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MIXED EMPIRICAL POINT RANDOM PROCESSES IN COMPACT METRIC SPACES. II

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YU. I. PETUNIN AND M. G. SEMEĬKO

Abstract. Models of finite simple mixed empirical ordered marked point processes in compact metric spaces are studied in the paper. The processes are constructed from simple samples drawn without replacement from a population. The notion of an ordered marked point process with independent and 1-dependent marks is introduced. Examples of ordered marked point processes with independent and 1-dependent marks are given.

Introduction

A model of a finite simple mixed empirical marked point process is studied in Section 4. We assume that the space of positions X is equipped with a probability measure P_x, while the space of marks $K = [a, b] \subset R^1$ is equipped with a probability measure P_k. An arbitrary trajectory of an ordered marked point process is treated as a simple random sample drawn without replacement from a population $Y = X \times K$ and according to the probability measure $P_{\overline{y}} = P_x \otimes P_k$. We introduce the notion of ordered marked point processes with independent and 1-dependent marks in Section 5. Examples of ordered marked point processes with independent and 1-dependent marks are given in Section 6.

4. Mixed empirical ordered marked point processes in compact metric spaces

We recall the definition of a trajectory introduced in Section 1.[1] According to this definition, a trajectory E^* of a finite simple ordered marked point process $(\mathcal{E}^*, \mathfrak{X}^*, P^*)$ in a bounded space

$$(Y = X \times K, \mathfrak{A}_Y = \mathfrak{A}_X \otimes \mathfrak{A}_K, \mathfrak{B}_Y = \mathfrak{B}_X \odot \mathfrak{B}_K)$$

is a thinned set of the Cartesian product $Y = X \times K$. If X is a compact metric space and the space of marks K coincides with an interval $[a, b] \subset R^1$, then the trajectory E^* consists of a finite number of points, namely

$$E^* = ([x_1; k_1], \dots, [x_i; k_i], \dots, [x_n; k_n]).$$

The phase space $Y = X \times K$ can be considered to be a compact metric space if the distance between points $[x_1; k_1]$ and $[x_2; k_2]$ is defined by

$$\rho([x_1; k_1], [x_2; k_2]) = \rho_X(x_1, x_2) + |k_1 - k_2|.$$

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[1]Editorial Note: This paper is a continuation of [7], with successive numbering of sections and formulas.

In what follows we assume that X is the population for the random variable x equipped with the probability measure P_x defined on the σ-algebra $\mathfrak{A}_X$. The space K is regarded as the population for the random variable k equipped with the probability measure P_k defined on the σ-algebra $\mathfrak{A}_K$. Therefore $(X, \mathfrak{A}_X, P_x)$ and $(K, \mathfrak{A}_K, P_k)$ are the sampling spaces for the random variables x and k.

We introduce the product of the probability measures $P_{\overline{y}} = P_x \otimes P_k$ on the σ-algebra of Borel sets $\mathfrak{A}_Y = \mathfrak{A}_X \otimes \mathfrak{A}_K$ in the phase space $Y = X \times K$. Then $(Y, \mathfrak{A}_Y, P_{\overline{y}})$ can be viewed as the sampling probability space of the two-dimensional random variable $\overline{y} = [x; k]$. This means that the model of a mixed empirical ordered point process studied in Section 3 of [7] can be applied to the ordered marked point process $(\mathcal{E}^*, \mathfrak{X}^*, P^*)$ in the ordered space $(Y = X \times K, \mathfrak{A}_Y, \mathfrak{B}_Y)$ if X is a compact metric space and $K = [a, b]$.

