\[ \sigma_k(h) = \frac{C_2^{1/2}}{T_k^{1/2}} (\ln (e^a + C/h))^{\alpha/2}, \]
\[ C_2 = \tilde{C}_2 + C \frac{T_{k+1} - T_k}{T_k} \left( \ln \left( e^a + \frac{1}{T_{k+1} - T_k} \right) \right)^\alpha. \]

Keeping the above notation, we have

(1) \[ \varepsilon_{ok} = \inf_{(T', \tau') \in B_k} \sup_{(T'', \tau'') \in B_k} m((T', \tau'), (T'', \tau'')) \]
\[ = \max \{(T_{k+1} - T_k)/2, (b - a)/2\}, \]

(2) \[ \delta_{ok} = \sup_{(T, \tau) \in B_k} \mathbb{E}(X(T, \tau))^{1/2} = C_1^{1/2}/T_k^{1/2}; \]

(3) \[ \sigma_k^{(-1)}(v) \text{ is the inverse function for } \sigma_k(h), \]
\[ \sigma_k^{(-1)}(v) = \frac{C}{\exp \left\{ (C_2/(v^2 T_k))^{1/\alpha} \right\} - \exp \{\alpha\}}. \]

(4) \[ t_{ok} = \sigma_k(\varepsilon_{ok}), \]

(5) \[ z_{ok} = \max \{t_{ok}, \delta_{ok}\}, \]

(6) \[ N_k(\varepsilon) \text{ is the minimal number of closed balls of radius } \varepsilon \text{ that cover } B_k. \]

In what follows we choose \( C = \sqrt{b - a} \) and assume that
\[ T_{k+1} - T_k > b - a > 1. \]

**Lemma 3.3.** Let
\[ X(T, \tau) = \hat{\rho}_T(\tau) - \rho(\tau) \]
and let \( c = \{c(T), T \in [A; +\infty)\} \) be a continuous function such that \( c(T) > 0 \). Put
\[ \gamma_k = \max_{T \in [T_k; T_{k+1}]} c(T). \]

If
\[ \sum_{k=1}^{\infty} \gamma_k z_{ok} \ln(T_{k+1} - T_k) < \infty, \text{ and } \]
\[ \int_{-\infty}^{+\infty} F^2(\lambda) \left( \ln(1 + |\lambda|) \right)^{2\alpha} d\lambda < \infty \text{ for some } \alpha > 2, \]

then
\begin{align*}
\mathbb{E} \exp \left\{ u \sup_{(T, \tau) \in T} |c(T) X(T, \tau)| \right\} \\
\leq 2R \left( \frac{ud\sqrt{2}}{1 - p} \right) \exp \left\{ \frac{1}{d} \sum_{k=1}^{\infty} \gamma_k z_{ok} \ln(T_{k+1} - T_k) + \frac{2\tilde{P}}{p^{2/\alpha} (1 - 2/\alpha)} \right\}
\end{align*}

for all real numbers \( p \) and \( u \) such that \( 0 < p < 1 \) and
\[ 0 < u < \frac{1 - p}{d\sqrt{2}}, \]

where
\[ d = \sum_{k=1}^{\infty} \gamma_k z_{ok}, \quad \tilde{P} = \sup_k \left( \ln \left( e^a + 2\sqrt{b - a}/(T_{k+1} - T_k) \right) \right). \]
Proof. Let

\[ r(v) = (\ln v)^f, \quad v > e, \ 1 < f < \alpha/2. \]

