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ON PRICING CONTINGENT CLAIMS IN A TWO INTEREST RATES JUMP-DIFFUSION MODEL VIA MARKET COMPLETIONS

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Abstract. This paper deals with the problem of hedging contingent claims in the framework of a two factors jump-diffusion model with different credit and deposit rates. The upper and lower hedging prices are derived for European options by means of auxiliary completions of the initial market.

1. Introduction

In well-known financial market models one considers a unique interest rate for both deposit and credit (see for instance the books by Elliott and Kopp [10], Karatzas and Shreve [12]). In reality, the credit rate is always higher than the deposit rate. Such a market constraint brings new difficulties in the problem of hedging contingent claims (see Bergman [4], Korn [13], Bart [3], and also Cvitanic and Karatzas [8], Cvitanić [6, 7], Föllmer and Kramkov [11], Karatzas and Shreve [12], Cvitanić, Pham, and Touzi [9], Soner and Touzi [18] regarding other market constraints).

In contrast with complete markets, there is no symmetry between seller and buyer positions in the case of a market with constraints. The fair price of the derivative security (option) is split to the upper and lower prices. Hence, the problem of hedging a given contingent claim is to find these prices. We consider the problem in a jump-diffusion setting and derive the formulas for the above prices in terms of parameters of the initial model.

We give an extension of the methodology of completions in a two interest rates jump-diffusion financial market and show how our results are applied in the Black–Scholes model (see Korn [13]) and in the Merton model (Merton [17]).

2. Description of the model and auxiliary results

Let $\{\Omega, \mathcal{F}, \mathrm{F} = (\mathcal{F}_t)_{t\geq 0}, \mathsf{P}\}$ be a standard stochastic basis. Suppose there are two risky assets S^i, $i = 1, 2$, whose prices are described by the equations

$$dS_t^i = S_{t^-}^i \left(\mu^i\, dt + \sigma^i\, dW_t - \nu^i\, d\Pi_t\right), \qquad i = 1, 2. \tag{2.1}$$

Here W is a standard Wiener process and Π is a Poisson process with positive intensity λ. The filtration F is generated by the independent processes W and Π, $\mu^i \in \mathbb{R}$, $\sigma^i > 0$, $\nu^i < 1$.

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We also assume that there are a deposit account B^1 and a credit account B^2 satisfying

$$dB_t^i = B_t^i r^i\,dt, \qquad i = 1, 2. \tag{2.2}$$

Denote by (B^1, B^2, S^1, S^2) the market described by the above assets. Any non-negative $\mathcal{F}_T$-measurable random variable f_T is called a *contingent claim* with the maturity time T. In the (B^1, B^2, S^1, S^2)-market, a *portfolio* $\pi = (\beta^1, \beta^2, \gamma^1, \gamma^2)$ is an $\mathcal{F}_t$-predictable process, where we denote respectively by β^i and γ^i the number of units of the i^{th} bond and i^{th} stock in the wealth. The value of the portfolio π is given by

$$V_t = \beta_t^1 B_t^1 + \beta_t^2 B_t^2 + \gamma_t^1 S_t^1 + \gamma_t^2 S_t^2 \quad \text{a.s.} \tag{2.3}$$

A portfolio π is called *self-financing* (SF) if it has the following property:

$$dV_t = \beta_t^1\,dB_t^1 + \beta_t^2\,dB_t^2 + \gamma_t^1\,dS_t^1 + \gamma_t^2\,dS_t^2 \quad \text{a.s.} \tag{2.4}$$

Such a portfolio will be called *admissible* if

$$V_t \geq 0 \quad \text{a.s.} \quad \text{for all } t \geq 0.$$

The set of admissible portfolios with initial capital x is denoted by $\mathcal{A}(x)$.

The seller has the obligation to deliver the claim f_T at maturity, and in return he receives an initial amount x. The amount x will grow to $X_T(x) \geq f_T$. The buyer is borrowing the initial amount $-y$, $y < 0$, which grows to $Y_T(y) \geq -f_T$ (at maturity, he receives the claim f_T and pays his debt Y_T). The seller and the buyer positions can be identified with the *wealth process* $X_t \geq 0$ and the *debt process* $Y_t \leq 0$ respectively. Moreover, the processes X and $-Y$ are the capitals of self-financing and admissible portfolios. Under the above conditions and (2.1)–(2.2) the wealth process X_t and the debt process Y_t have the form

$$dX_t = X_{t^-}\left[\left(1 - \alpha_t^1 - \alpha_t^2\right)^+ r^1\,dt - \left(1 - \alpha_t^1 - \alpha_t^2\right)^- r^2\,dt + \alpha_t^1 \frac{dS_t^1}{S_{t^-}^1} + \alpha_t^2 \frac{dS_t^2}{S_{t^-}^2}\right], \tag{2.5}$$

$$dY_t = Y_{t^-}\left[\left(1 - \alpha_t^1 - \alpha_t^2\right)^+ r^2\,dt - \left(1 - \alpha_t^1 - \alpha_t^2\right)^- r^1\,dt + \alpha_t^1 \frac{dS_t^1}{S_{t^-}^1} + \alpha_t^2 \frac{dS_t^2}{S_{t^-}^2}\right]. \tag{2.6}$$

Here $\alpha_t^i = \gamma_t^i S_{t^-}^i / X_{t^-}^i$ (resp. $\gamma_t^i S_{t^-}^i / Y_{t^-}^i$), $i = 1, 2$, is the proportion of cash invested on the i^{th} stock in the wealth process (resp. debt process), and

$$a^+ = \max\{0, a\}, \qquad a^- = -\min\{0, a\}.$$

Note that throughout the paper α will be also called a strategy.