Let G_1 and G_2 be two independent stochastic experiments corresponding to the probability spaces $(X, \mathfrak{A}_X, P_x)$ and $(K, \mathfrak{A}_K, P_k)$, respectively. Then $\overline{G} = (G_1, G_2)$ is a "compound" stochastic experiment corresponding to the probability space $(Y, \mathfrak{A}_Y, P_{\overline{y}})$. A number $n \in Z_+$ is drawn randomly (according to the probability distribution determined by the generating sequence $\{p_n^*\}$). Any trajectory

$$E^* = ([x_1; k_1], \dots, [x_i; k_i], \dots, [x_j; k_j], \dots, [x_n; k_n])$$

of a size n of the ordered marked point process is a result of n independent "compound" stochastic experiments $\overline{G} = (G_1, G_2)$. Each experiment $\overline{G}$ is a random sampling without replacement of a marked pair $[x; k]$ from the phase space $Y = X \times K$, where the positions x_i belong to the space X (this corresponds to the experiment G_1), while the marks k_i belong to the space K (this corresponds to the experiment G_2). Then the trajectory E^* can be viewed as a simple sample ($x_i \neq x_j$, $k_i \neq k_j$, $i \neq j$) of a finite size n drawn randomly and without replacement from the population $Y = X \times K$ for the two-dimensional random variable $\overline{y} = [x; k]$ with the joint probability measure $P_{\overline{y}} = P_x \otimes P_k$. Thus the random elements

$$[x_1; k_1], \dots, [x_i; k_i], \dots, [x_n; k_n]$$

are independent and identically distributed according to the probability measure $P_{\overline{y}}$. Moreover the mark k_i does not depend on the position x_i for any marked pair $[x_i; k_i]$, $i = 1, \dots, n$. The projection of the ordered marked point process $(\mathcal{E}^*, \mathfrak{X}^*, P^*)$ to the space of positions X is a simple mixed empirical ordered point process

$$(\mathcal{E}, \mathfrak{X}, P) = \mathrm{pr}_X(\mathcal{E}^*, \mathfrak{X}^*, P^*)$$

in the bounded space of positions $(X, \mathfrak{A}_X, \mathfrak{B}_X)$ (see [1]) whose trajectories are simple samples

$$E = \mathrm{pr}_X E^* = (x_1, \dots, x_i, \dots, x_j, \dots, x_n), \qquad x_i \neq x_j,\ i \neq j,$$

of size n drawn from the population X for the random variable x with probability measure P_x, while the projection of the ordered marked point process to the space of marks K consists of trajectories $(k_1, \dots, k_i, \dots, k_j, \dots, k_n)$, $k_i \neq k_j$, $i \neq j$, that can be viewed as samples of size n drawn randomly and without replacement from the population $K = [a, b]$ for the random variable k with probability measure P_k.

The process $(\mathcal{E}^*, \mathfrak{X}^*, P^*)$ is called a strong simple mixed empirical ordered marked point process in the ordered space $(Y, \mathfrak{A}_y, \mathfrak{B}_y)$ (see Section 1).

The joint probability distribution $P_{\overline{y}}$ of an arbitrary marked pair, denoted also by $\overline{y} = [x; k]$ and belonging to the trajectory E^* on the metric space $Y = X \times K$, can be expressed in terms of the probability measure P^* of the ordered marked point process

$(\mathcal{E}^*, \mathfrak{X}^*, P^*)$. Namely,

$$P_{\overline{y}}(B_Y) = P_{[x;k]}(B_X \times B_K) = P_{[x;k]}\{[x;k] \in B_X \times B_K\}$$
$$(8) \qquad = P^*\{E^* = ([x;k])\colon [x;k] \in B_X \times B_K \mid \mathcal{E}_1^*\}$$
$$= P^*\{E^* = ([x;k])\colon N^*(E^*, B_X \times B_K) = 1 \mid \mathcal{E}_1^*\}$$

for all $B_X \in \mathfrak{C}_X$ and $B_K \in \mathfrak{C}_K$.