Since

\[ r(xy) = (\ln x + \ln y)^f \leq 2^{f-1} (\ln x)^f + (\ln y)^f \]

and

\[ N_k(\sigma^{-1}_k(v)) \leq \left( \frac{T_{k+1} - T_k}{2 \sigma_k^{-1}(v)} + 1 \right) \left( \frac{b - a}{2 \sigma_k^{-1}(v)} + 1 \right) \leq \frac{(T_{k+1} - T_k)(b - a)}{(\sigma_k^{-1}(v))^2} \]

for \( v < t_{0k} \), we have

\[
\frac{1}{p_{0k}} \int_0^{p_{0k}} r \left( N_k(\sigma_k^{-1}(v)) \right) dv \leq \frac{1}{p_{0k}} \int_0^{p_{0k}} r \left( \frac{(b - a)(T_{k+1} - T_k)}{(\sigma_k^{-1}(v))^2} \right) dv \\
\leq \frac{1}{p_{0k}} \int_0^{p_{0k}} \left( \ln \left( \frac{(b - a)(T_{k+1} - T_k)}{C^2} \right) f \cdot \frac{2^f}{p_{0k}} \frac{C_2^{f/\alpha}}{T_k^{f/\alpha}} \int_0^{p_{0k}} v^{-2f/\alpha} dv \right) dv \\
= 2^{f-1} \left[ \left( \ln \left( \frac{(b - a)(T_{k+1} - T_k)}{C^2} \right) f \right) + \frac{2^f}{p_{0k}} \frac{C_2^{f/\alpha}}{T_k^{f/\alpha}} \int_0^{p_{0k}} v^{-2f/\alpha} dv \right] \\
= 2^{f-1} \left[ \left( \ln \left( \frac{(b - a)(T_{k+1} - T_k)}{C^2} \right) f \right) + \frac{2^f C_2^{f/\alpha}}{T_k^{f/\alpha} p_{0k}^{1-2f/\alpha}} \right] \\
= 2^{f-1} \left[ \left( \ln \left( \frac{(b - a)(T_{k+1} - T_k)}{C^2} \right) f \right) + \frac{2^f C_2^{f/\alpha}}{T_k^{f/\alpha} (p_{0k}^{-2f/\alpha})} \right].
\]

Since

\[ t_{0k} = \sigma_k(\varepsilon_{0k}) = \frac{C_2^{1/2}}{T_k^{1/2} \left( e^{\alpha} + \frac{2C}{T_{k+1} - T_k} \right)^{\alpha/2}}, \]

we get

\[
\frac{1}{p_{0k}} \int_0^{p_{0k}} r \left( N_k(\sigma_k^{-1}(v)) \right) dv \\
\leq 2^{f-1} \left[ \left( \ln(T_{k+1} - T_k) \right)^f + \frac{2^f C_2^{f/\alpha}}{T_k^{f/\alpha} p_{0k}^{2f/\alpha} \left( 1 - \frac{2f}{\alpha} \right)} \frac{T_k^{f/\alpha} \left( e^{\alpha} + \frac{2\sqrt{b-a}}{T_{k+1} - T_k} \right)^f}{C_2^{f/\alpha}} \right] \\
= 2^{f-1} \left[ \left( \ln(T_{k+1} - T_k) \right)^f + \frac{2^f C_2^{f/\alpha}}{p_{0k}^{2f/\alpha} \left( 1 - \frac{2f}{\alpha} \right)} \left( e^{\alpha} + \frac{2\sqrt{b-a}}{T_{k+1} - T_k} \right)^f \right]
\]

for \( 1 < f < \alpha/2 \).
Note that $\ln r^{-1}(z) = z^{1/f}$. Letting $f \uparrow 1$ we obtain

$$
\prod_{k=1}^{\infty} \left( r^{-1} \left( \frac{1}{p_{0k}} \int_0^{p_{0k}} r \left( N_k(\sigma_k^{-1}(v)) \right) \, dv \right) \right)^{\gamma_k z_{0k}/d}
$$

$$
= \exp \left\{ \sum_{k=1}^{\infty} \frac{\gamma_k z_{0k}}{d} \left( \frac{1}{p_{0k}} \int_0^{p_{0k}} r \left( N_k(\sigma_k^{-1}(v)) \right) \, dv \right)^{1/f} \right\}
$$