In this paper, a portfolio π with initial capital x is called a *hedge* for the seller if the corresponding wealth process satisfies $X_T^{x,\pi} \geq f_T$ P-a.s. Similarly a portfolio π is a hedge for the buyer if the debt process is such that $Y_T^{y,\pi} \geq -f_T$ P-a.s.

For the seller, we say that a hedge π^* is *minimal* if $X_t^{\pi} \geq X_t^{\pi^*}$ P-a.s., for all t and for any other hedge π. For the buyer, a hedge π' is *minimal* if $Y_t^{\pi} \geq Y_t^{\pi'}$ P-a.s., for all t and for any other hedge π.

Let us consider the special case where the financial market has the same deposit and credit rates: $r^1 = r^2 = r$, and hence, $B^1 = B^2 = B$. In the framework of such a (B, S^1, S^2)-market, the capital (resp. debt) generated by an admissible portfolio process $\pi := (\beta, \gamma^1, \gamma^2)$ is described as follows:

$$X_t = \beta_t B_t + \gamma_t^1 S_t^1 + \gamma_t^2 S_t^2 \quad \text{a.s.}, \tag{2.7}$$

$$\frac{dX_t}{X_{t^-}} = \frac{dY_t}{Y_{t^-}} = \left[\left(1 - \alpha_t^1 - \alpha_t^2\right) r\,dt + \alpha_t^1 \frac{dS_t^1}{S_{t^-}^1} + \alpha_t^2 \frac{dS_t^2}{S_{t^-}^2}\right]. \tag{2.8}$$

If $\sigma^1\nu^2 \neq \sigma^2\nu^1$, then the parameters

$$\phi = -\frac{(\mu^1 - r)\,\nu^2 - (\mu^2 - r)\,\nu^1}{\sigma^1\nu^2 - \sigma^2\nu^1}, \tag{2.9}$$
$$\psi = \frac{(\mu^1 - r)\,\sigma^2 - (\mu^2 - r)\,\sigma^1}{\sigma^2\nu^1 - \sigma^1\nu^2}\lambda^{-1} - 1$$

define (see Melnikov et al. [15]) a density Z of a unique martingale measure P^* in the (B, S^1, S^2)-market as a stochastic exponent

$$Z_t = \mathcal{E}_t(N) = \exp\left\{\phi W_t - \frac{\phi^2}{2}t + (\lambda - \lambda^*)t + (\ln\lambda^* - \ln\lambda)\Pi_t\right\}, \tag{2.10}$$

where $N_t = \phi W_t + \psi(\Pi_t - \lambda t)$. Under such a measure, the given Poisson process Π has intensity $\lambda^* = \lambda(1+\psi)$, and $W_t^* = W_t - \phi t$ is a Wiener process.

We consider contingent claims of the form $f_T := f(S_T^1)$.

It is well known that the fair price of the call option $(S_T^1 - K)^+$ in this (B, S^1, S^2)-market is given by the formula

(2.11)
$$\begin{aligned} E^* e^{-rT}(S_T^1 - K)^+ &= \sum_{n=0}^{\infty}\left(e^{-\lambda^* T}\frac{(\lambda^* T)^n}{n!}C_{BS}\left[S_0^1\left(1-\nu^1\right)^n e^{\nu^1\lambda^* T}, r, \sigma^1, K, T\right]\right) \\ &= C_r(0), \end{aligned}$$

where E^* is the expected value under the risk neutral measure, and

$$C_{BS}(x, r, \sigma, K, T) = x\Phi(d_1) - Ke^{-rT}\Phi(d_2),$$
$$d_1 = \frac{\ln(x/K) + T\left(r + \sigma^2/2\right)}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T},$$
$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2/2}\,dt.$$

(See Aase [1], Bardhan and Chao [2], Colwell and Elliott [5], Mercurio and Runggaldier [16], Melnikov et al. [15].) We derive the well-known price of a put option from the call-put parity.

The next lemma studies the monotonic properties of C_r (see (2.11)) as a function of r.

Lemma 2.1. *If the following inequalities are fulfilled:*

2.1) $\partial\lambda^*/\partial r \geq 0$, *or*
2.2) $\partial\lambda^*/\partial r \leq 0$ *and* $\nu^1 \geq 0$, *or*
2.3) $\partial\lambda^*/\partial r \leq 0, \nu^1 \leq 0$ *and*

$$\frac{\nu^1\frac{\partial\lambda^*}{\partial r}}{1 + \nu^1\frac{\partial\lambda^*}{\partial r}} \leq \Phi(d_2(0)),$$

then $\rho_C := \partial C/\partial r$ *is positive.*

If the next inequalities are satisfied:

2.1′) $\partial\lambda^*/\partial r \leq 0$, *or*
2.2′) $\partial\lambda^*/\partial r \geq 0$ *and* $\nu^1 \leq 0$, *or*
2.3′) $\partial\lambda^*/\partial r \geq 0$, $\nu^1 \geq 0$ *and*

$$\Phi(d_2(0)) \leq \frac{1}{1 + \nu^1\frac{\partial\lambda^*}{\partial r}},$$

then $\rho_P := \partial P/\partial r$ *is negative.*

The proof of this lemma is provided in the Appendix.