If one considers relation (8) for $B_K \equiv K$ and then for $B_X \equiv X$, then one obtains the marginal probability distributions $P_x(B_X)$ and $P_k(B_K)$ of the random variable (position) x on the metric space X and random variable (mark) k on the interval $K = [a, b]$ in terms of the probability measure P^*. Indeed,

$$P_x(B_X) = P_x\{x \in B_X\} = P_{[x;k]}(B_X \times K)$$
$$= P^*\{E^* = ([x;k])\colon N^*(E^*, B_X \times K) = 1 \mid \mathcal{E}_1^*\},$$
$$P_k(B_K) = P_k\{k \in B_K\} = P_{[x;k]}(X \times B_K)$$
$$= P^*\{E^* = ([x;k])\colon N^*(E^*, X \times B_K) = 1 \mid \mathcal{E}_1^*\}$$

for all $B_X \in \mathfrak{C}_X$ and $B_K \in \mathfrak{C}_K$.

This means that $P_x(B_X)$ is a probability distribution of the position x if its mark is $k \in K$, while $P_k(B_K)$ is the probability distribution of the mark k if its position is $x \in X$.

Since $P_{\overline{y}}$ is the product of the measures P_x and P_k, the probability

$$l = P_{\overline{y}}(B_X \times B_K) = P_{[x;k]}\{[x;k] \in B_X \times B_K\} = P_{[x;k]}(B_X \times K)P_{[x;k]}(X \times B_K)$$
$$= P_x(B_X)P_k(B_K) = pr$$

is well defined for an arbitrary rectangle $B_X \times B_K \in \mathfrak{C}_Y$, where

$$p = P_x(B_X) \quad \text{and} \quad r = P_k(B_K).$$

If n is fixed ($N^*(E^*, Y) = n$, $n \in Z_+$), the conditional distribution of the counting measure $N^*(E^*, B_X \times B_K)$, $B_X \times B_K \in \mathfrak{C}_Y$, of the ordered marked point process $(\mathcal{E}^*, \mathfrak{X}^*, P^*)$ is the binomial $B(n, l)$ distribution with parameter $l = P_{\overline{y}}(B_X \times B_K)$, that is,

$$(9) \quad P^*\{E^*\colon N^*(E^*, B_X \times B_K) = k \mid \mathcal{E}_n^*\} = \binom{n}{k} l^k (1-l)^{n-k} = \binom{n}{k} (pr)^k (1-pr)^{n-k},$$

where $k = 0, 1, \ldots, n$.

Using (2), (8), and conditional distribution (9), we evaluate the distribution of the counting measure $N^*(E^*, B_X \times B_K)$:

$$P^*\bigl\{E^*\colon N^*(E^*, B_X \times B_K) = k\bigr\} = \sum_{n=k}^{\infty} p_n^* \binom{n}{k} (pr)^k (1-pr)^{n-k},$$

where $k \in Z_+$ and $\{p_n^*\}$ is a generating sequence of the distribution P^*.

5. Finite simple ordered marked point processes with independent and 1-dependent marks

Consider a finite simple ordered marked point process $(\mathcal{E}^*, \mathfrak{X}^*, P^*)$ in the ordered space

$$(Y = X \times K, \mathfrak{A}_Y = \mathfrak{A}_X \otimes \mathfrak{A}_K, \mathfrak{B}_Y = \mathfrak{B}_X \odot \mathfrak{B}_K),$$

where X is a compact metric space of positions. The space of marks $K = [a, b] \subset R^1$ is treated as the population of marks k with an absolutely continuous distribution P_k. Looking at the trajectory $E^* = ([x_1; k_1], \ldots, [x_n; k_n])$ of an ordered marked point process, one can see a difference between the sequence $E = (x_1, \dots, x_n)$ of positions and that of marks $(k_1, \dots, k_n)$. Namely, if E is a realization of a finite simple ordered point process

$$(\mathcal{E}, \mathfrak{X}, P) = \mathrm{pr}_X(\mathcal{E}^*, \mathfrak{X}^*, P^*)$$

in the ordered space of positions $(X, \mathfrak{A}_X, \mathfrak{B}_X)$ (see [1]), then $(k_1, \dots, k_n)$ is a simple random sample of a random size n drawn from the population K.