$$
\leq \exp \left\{ 2^{1-1/f} \sum_{k=1}^{\infty} \frac{\gamma_k z_{0k}}{d} \left[ \ln(T_{k+1} - T_k) + \frac{2}{p^{2/\alpha} (1 - 2/\alpha)^{1/f}} \left( \ln \left( e^\alpha + \frac{2\sqrt{b-a}}{T_{k+1} - T_k} \right) \right) \right] \right\}
$$

$$
\leq \exp \left\{ \frac{1}{d} \sum_{k=1}^{\infty} \gamma_k z_{0k} \ln(T_{k+1} - T_k) + \frac{2\tilde{P}}{p^{2/\alpha} (1 - 2/\alpha)} \right\},
$$

where

$$
\tilde{P} = \sup_k \left( \ln \left( e^\alpha + \frac{2\sqrt{b-a}}{T_{k+1} - T_k} \right) \right).
$$

The latter inequality together with (1) completes the proof of the theorem. \(\square\)

**Theorem 3.1.** Let $X(T, \tau) = \hat{\rho}_T(\tau) - \rho(\tau)$ and let $c = \{c(T), T \in [A; +\infty)\}$ be a continuous function such that $c(T) > 0$. Put

$$
\gamma_k = \max_{T \in [T_k, T_{k+1}]} c(T).
$$

If

1. $\sum_{k=1}^{\infty} \gamma_k z_{0k} \ln(T_{k+1} - T_k) < \infty$, and
2. $\int_{-\infty}^{+\infty} f^2(\lambda) (\ln(1 + |\lambda|))^{2\alpha} \, d\lambda < \infty$ for some $\alpha > 2$,

then

$$
P \left\{ \sup_{(T, \tau) \in T} |c(T)X(T, \tau)| > x \right\}
$$

$$
\leq 2e \exp \left\{ -\frac{x}{d\sqrt{2}} + \frac{2\tilde{P}}{d} \left( \frac{x}{d\sqrt{2}} \right)^{1/2} \right\} \Phi_5,
$$

where

$$
d = \sum_{k=1}^{\infty} \gamma_k z_{0k}, \quad \tilde{P} = \sup_k \left( \ln \left( e^\alpha + \frac{2\sqrt{b-a}}{T_{k+1} - T_k} \right) \right),
$$

$$
\Phi_5 = \exp \left\{ \frac{1}{d} \sum_{k=1}^{\infty} \gamma_k z_{0k} \ln(T_{k+1} - T_k) \right\}
$$

for any $x > d\sqrt{2}$. 
Proof. The above inequality can easily be proved by substituting $p = d\sqrt{2}/x$, $x > d\sqrt{2}$, in (13) and applying the Chebyshev inequality and Theorem 2.4. Indeed

$$\begin{align*}
P\left\{ \sup_{(T,\tau)\in\mathbf{T}} |c(T)X(T,\tau)| > x \right\} & \leq 2\exp\left\{ -\frac{x(1-p)}{d\sqrt{2}} + \frac{x\sqrt{2}}{d} \right\} \exp\left\{ \frac{2\tilde{P}}{p^{2/\alpha}(1-2/\alpha)} \right\} \tilde{\Phi}_5 \\
& = 2\exp\left\{ -\frac{x}{d\sqrt{2}} \left( 1 - \frac{d\sqrt{2}}{x} \right) \right\} \left( 1 + \frac{x\sqrt{2}}{d} \left( 1 - \frac{d\sqrt{2}}{x} \right) \right)^{1/2} \tilde{\Phi}_5 \\
& \times \exp\left\{ \frac{2\tilde{P}}{(1-2/\alpha)} \left( \frac{x}{d\sqrt{2}} \right)^{2/\alpha} \right\} \tilde{\Phi}_5 \\
& \leq 2e\exp\left\{ -\frac{x}{d\sqrt{2}} + \frac{2\tilde{P}}{(1-2/\alpha)} \left( \frac{x}{d\sqrt{2}} \right)^{2/\alpha} \right\} \left( \frac{x\sqrt{2}}{d} \right)^{1/2} \tilde{\Phi}_5. \quad \square
\end{align*}$$