Let us now turn to the (B^1, B^2, S^1, S^2)-market.

3. Main results and pricing formulas

To study the hedging problem in the framework of (B^1, B^2, S^1, S^2)-market we define a variety of (B, S^1, S^2) (or (B^d, S^1, S^2))-markets with the interest rates $r = r^d = r^1 + d$, where $d = (d_t)$ is a predictable process such that $d_t \in [0, r^2 - r^1]$. Consider first the position of a seller. From his/her viewpoint, the investor wishes to find the minimal initial amount possible to invest in generating a wealth process matching at least f_T. Such a price is provided by the initial capital of the minimal hedge (if it exists) against the claim f_T. In the (B^1, B^2, S^1, S^2)-market, the upper hedging price (or seller price) will be given by the following Statement (see Korn [13] for the Black–Scholes model).

Statement 3.1. *Let $d = (d_t)$ be a predictable process with values in the interval*

$$\left[0, r^2 - r^1\right].$$

Assume that $\alpha_d := (\alpha^1, \alpha^2)$, the optimal hedging strategy against the claim f_T in the (B^d, S^1, S^2)-market, satisfies the condition

$$\left(r^2 - r^1 - d_t\right)\left(1 - \alpha_t^1 - \alpha_t^2\right)^- + d_t\left(1 - \alpha_t^1 - \alpha_t^2\right)^+ = 0. \tag{3.1}$$

Then $C_{r^d}(0)$ (resp. $P_{r^d}(0)$), the initial price of the minimal hedge in (B^d, S^1, S^2) against f_T, is equal to C_+ (resp. P_+), the initial price of the minimal hedging strategy in

$$\left(B^1, B^2, S^1, S^2\right).$$

Namely

$$C_{r^d}(0) = C_+ \quad (\textit{resp. } P_{r^d}(0) = P_+).$$

Before giving the proof of this statement, we show that the set of solutions of the equation (3.1) is non-empty at least for the European put and call options.

Example. Let $f_T = (S_T^1 - K)^+$; the European call price (2.11) can be expressed as follows:

$$\begin{aligned} C_{r^d}(t) = {} & S_t^1 \left(1 - \nu^1\right)^n e^{\nu^1 \lambda^*(T-t)} \left(\sum_0^\infty \frac{(\lambda^*(T-t))^n}{n!} e^{-\lambda^*(T-t)} \Phi(d_1)\right) \\ & - K e^{-r^d(T-t)} \left(\sum_0^\infty \frac{(\lambda^*(T-t))^n}{n!} e^{-\lambda^*(T-t)} \Phi(d_2)\right). \end{aligned}$$

For any time t the seller borrows money: $1 - \alpha_t^1 - \alpha_t^2 < 0$. In the 1^{st} term of C_{r^d} the coefficient of S^1 is the number of units of stock needed, the 2^{nd} term, always non-positive (assume it is negative), is invested in a bank account. Taking the latter into account ($(1 - \alpha_t^1 - \alpha_t^2)^+ = 0$) in relation (3.1) yields to

$$\left(r^2 - r^1 - d_t\right)\left(1 - \alpha^1 - \alpha^2\right)^- = 0.$$

Since $(1 - \alpha^1 - \alpha^2)^- > 0$, we derive $(r^2 - r^1 - d_t) = 0$. Hence $r^2 - r^1 = d_t$, and the pair $(r^2 - r^1, \alpha_{r^2 - r^1})$ satisfies the relation (3.1).

Proof of Statement 3.1. Let us first show that under relation (3.1), the minimal hedging strategy (α) in the (B^d, S^1, S^2)-market is a hedging strategy in the (B^1, B^2, S^1, S^2)-market. Let C_{r^d} be the initial capital associated to that hedge in the (B^d, S^1, S^2)-market.

If α satisfies (3.1), then we rewrite the latter equation as follows:

$$\begin{aligned}&\left(r^2-r^1-d_t\right)\left(1-\alpha_t^1-\alpha_t^2\right)^-+d_t\left(1-\alpha_t^1-\alpha_t^2\right)^+\\&\quad=r^2\left(1-\alpha_t^1-\alpha_t^2\right)^--\left(r^1+d_t\right)\left(1-\alpha_t^1-\alpha_t^2\right)^-+d_t\left(1-\alpha_t^1-\alpha_t^2\right)^+\\&\quad=r^2\left(1-\alpha_t^1-\alpha_t^2\right)^--r^d\left(1-\alpha_t^1-\alpha_t^2\right)-r^1\left(1-\alpha_t^1-\alpha_t^2\right)^+\\&\quad=0.\end{aligned}$$

Hence,

$$r^d\left(1-\alpha_t^1-\alpha_t^2\right)=r^1\left(1-\alpha_t^1-\alpha_t^2\right)^+-r^2\left(1-\alpha_t^1-\alpha_t^2\right)^-$$

and

$$\begin{aligned}\frac{dX_t^{\alpha,d}}{X_{t^-}^{\alpha,d}}&=r^d\left(1-\alpha_t^1-\alpha_t^2\right)\,dt+\alpha_t^1\frac{dS_t^1}{S_{t^-}^1}+\alpha^2\frac{dS_t^2}{S_{t^-}^2}\\&=\left(r^1(1-\alpha_t^1-\alpha_t^2)^+-r^2(1-\alpha_t^1-\alpha_t^2)^-\right)\,dt+\alpha^1\frac{dS_t^1}{S_{t^-}^1}+\alpha^2\frac{dS_t^2}{S_{t^-}^2}\\&=\frac{dX_t^\alpha}{X_{t^-}^\alpha}.\end{aligned}$$