Given an arbitrary number $n_0 \in \mathbb{N}$, vectors $(x_1, \dots, x_{n_0}) = E \subset \mathcal{E}_{n_0}$, and sets $B_K^{(1)}, \dots, B_K^{(n_0)} \subset \mathfrak{B}_K$, where $B_K^{(i)} B_K^{(j)} = \varnothing$, $i, j = 1, \dots, n_0$, $i \neq j$, one can evaluate the conditional joint distribution

$$P_{k_1 \dots k_{n_0}}(B_K^{(1)}, \dots, B_K^{(n_0)} \mid \mathcal{E}_{n_0})$$

of marks $k_1, \dots, k_{n_0}$ corresponding to given positions $(x_1, \dots, x_{n_0}) = E$, namely

$$\begin{aligned} P_{k_1 \dots k_{n_0}}\left(B_K^{(1)}, \dots, B_K^{(n_0)} \mid \mathcal{E}_{n_0}\right) &= P_{k_1 \dots k_{n_0}}\left(B_K^{(1)}, \dots, B_K^{(n_0)} \mid (x_1, \dots, x_{n_0})\right) \\ &= P_{k_1 \dots k_{n_0}}\left\{k_1 \in B_K^{(1)}, \dots, k_{n_0} \in B_K^{(n_0)} \mid (x_1, \dots, x_{n_0})\right\} \\ &= P^*\Big\{E^* \colon N^*\big(E^*, X \times B_K^{(1)}\big) = 1, \dots, \\ &\qquad N^*\big(E^*, X \times B_K^{(n_0)}\big) = 1 \mid (x_1, \dots, x_{n_0})\Big\} \\ &= \mu\left(B_K^{(1)}, \dots, B_K^{(n_0)} \mid x_1, \dots, x_{n_0}\right), \end{aligned} \tag{10}$$

where $\mu\big(B_K^{(1)}, \dots, B_K^{(n_0)} \mid x_1, \dots, x_{n_0}\big)$ is a probability measure defined on the σ-algebra $\mathfrak{A}_K^{(n_0)} = \mathfrak{A}_K \times \cdots \times \mathfrak{A}_K$ (n_0 times) and where $x_1, \dots, x_{n_0}$ are elements of the σ-algebra $\mathfrak{A}_X$. Putting $B_K^{(j)} \equiv K$ in (10) for all $j = 1, \dots, n_0$, $j \neq i$, we get the conditional probability distribution of the mark k_i, $i = 1, \dots, n_0$:

$$\begin{aligned} P_{k_i}\left(B_K^{(i)} \mid \mathcal{E}_{n_0}\right) &= P_{k_i}\left\{k_i \in B_K^{(i)} \mid (x_1, \dots, x_{n_0})\right\} \\ &= P_{k_1 \dots k_{i-1} k_i k_{i+1} \dots k_{n_0}}\left(K, \dots, K, B_K^{(i)}, K, \dots, K \mid (x_1, \dots, x_{n_0})\right) \\ &= P^*\left\{E^* \colon N^*\big(E^*, X \times B_K^{(i)}\big) = 1 \mid (x_1, \dots, x_{n_0})\right\} = \mu_i\left(B_K^{(i)} \mid x_1, \dots, x_{n_0}\right), \end{aligned}$$

where $\mu_i\big(B_K^{(i)} \mid x_1, \dots, x_{n_0}\big)$ is a probability measure defined on the σ-algebra $\mathfrak{A}_K$ and where $x_1, \dots, x_{n_0}$ are elements of the σ-algebra $\mathfrak{A}_X$. Given an arbitrary set $B_K^{(i)} \in \mathfrak{B}_K$, the function $\mu_i\big(B_K^{(i)} \mid x_1, \dots, x_{n_0}\big)$, $x_1, \dots, x_{n_0} \in X$, is measurable with respect to the σ-algebra $\mathfrak{A}_X$ (see [5]).