Theorem 3.2. Let a function $c(T) = T^{1/2}/(\ln T)^{\beta}$ be defined for all $T > e^m$ (here $m > 4$ is a fixed number) and let $2 < \beta < m/2$. If

$$\int_{-\infty}^{+\infty} f^2(\lambda) (\ln(1+|\lambda|))^{2\alpha} d\lambda < \infty$$

for some $\alpha > 2$, then

$$P\left\{ \sup_{(T,\tau)\in\mathbf{T}} |c(T)X(T,\tau)| > x \right\} \leq 2e\exp\left\{ -\frac{x}{d\sqrt{2}} + D_\alpha \left( \frac{x}{d\sqrt{2}} \right)^{2/\alpha} \right\} \left( \frac{x\sqrt{2}}{d} \right)^{1/2} D$$

for any $x > d\sqrt{2}$, where

$$D_\alpha = \frac{2}{(1-2/\alpha)} \left( \ln e^{\alpha} + \frac{2\sqrt{\alpha}}{\pi^{3/2}(e-1)} \right), \quad D = \exp\left\{ \sum_{k=1}^{\infty} \frac{1}{(m+k+1)^{\beta-1}} \right\},$$

$$d = C_0 e^{1/2} \sum_{k=1}^{\infty} \frac{1}{(m+k+1)^{\beta-1}},$$

and $C_0$ is a known constant.

Proof. Theorem 3.2 follows from Theorem 3.1. Note that the function $c(T) > 0$ is increasing for $\beta < \ln T/2$. Since $\beta > 2$, one can choose $A = e^m$ for some $m > 4$. Now we check the assumptions of Theorem 3.1 and obtain bounds for the probability

$$P\left\{ \sup_{(T,\tau)\in\mathbf{T}} |c(T)X(T,\tau)| > x \right\}.$$

First we choose the points $T_k$ of the partition such that $T_k = e^{m+k}$, $k = 1, 2, \ldots$. It is clear that

$$T_{k+1} - T_k = e^{m+k}(e-1) > 1.$$
The correlation function of a stochastic process.

\[ m > T \text{ for some } \]

whence a stochastic process with correlation function and assume that \( m > T > e \) where \( \xi \), Assume that the stochastic process \( \xi \) is real-valued, mean square continuous, stationary, Gaussian.

Theorem 3.2 allows one to construct a criterion for testing a hypothesis concerning the correlation function of a stochastic process.

4. We treat the correlogram \( \hat{\rho}_T(\tau) \) as an estimator for the correlation function and assume that \( T > e^m \) for some \( m > 4 \). We treat the correlogram \( \hat{\rho}_T(\tau) \) as an estimator for the correlation function and assume that \( T > e^m \) for some \( m > 4 \).

Thus

\[ d = \sum_{k=1}^{\infty} \gamma_k z_{ok} = C_0 e^{1/2} \sum_{k=1}^{\infty} \frac{1}{(m+k+1)^{\beta}} < \infty \text{ for } \beta > 1, \]

that is, assumption 1) of Theorem 3.1 holds.

Now we estimate \( \tilde{\Phi}_5 \) and \( \tilde{P} \). We have

\[ \tilde{\Phi}_5 = \exp \left\{ \frac{1}{d} \sum_{k=1}^{\infty} \gamma_k z_{ok} \ln(T_{k+1} - T_k) \right\} \leq \exp \left\{ \sum_{k=1}^{\infty} \frac{1}{(m+k+1)^{\beta}} \right\}, \]

\[ \tilde{P} = \sup_{k \geq m} \left( \ln \left( e^\alpha + \frac{2\sqrt{b-a}}{e^m(e-1)} \right) \right) = \left( \ln \left( e^\alpha + \frac{2\sqrt{b-a}}{e^m(e-1)} \right) \right), \]