Therefore, the wealth processes $X_t^{\alpha,d}(C_{r^d})$ and $X_t^\alpha(C_{r^d})$ on the (B^d,S^1,S^2)- and (B^1,B^2,S^1,S^2)-markets, respectively, coincide, and in particular

$$X_T^{\alpha,d}(C_{r^d})=X_T^\alpha(C_{r^d})=f(S_T^1).$$

Hence, the minimal hedge α in the (B^d,S^1,S^2)-market against f_T is a hedge in the (B^1,B^2,S^1,S^2)-market when relation (3.1) holds.

We now show that the above strategy α with initial capital C_{r^d} is minimal among the hedges against $f(S_T^1)$ in the (B^1,B^2,S^1,S^2)-market. For that purpose, we will show that in the (B^1,B^2,S^1,S^2)-market, the initial capital of an arbitrary strategy (α^a) hedging f_T is greater than or equal to the initial capital of the minimal hedge α (C_{r^d}):

$$C_{r^d}:=E^{d,*}\left[f(S_T^1)e^{-r^dT}\right]\le x,$$

where $E^{d,*}$ is the expected value under the martingale measure $P^{d,*}$ (see relation (2.10)) in the (B^d,S^1,S^2)-market, and x represents the initial capital of α^a, an arbitrary strategy in the $(B^1,B^2,S^1,S^2)-market.$

Let $X_t^{\alpha^a}$ be the wealth process generated by α^a in the (B^1,B^2,S^1,S^2)-market. We will prove that $E^{d,*}\left[f_Te^{-r^dT}\right]\le E^{d,*}\left[X_T^{\alpha^a}e^{-r^dT}\right]\le x$.

Consider the discounted wealth process $\widetilde{X}_t:=X_t^{\alpha^a}e^{-r^dt}$; then by using Itô's formula we obtain

$$\begin{aligned}d\widetilde{X}_t&=-r^de^{-r^dt}X_{t^-}^{\alpha^a}\,dt+e^{-r^dt}\,dX_t^{\alpha^a}=e^{-r^dt}X_{t^-}^{\alpha^a}\left[\frac{dX_t^{\alpha^a}}{X_{t^-}^{\alpha^a}}-r^d\,dt\right]\\&=e^{-r^dt}X_{t^-}^{\alpha^a}\Bigg[\left(\left(1-\alpha_t^{1,a}-\alpha_t^{2,a}\right)^+r^1-(1-\alpha_t^{1,a}-\alpha_t^{2,a})^-r^2-r^d\right)\,dt\\&\qquad\qquad+\alpha_t^{1,a}\frac{dS_t^1}{S_{t^-}^1}+\alpha_t^{2,a}\frac{dS_t^2}{S_{t^-}^2}\Bigg].\end{aligned}$$

Note that from the construction of the martingale measure (see Melnikov et al. [15]), $W_t^{d,*}=W_t^d-\phi t$ is a $P^{d,*}$-Wiener process, and $(\Pi_t-\lambda^*t)$ is a $P^{d,*}$-martingale. Therefore we can rewrite the dynamics of the stocks as follows:

$$\frac{dS^1_{t^-}}{S^1_{t^-}} = r^d\,dt + \sigma^1\,dW^{d,*} - \nu^1\,d(\Pi_t - \lambda^* t),$$
$$\frac{dS^2_{t^-}}{S^2_{t^-}} = r^d\,dt + \sigma^2\,dW^{d,*} - \nu^2\,d(\Pi_t - \lambda^* t),$$

and

$$d\widetilde{X}_t = e^{-r^d t} X^{\alpha^a}_{t^-}\Bigg[\bigg(\Big(1-\alpha^{1,a}_t-\alpha^{2,a}_t\Big)^+ r^1 - \Big(1-\alpha^{1,a}_t-\alpha^{2,a}_t\Big)^- r^2 - \Big(1-\alpha^{1,a}_t-\alpha^{2,a}_t\Big) r^d\bigg)\,dt + \alpha^{1,a}_t\left(\sigma^1\,dW^{d,*} - \nu^1\,d(\Pi_t-\lambda^* t)\right) + \alpha^{2,a}_t\left(\sigma^2\,dW^{d,*} - \nu^2\,d(\Pi_t-\lambda^* t)\right)\Bigg].$$

Since $r^d = r^1 + d$, it follows that

$$d\widetilde{X}_t = X^{\alpha^a}_{t^-} e^{-r^d t}\Bigg[\bigg(\Big(1-\alpha^{1,a}_t-\alpha^{2,a}_t\Big)^- \left(r^1-r^2\right) - d\Big(1-\alpha^{1,a}_t-\alpha^{2,a}_t\Big)\bigg)\,dt + \Big(\alpha^{1,a}_t\sigma^1 + \alpha^{2,a}_t\sigma^2\Big)\,dW^{d,*}_t - \Big(\alpha^{1,a}_t\nu^1 + \alpha^{2,a}_t\nu^2\Big)\,d(\Pi_t-\lambda^* t)\Bigg]. \tag{3.2}$$