If the neighboring positions

$$(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_{n_0})$$

have little influence on the mark k_i, $i = 1, \dots, n_0$, that is, if the mark k_i does not depend on the neighboring positions, then

$$\begin{aligned} &P^*\left\{E^* \colon N^*\big(E^*, X \times B_K^{(i)}\big) = 1 \mid (x_1, \dots, x_{n_0})\right\} \\ &\qquad = P^*\left\{E^* \colon N^*\big(E^*, X \times B_K^{(i)}\big) = 1 \mid x_i\right\} = \mu_i\left(B_K^{(i)} \mid x_i\right), \end{aligned}$$

where

$$\mu_i\left(B_K^{(i)} \mid x_i\right) = P_{k_i}\left(B_K^{(i)} \mid x_i\right) = P_{k_i}\left\{k_i \in B_K^{(i)} \mid x_i\right\}$$

is the probability measure of the transformation of the space X to the space K (see [2]), that is, μ_i is the probability distribution of the mark k_i that depends on its position x_i.

Also if the influence of positions $(x_1, \dots, x_{n_0})$ on the mark k_i, $i = 1, \dots, n_0$, is rather inessential, that is, if the mark k_i does not depend on positions $(x_1, \dots, x_{n_0})$, then

$$\begin{aligned} P^* &\left\{E^* \colon N^*\left(E^*, X \times B_K^{(i)}\right) = 1 \mid (x_1, \dots, x_{n_0})\right\} \\ &= P^*\left\{E^* \colon N^*(E^*, X \times B_K^{(i)}) = 1\right\} = \mu_i\left(B_K^{(i)}\right), \end{aligned}$$

where

$$\mu_i\left(B_K^{(i)}\right) = P_{k_i}\left(B_K^{(i)}\right) = P_{k_i}\left\{k_i \in B_K^{(i)}\right\}$$

is a probability distribution of the mark k_i, $i = 1, \dots, n_0$.

Some applications of marked point processes in stochastic geometry are based on the assumption that the marks are identically distributed random variables (see [4, 6]). Note that this property is involved in the construction of a mixed empirical ordered marked point process (see Section 4). Moreover

$$\mu_i\left(B_K^{(i)} \mid x_1, \dots, x_{n_0}\right) = \mu\left(B_K^{(i)} \mid x_1, \dots, x_{n_0}\right), \qquad i = 1, \dots, n_0,$$

$$\mu_i\left(B_K^{(i)} \mid x_i\right) = \mu\left(B_K^{(i)} \mid x_i\right), \qquad i = 1, \dots, n_0,$$

$$\mu_i\left(B_K^{(i)}\right) = \mu\left(B_K^{(i)}\right), \qquad i = 1, \dots, n_0.$$

Definition 14. A finite simple ordered marked point process $(\mathcal{E}^*, \mathfrak{X}^*, P^*)$ in a bounded space $(X \times K, \mathfrak{A}_X \otimes \mathfrak{A}_K, \mathfrak{B}_X \odot \mathfrak{B}_K)$ is called a process with independent μ-marks if

$$B_K^{(1)}, \dots, B_K^{(n_0)} \subset \mathfrak{B}_K$$

for all $n_0 \in \mathbb{N}$ where

$$B_K^{(i)} B_K^{(j)} = \varnothing, \qquad i, j = 1, \dots, n_0,\ i \neq j,$$

and the marks $k_1, \dots, k_{n_0}$ of a random n_0-dimensional vector $E^* = ([x_i; k_i] \colon i = 1, \dots, n_0)$ are jointly independent random variables in the space K, have the probability distribution $\mu(B_K)$, and are such that $k_1, \dots, k_{n_0}$ do not depend on their positions $(x_1, \dots, x_{n_0})$ in the space X,

$$\mu\left(B_K^{(1)}, \dots, B_K^{(n_0)} \mid x_1, \dots, x_{n_0}\right) = \prod_{i=1}^{n_0} \mu\left(B_K^{(i)}\right).$$