whence

\[ P \left\{ \sup_{(T, \tau) \in T} |c(T)X(T, \tau)| > x \right\} \leq 2e \exp \left\{ \frac{1}{d} \right\} \left( \ln \left( e^\alpha + \frac{2\sqrt{b-a}}{e^m(e-1)} \right) \right) \]

for

\[ D_\alpha = \frac{2}{(1 - 2/\alpha)} \left( \ln \left( e^\alpha + \frac{2\sqrt{b-a}}{e^m(e-1)} \right) \right), \quad D = \exp \left\{ \sum_{k=1}^{\infty} \frac{1}{(m+k+1)^{\beta}} \right\}. \]

Theorem 3.2 allows one to construct a criterion for testing a hypothesis concerning the correlation function of a stochastic process.

Let \( \xi = \{\xi(t), t \geq 0\} \) be a real-valued, mean square continuous, stationary, Gaussian stochastic process with \( E \xi(t) = 0 \). Denote its spectral density by \( f(\lambda) \) and its correlation function by

\[ \rho(\tau) = E \xi(t + \tau)\xi(t), \quad a \leq \tau \leq b. \]

Assume that the stochastic process \( \xi \) is observed on the interval \([0, \bar{T} + b]\), where \( \bar{T} > e^m \) for some \( m > 4 \). We treat the correlogram \( \hat{\rho}_T(\tau) \) as an estimator for the correlation function and assume that \( T > e^m \) for some \( m > 4 \).
Let \( H \) be the hypothesis that the correlation function of the stochastic process \( \xi(t) \) coincides with \( \rho(\tau) \) for all \( a \leq \tau \leq b \). To test the hypothesis \( H \) we propose the following criterion.

**Criterion 3.1.** Given \( \gamma, 0 < \gamma < 1 \), find \( x_\gamma \), such that

\[
A(x_\gamma) = 2e \exp \left\{ -\frac{x_\gamma^2}{d\sqrt{2}} + D_\alpha \left( \frac{x_\gamma}{d\sqrt{2}} \right)^{2/\alpha} \right\} \left( \frac{x_\gamma\sqrt{2}}{d} \right)^{1/2} = \gamma,
\]

where \( \alpha > 2 \) satisfies

\[
\int_{-\infty}^{+\infty} f^2(\lambda)(\ln(1 + |\lambda|))^{2\alpha} d\lambda < \infty,
\]

\[
D_\alpha = 2 \left( \frac{\ln(\epsilon^\alpha + \frac{2\sqrt{\beta-1}}{\epsilon^\alpha})}{(1 - 2/\alpha)} \right), \quad D = \exp \left\{ \frac{\sum_{k=1}^\infty 1}{\sum_{k=1}^\infty (m+k+1)^\beta} \right\}, \quad 2 < \beta < \frac{m}{2},
\]

and the number \( d \) is defined in Theorem 3.2. The hypothesis \( H \) is accepted if

\[
\sup_{\epsilon^m < T < T, a \leq \tau \leq b} \frac{T^{1/2}}{\ln T} |\hat{\rho}_T(\tau) - \rho(\tau)| < x_\gamma
\]

for \( \epsilon^m < T < \tilde{T} \) and \( a \leq \tau \leq b \). Otherwise the hypothesis is rejected.

Note that the error of the first kind does not exceed \( \gamma \) for this criterion.

4. **Concluding remarks**

The bounds for the distribution of the supremum of a square Gaussian process are found and a criterion for testing a hypothesis pertaining to the correlation function of a stationary Gaussian stochastic process is constructed in this paper. The criterion is useful for sufficiently large \( T \) (say, for \( T > \epsilon^m \), where \( m > 4 \)). Its error of the first kind does not exceed \( \gamma \). If one simultaneously uses Criterion 3.1 and other criteria constructed in other papers (say, in [9]), then one can decrease essentially the error of the second kind when testing hypotheses concerning the correlation function of a Gaussian stochastic process by using observations of its trajectory on a finite interval of an arbitrary length.

**Bibliography**


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