Now, notice that

$$\int_0^t X^{\alpha^a}_{u^-} e^{-r^d u}\left(\left(1-\alpha^{1,a}_u-\alpha^{2,a}_u\right)^-\left(r^1-r^2\right) - d\left(1-\alpha^{1,a}_u-\alpha^{2,a}_u\right)\right)du \le 0,$$

and

$$\int_0^t X^{\alpha^a}_{u^-} e^{-r^d u}\left(\alpha^{1,a}_u\sigma^1 + \alpha^{2,a}_u\sigma^2\right)dW^{d,*}_u - \left(\alpha^{1,a}_u\nu^1 + \alpha^{2,a}_u\nu^2\right)d(\Pi_u - \lambda^* u)$$

is a $P^{d,*}$-local martingale. Without loss of generality we assume the latter is a $P^{d,*}$-martingale. Whence upon first integrating the relation (3.2) and then taking the $P^{d,*}$-expectation, we obtain, for all t in $[0,T]$,

$$E^{d,*}[\widetilde{X}_t] = E^{d,*}[X^{\alpha^a}_t e^{-r^d t}] \le x. \tag{3.3}$$

The strategy α^a is a hedge for f_T and yields to $X^{\alpha^a}_T e^{-r^d T} = \widetilde{X}_T \ge f_T e^{-r^d T}$, henceforth

$$C_{r^d} = E^{d,*}\left[f_T e^{-r^d T}\right] \le E^{d,*}\left[X^{\alpha^a}_T e^{-r^d T}\right] \le x,$$

where x is the initial capital of an arbitrary hedge for f_T in the (B^1, B^2, S^1, S^2)-market. Further, provided relation (3.1) is fulfilled, C_{r^d} is an initial price of a hedge for f_T in the latter market. Therefore, $C_{r^d} = C_+$, where C_+ is the initial capital of the minimal hedge in the (B^1, B^2, S^1, S^2)-market.

The proof is similar for the put case; hence $P_{r^d} = P_+$. □

Secondly, we study the position of a buyer in the following statement.

Statement 3.2. *Let $d = (d_t)$ be a predictable process in $[0, r^2 - r^1]$, and assume that α_d, the minimal hedging strategy against f_T in the (B^d, S^1, S^2)-market satisfies the equation*

$$\left(r^2 - r^1 - d_t\right)\left(1-\alpha^1_t-\alpha^2_t\right)^+ + d_t\left(1-\alpha^1_t-\alpha^2_t\right)^- = 0. \tag{3.4}$$

Then

(1) *the strategy α_d is a hedge against $-f_T$ in (B^1, B^2, S^1, S^2);*

(2) *furthermore, α_d provides the minimal hedge against $-f_T$ in the (B^1, B^2, S^1, S^2)-market.*

In order to proof Statement 3.2, we first state the following lemma.

Lemma 3.3. *The minimal hedging strategy (for a seller) against f_T in (B^d, S^1, S^2) is the minimal hedging strategy (for a buyer) against $-f_T$ in the same market.*

Proof. In the unconstrained (B^d, S^1, S^2)-market, the stochastic differential equations of the debt and wealth process generated by a strategy α satisfy (see (2.8))

$$\frac{dX_t^\alpha}{X_{t^-}^\alpha} = \frac{dY_t^\alpha}{Y_{t^-}^\alpha}.$$

If α_d is a hedge (for the seller) against f_T in (B^d, S^1, S^2), then $X_T^{\alpha_d, x} = f_T$. Now, taking $y = -x$ as the initial price for the debt process yields to

$$Y_T^{\alpha_d, y} = -X_T^{\alpha_d, x} = -f_T.$$

Henceforth, α_d is a hedge against $-f_T$ in (B^d, S^1, S^2) (see hedge for a buyer). □

Proof of Statement 3.2. Let α, the minimal hedge against $-f_T$ in the (B^d, S^1, S^2)-market, with initial debt $-C_{r^d}$, satisfy the relation (3.4).

We rewrite the relation (3.4) as follows:

$$\begin{aligned}
&\left(r^2 - r^1 - d\right)\left(1 - \alpha^1 - \alpha^2\right)^+ + d\left(1 - \alpha^1 - \alpha^2\right) \\
&\quad = r^2\left(1 - \alpha^1 - \alpha^2\right)^+ - r^d\left(1 - \alpha^1 - \alpha^2\right) - r^1\left(1 - \alpha^1 - \alpha^2\right)^- = 0, \\
&r^d\left(1 - \alpha^1 - \alpha^2\right) = r^2\left(1 - \alpha^1 - \alpha^2\right)^+ - r^1\left(1 - \alpha^1 - \alpha^2\right)^-.
\end{aligned}$$

Denote by Y^d and Y the debt processes generated by α in the (B^d, S^1, S^2)- and (B^1, B^2, S^1, S^2)-markets, respectively; then

$$\frac{dY_t^d}{Y_{t^-}^d} = \frac{dY_t}{Y_{t^-}}.$$

From the above equality, taking $-C_{r^d}$ as initial price yields to α being a hedge against $-f_T$ in the (B^1, B^2, S^1, S^2)-market.