Definition 15. A finite simple ordered marked point process $(\mathcal{E}^*, \mathfrak{X}^*, P^*)$ in a bounded space $(X \times K, \mathfrak{A}_X \otimes \mathfrak{A}_K, \mathfrak{B}_X \odot \mathfrak{B}_K)$ is called a process with 1-dependent μ-marks if

$$B_K^{(1)}, \dots, B_K^{(n_0)} \subset \mathfrak{B}_K$$

for all $n_0 \in \mathbb{N}$ where

$$B_K^{(i)} B_K^{(j)} = \varnothing, \qquad i, j = 1, \dots, n_0,\ i \neq j,$$

the marks $k_1, \dots, k_{n_0}$ of a random n_0-dimensional vector $E^* = ([x_i; k_i] \colon i = 1, \dots, n_0)$ are conditionally jointly independent random variables [3]:

$$\mu\left(B_K^{(1)}, \dots, B_K^{(n_0)} \mid x_1, \dots, x_{n_0}\right) = \prod_{i=1}^{n_0} \mu_i\left(B_K^{(i)} \mid x_i\right) = \prod_{i=1}^{n_0} \mu\left(B_K^{(i)} \mid x_i\right)$$

such that every mark k_i, $i = 1, \dots, n_0$, has conditional probability measure $\mu_i(B_K^{(i)} \mid x_i)$ that depends on a single parameter, namely on its position $x_i \in X$. Thus this conditional probability measure belongs to a one-parameter family of probability distributions $\{\mu(B_K \mid x) \colon x \in X\}$:

$$\mu_i\left(B_K^{(i)} \mid x_i\right) = \mu\left(B_K^{(i)} \mid x_i\right).$$

Definition 16. A finite simple ordered marked point process $(\mathcal{E}^*, \mathfrak{X}^*, P^*)$ in a bounded space $(X \times K, \mathfrak{A}_X \otimes \mathfrak{A}_K, \mathfrak{B}_X \odot \mathfrak{B}_K)$ is called a process with 1-dependent marks if

$$B_K^{(1)}, \dots, B_K^{(n_0)} \subset B_K$$

for all $n_0 \in \mathbb{N}$ where

$$B_K^{(i)} B_K^{(j)} = \varnothing, \qquad i, j = 1, \dots, n_0,\ i \neq j,$$

the marks $k_1, \dots, k_{n_0}$ of a random n_0-dimensional vector $E^* = ([x_i; k_i] \colon i = 1, \dots, n_0)$ are conditionally jointly independent random variables [3],

$$\mu\left(B_K^{(1)}, \dots, B_K^{(n_0)} \mid x_1, \dots, x_{n_0}\right) = \prod_{i=1}^{n_0} \mu_i\left(B_K^{(i)} \mid x_i\right),$$

and the conditional probability distribution $\mu_i\bigl(B_K^{(i)} \mid x_i\bigr)$ of any mark k_i, $i = 1, \dots, n_0$, in the space K depends only on its position $x_i \in X$.

6. Examples of ordered marked point processes with independent and 1-dependent marks

Example 1 ([2])**.** It is often the case in hydrology that atmospheric precipitates lead to the peak exceedances above the basic level of a river. Let $(x_1, \dots, x_n)$ be the random moments of peak exceedances above the basic level in the interval $X = [O, T] \subset R_+^1$. One can assume that the points $(x_1, \dots, x_n)$ form a realization E of some ordered point process $(\mathcal{E}, \mathfrak{X}, P)$ in the space of positions X and that n is an integer nonnegative random variable (the number of peak exceedances above the basic level in the interval X) that does not depend on $(x_1, \dots, x_n)$. The height above the basic flow at the moment x_i is denoted by k_i, $i = 1, \dots, n$. We also assume that the flow drops below the base level between two successive exceedances. It is clear that k_i treated as a random variable (mark) assuming values in the space of marks $K = (0, b]$ depends on the position x_i only, that is, $k_i = k_i(x_i)$, and does not depend on other exceedances $k_1, \dots, k_{i-1}, k_{i+1}, \dots, k_n$. The pairs $[x_i; k_i]$ form a trajectory $E^* = ([x_1; k_1], \dots, [x_n; k_n])$ of some ordered marked point process $(\mathcal{E}^*, \mathfrak{X}^*, P^*)$ in the phase space $X \times K = [O, T] \times (0, b]$. If all marks k_i have the same conditional probability measure $\mu_i\bigl(B_K^{(i)} \mid x_i\bigr)$ belonging to a one parameter family of probability distributions $\mu_i\bigl(B_K^{(i)} \mid x_i\bigr) = \mu\bigl(B_K^{(i)} \mid x_i\bigr)$, $i = 1, 2, \dots$, then $(\mathcal{E}^*, \mathfrak{X}^*, P^*)$ is an ordered marked point process with 1-dependent μ-marks.