To prove the minimality of the above strategy, we consider an arbitrary strategy α^a with initial debt process y, let $Y_t^{\alpha^a}$ be the debt process generated by α^a, and denote $\widetilde{Y_t^{\alpha^a}} = e^{-r^d t} Y_t^{\alpha^a}$. Using Itô's formula we derive

$$d\left(\widetilde{Y_t^{\alpha^a}}\right) = Y_{t^-}^{\alpha^a} e^{-r^d t}\left(-r^d\,dt + \frac{dY_t^{\alpha^a}}{Y_{t^-}^{\alpha^a}}\right),$$

$$\begin{aligned}
d\widetilde{Y}_t^{\alpha^a} = e^{-r^d t} Y_{t^-}^{\alpha^a} \Bigg[&\Bigg(\left(1 - \alpha_t^{1,a} - \alpha_t^{2,a}\right)^+ r^2 - \left(1 - \alpha_t^{1,a} - \alpha_t^{2,a}\right)^- r^1 \\
&\qquad - \left(1 - \alpha_t^{1,a} - \alpha_t^{2,a}\right) r^d \Bigg) dt \\
&+ \alpha_t^{1,a}\left(\sigma^1\,dW^{d,*} - \nu^1\,d(\Pi_t - \lambda^* t)\right) \\
&\qquad + \alpha_t^{2,a}\left(\sigma^2\,dW^{d,*} - \nu^2\,d(\Pi_t - \lambda^* t)\right) \Bigg].
\end{aligned} \tag{3.5}$$

Notice that from the drift term

$$\begin{aligned}
&\left(\left(1 - \alpha_t^{1,a} - \alpha_t^{2,a}\right)^+ r^2 - \left(1 - \alpha_t^{1,a} - \alpha_t^{2,a}\right)^- r^1 - \left(1 - \alpha_t^{1,a} - \alpha_t^{2,a}\right) r^d\right) \\
&\quad = \left(\left(1 - \alpha_t^{1,a} - \alpha_t^{2,a}\right)^+ \left(r^2 - r^d\right) - \left(1 - \alpha_t^{1,a} - \alpha_t^{2,a}\right)^- \left(r^1 - r^d\right)\right) \geq 0.
\end{aligned}$$

Hence,

$$\int_0^t e^{-r^d u} Y_{u^-}^{\alpha^a} \Big(\left(1-\alpha_u^{1,a}-\alpha_u^{2,a}\right)^+ \left(r^2-r^d\right) - \left(1-\alpha_u^{1,a}-\alpha_u^{2,a}\right)^- \left(r^1-r^d\right)\Big)\, du \le 0. \tag{3.6}$$

Further, we can assume without loss of generality that

$$\int_0^t e^{-r^d u} Y_{u^-}^{\alpha^a} \left(\alpha_u^{1,a}\left(\sigma^1\, dW_u^{d,*} - \nu^1\, d(\Pi_u - \lambda^* u)\right) + \alpha_u^{2,a}\left(\sigma^2\, dW_u^{d,*} - \nu^2\, d(\Pi_u - \lambda^* u)\right)\right)$$

is a $P^{d,*}$-martingale. Consequently

$$\int_0^T d\widetilde{Y_u^{\alpha^a}} \le \int_0^T e^{-r^d u} Y_{u^-}^{\alpha^a} \Big(\alpha_u^{1,a}\left(\sigma^1\, dW_u^{d,*} - \nu^1\, d(\Pi_u - \lambda^* u)\right) + \alpha_u^{2,a}\left(\sigma^2\, dW_u^{d,*} - \nu^2\, d(\Pi_u - \lambda^* u)\right)\Big). \tag{3.7}$$

Since α^a is a hedge, it follows that $Y_T^{\alpha^a} \ge -f_T$, and we derive

$$-C_d = E^{d,*}\left[e^{-r^d T}(-f_T)\right] \le E^{d,*}\left[\widetilde{Y_T^{\alpha^a}}\right],$$

from the relations (3.5), (3.6), and (3.7)

$$E^{d,*}\left[\widetilde{Y_T^{\alpha^a}}\right] = E^{d,*}\left[Y_T^{\alpha^a} e^{-r^d T}\right] \le y.$$

Hence $-C_d \le y$ for any arbitrary strategy with initial debt y. Since the pair $(\alpha, -C_d)$ is a hedge against $-f_T$ in the (B^1, B^2, S^1, S^2)-market, it provides the minimal hedge.

The proof holds for both put and call options. □

Let us give an approximation of the arbitrage-free prices of the claim $f_T = (S_T^1 - K)^+$. The key ingredient of the method relies on the following. Taking the supremum (resp. infimum) over the auxiliary markets of the actual prices, we find some natural approximations for the upper and lower hedging prices of the claim, and hence we approximate the arbitrage-free interval of prices by taking

$$\left[\inf_{d\in[0,r^2-r^1]} C_{r^d}, \sup_{d\in[0,r^2-r^1]} C_{r^d}\right].$$

Exploiting the call-put parity, a similar method is used for $f_T = (K - S_T^1)^+$.

We have considered auxiliary markets of the form (B^d, S^1, S^2) with constant d and therefore with constant interest rates $r^d = r^1 + d$.

To formulate our pricing results we introduce the following conditions derived from Lemma 2.1:

(I) $\nu^1 \partial\lambda^*/\partial r \le 0$;

(II) $\nu^1 \ge 0$ and $\partial\lambda^*/\partial r \ge 0$ and

$$\Phi(d_2(0)) \le \frac{1}{1+\nu^1 \frac{\partial\lambda^*}{\partial r}};$$

(III) $\nu^1 \le 0$ and $\partial\lambda^*/\partial r \le 0$ and

$$\frac{\nu^1 \frac{\partial\lambda^*}{\partial r}}{1+\nu^1 \frac{\partial\lambda^*}{\partial r}} \le \Phi(d_2(0)).$$

Under these conditions, combining Lemma 2.1, Statement 3.1, and Statement 3.2, we arrive at the following theorem.

Theorem 3.4. *If condition* (I) *or* (II) *or* (III) *is fulfilled, then the following pricing formulas hold:*

$$\begin{aligned}
\sup_{d\in[0,r^2-r^1]} C_{r^d} &\geq \sup_{d\in[0,r^2-r^1],\, d \text{ is const.}} C_{r^d} \geq C_{r^2},\\
\sup_{d\in[0,r^2-r^1]} P_{r^d} &\geq \sup_{d\in[0,r^2-r^1],\, d \text{ is const.}} P_{r^d} \geq P_{r^1},\\
\inf_{d\in[0,r^2-r^1]} C_{r^d} &\leq \inf_{d\in[0,r^2-r^1],\, d \text{ is const.}} C_{r^d} \leq C_{r^1},\\
\inf_{d\in[0,r^2-r^1]} P_{r^d} &\leq \inf_{d\in[0,r^2-r^1],\, d \text{ is const.}} P_{r^d} \leq P_{r^2}.
\end{aligned} \tag{3.8}$$

Corollary 3.5 (See Korn [13]). *The Black–Scholes model satisfies* (3.8) *since conditions* (I), (II) *and* (III) *are fulfilled:*

$$\nu^1 = 0, \qquad \nu^1 \frac{\partial \lambda^*}{\partial r} = 0$$

and $0 \leq \Phi(d_2) \leq 1$.

Corollary 3.6. *The pure jump case (the Merton model) satisfies* (3.8) *since condition* (I) *holds:*

$$\nu^1 \frac{\partial \lambda^*}{\partial r} = -1.$$

4. Appendix

Proof of Lemma 2.1. a) *The case of a call option.* For convenience we will use a different representation of $\partial C/\partial r$ depending on whether $\partial\lambda^*/\partial r$ is positive or negative. Differentiating (2.11) yields to

$$\begin{aligned}
\frac{\partial C}{\partial r} &= T\frac{\partial \lambda^*}{\partial r}\sum_{n\geq 0}\frac{(\lambda^* T)^n}{n!}e^{-\lambda^* T}A(n)\\
&\quad + KTe^{-rT}\sum_{n\geq 0}\frac{(\lambda^* T)^n}{n!}e^{-\lambda^* T}\Phi(d_2(n)) \quad \text{if } \frac{\partial\lambda^*}{\partial r}\geq 0,
\end{aligned} \tag{4.1}$$

and

$$\begin{aligned}
\frac{\partial C}{\partial r} &= T\frac{\partial \lambda^*}{\partial r}\left(1-\nu^1\right)\sum_{n\geq 0}\frac{(\lambda^* T)^n}{n!}e^{-\lambda^* T}B(n)\\
&\quad - \nu T\frac{\partial\lambda^*}{\partial r}Ke^{-rT}\sum_{n\geq 0}\frac{(\lambda^* T)^n}{n!}e^{-\lambda^* T}\Big(\Phi(d_2(n+1)) - \Phi(d_2(n))\Big)\\
&\quad + KTe^{-rT}\sum_{n\geq 0}\frac{(\lambda^* T)^n}{n!}e^{-\lambda^* T}\Phi(d_2(n)) \quad \text{if } \frac{\partial\lambda^*}{\partial r}\leq 0,
\end{aligned} \tag{4.2}$$

where $A(n)$ and $B(n)$ have the following expressions:

$$\begin{aligned}
A(n) &= S_0\left(1-\nu^1\right)^{n+1}e^{\nu^1\lambda^* T}\big(\Phi(d_1(n+1)) - \Phi(d_1(n))\big)\\
&\quad - Ke^{-rT}\big(\Phi(d_2(n+1)) - \Phi(d_2(n))\big),\\
B(n) &= S_0\left(1-\nu^1\right)^{n}e^{\nu^1\lambda^* T}\big(\Phi(d_1(n+1)) - \Phi(d_1(n))\big)\\
&\quad - Ke^{-rT}\big(\Phi(d_2(n+1)) - \Phi(d_2(n))\big);
\end{aligned}$$

we denote σ^1 by σ, and

$$d_2(n) = \frac{\ln[S/K] + n\ln\left(1-\nu^1\right) + \nu^1\lambda^* T + \left(r - \sigma^2/2\right)T}{\sigma\sqrt{T}},$$
$$d_1(n) = d_2(n) + \sigma\sqrt{T}.$$

One can easily show that $A(n) \geq 0$ and $B(n) \leq 0$.