Example 2 (Mixed empirical Poisson semispherical segment process)**.** Denote by $\mathcal{A}$ an ordered semispherical segment stochastic process defined on the unit two-dimensional Euclidean sphere S^2 with trajectories

$$E_{\mathcal{A}} = \bigl(Q_1(u_1(\varphi_1, \theta_1), a_1), \dots, Q_n(u_n(\varphi_n, \theta_n), a_n)\bigr),$$

where $u_i(\varphi_i, \theta_i)$ are the centers of segments, (φ_i, θ_i) are spherical coordinates of the centers, a_i are their angle diameters, $a_i \in K = [O, A]$, and $A \ll \pi$, $i = 1, \dots, n$ (see [4]). We regard an ordered semispherical segment stochastic process $\mathcal{A}$ as an ordered marked point stochastic process $(\mathcal{E}^*_{\mathcal{A}}, \mathfrak{X}^*_{\mathcal{A}}, P^*_{\mathcal{A}})$ with trajectories $E^*_{\mathcal{A}} = ([u_1; a_1], \dots, [u_n; a_n])$ in the ordered space $(S^2 \times K, \mathfrak{A}_{S^2} \otimes \mathfrak{A}_K, \mathfrak{B}_{S^2} \odot \mathfrak{B}_K)$. We assume that every trajectory $E^*_{\mathcal{A}}$ is obtained as a result of the following random experiment. Let G_1 and G_2 be two independent random experiments corresponding to the probability spaces $(S^2, \mathfrak{A}_{S^2}, P_u)$ and $(K, \mathfrak{A}_K, P_a)$, respectively. Then $\overline{G} = (G_1, G_2)$ is a "compound" random experiment corresponding to the probability space $(S^2 \times K, \mathfrak{A}_{S^2} \otimes \mathfrak{A}_K, P_u \otimes P_a)$ (see Section 4).

A number $n \in Z_+$ is drawn randomly according to the probability distribution generated by the Poisson sequence

$$\left\{ p_n : p_n = \frac{\lambda^n}{n!} e^{-\lambda}, \lambda > 0 \right\}$$

(see Section 3). Then every trajectory $E^*_{\mathcal{A}}$ of the ordered marked point process of size n is obtained as a result of n independent repetitions of the "compound" experiment $\overline{G}$ being the random sampling without replacement of a marked pair $[u_i; a_i]$ from the phase space $S^2 \times K$: the positions u_i are drawn from the space S^2 (experiment G_1), while the marks k_i are drawn from the space K (experiment G_2). Thus the process $(\mathcal{E}^*_{\mathcal{A}}, \mathfrak{X}^*_{\mathcal{A}}, P^*_{\mathcal{A}})$ is a a finite strictly simple mixed empirical Poisson ordered marked point process with independent P_a-marks.

The stochastic process $\mathcal{A}$ corresponding to the ordered marked point process

$$(\mathcal{E}^*_{\mathcal{A}}, \mathfrak{X}^*_{\mathcal{A}}, P^*_{\mathcal{A}})$$

is called a mixed empirical Poisson stochastic process of segments.

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Faculty for Cybernetics, National Taras Shevchenko University, Academician Glushkov Avenue, 6, Kyiv 03127, Ukraine

E-mail address: vm214@dcp.kiev.ua

Department of Higher Mathematics, Kyiv National University for Economy, Peremogy Avenue, 54/1, Kyiv, 03057, Ukraine

E-mail address: semejko@ukr.net

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