We only give the proof that $A(n) \geq 0$, since a similar method can be used to show that $B(n) \leq 0$. We have

$$A(n) = S_0\left(1-\nu^1\right)^{n+1} e^{\nu^1\lambda^* T}\big(\Phi(d_1(n+1)) - \Phi(d_1(n))\big) - Ke^{-rT}\big(\Phi(d_2(n+1)) - \Phi(d_2(n))\big),$$

where $d_i(n+1) = d_i(n) + \ln(1-\nu)/(\sigma\sqrt{T})$, $i = 1, 2$, and

$$\begin{aligned}
A(n) &= \frac{1}{\sqrt{2\pi}}\left(S_0\left(1-\nu^1\right)^{n+1} e^{\nu^1\lambda^* T}\int_{d_1(n)}^{d_1(n+1)} e^{-x^2/2}\,dx - Ke^{-rT}\int_{d_2(n)}^{d_2(n+1)} e^{-x^2/2}\,dx\right)\\
&= \frac{1}{\sqrt{2\pi}}\Bigg(S_0\left(1-\nu^1\right)^{n+1} e^{\nu^1\lambda^* T}\int_0^{\frac{\ln\left(1-\nu^1\right)}{\sigma\sqrt{T}}} e^{-(x+d_1(n))^2/2}\,dx\\
&\qquad\qquad - Ke^{-rT}\int_0^{\frac{\ln\left(1-\nu^1\right)}{\sigma\sqrt{T}}} e^{-(x+d_2(n))^2/2}\,dx\Bigg)\\
&= \frac{1}{\sqrt{2\pi}}\int_0^{\frac{\ln\left(1-\nu^1\right)}{\sigma\sqrt{T}}}\left(S_0\left(1-\nu^1\right)^{n+1} e^{\nu^1\lambda^* T} e^{-(x+d_1(n))^2/2} - Ke^{-rT}e^{-(x+d_2(n))^2/2}\right)dx\\
&= \frac{1}{\sqrt{2\pi}}\int_0^{\frac{\ln\left(1-\nu^1\right)}{\sigma\sqrt{T}}} S_0\left(1-\nu^1\right)^{n+1} e^{\nu^1\lambda^* T} e^{-(x+d_1(n))^2/2}\left(1 - \frac{e^{x\sigma\sqrt{T}}}{(1-\nu^1)}\right)dx.
\end{aligned}$$

If $\ln(1-\nu^1)$ is negative then

$$1 - \frac{e^{x\sigma\sqrt{T}}}{(1-\nu^1)}$$

is also negative. Hence, $A(n)$ is positive. Similarly, if $\ln(1-\nu^1)$ is positive then

$$1 - \frac{e^{x\sigma\sqrt{T}}}{(1-\nu^1)}$$

is also positive. Hence, $A(n)$ is always positive. Consequently:

1) For $\partial\lambda^*/\partial r \geq 0$, from the sign of $A(n)$ and equation (4.1) we obtain $\partial C/\partial r > 0$.

2) Similarly for $\partial\lambda^*/\partial r \leq 0$, since $B(n) \leq 0$ and from equation (4.2), we only need to find the sign of

$$\begin{aligned}
&-\nu^1 T\frac{\partial\lambda^*}{\partial r}Ke^{-rT}\sum_{n\geq 0}\frac{(\lambda^* T)^n}{n!}e^{-\lambda^* T}\big(\Phi(d_2(n+1)) - \Phi(d_2(n))\big)\\
&\qquad\qquad + KTe^{-rT}\sum_{n\geq 0}\frac{(\lambda^* T)^n}{n!}e^{-\lambda^* T}\Phi(d_2(n)).
\end{aligned} \tag{4.3}$$

Expression (4.3) can be transformed as

$$KTe^{-rt}\sum_{n\geq 0}\frac{(\lambda^* T)^n}{n!}\left(\Phi(d_2(n))\left(1 + \nu^1\frac{\partial\lambda^*}{\partial r}\right) - \nu^1\frac{\partial\lambda^*}{\partial r}\Phi(d_2(n+1))\right). \tag{4.4}$$

To guarantee the positivity of the above expression, it is sufficient to prove that

$$X = \left(\Phi(d_2(n))\left(1+\nu^1\frac{\partial\lambda^*}{\partial r}\right) - \nu^1\frac{\partial\lambda^*}{\partial r}\Phi(d_2(n+1))\right)$$

is positive.

Let us now consider two cases $\nu^1 \geq 0$ or $\nu^1 \leq 0$ and note that from the expression of $d_2(n)$ the following always holds:

$$\nu^1\Big(\Phi(d_2(n+1)) - \Phi(d_2(n))\Big) \leq 0.$$

(a) If $\nu^1 \leq 0$, then from the previous relation, Φ is a non-decreasing function of n and $(1+\nu^1\partial\lambda^*/\partial r) > 0$; therefore

$$\left(\Phi(d_2(0))\left(1+\nu^1\frac{\partial\lambda^*}{\partial r}\right) - \nu^1\frac{\partial\lambda^*}{\partial r}\right) < X.$$

Hence, if

$$\Phi(d_2(0)) \geq \frac{\nu^1\frac{\partial\lambda^*}{\partial r}}{1+\nu^1\frac{\partial\lambda^*}{\partial r}},$$

then $X > 0$ and $\partial C/\partial r > 0$.

(b) If $\nu^1 \geq 0$, then Φ is a non-increasing function of n, i.e., $\Phi(d_2(n+1)) \leq \Phi(d_2(n))$, and $\Phi(d_2(n+1)) < X$. Therefore, X is non-negative and $\partial C/\partial r > 0$.

b) *The case of a put option.* For the put option, ρ is given by the following:

$$\begin{aligned}\frac{\partial P}{\partial r} = {} & T\frac{\partial\lambda^*}{\partial r}\sum_{n\geq 0}\frac{(\lambda^*T)^n}{n!}e^{-\lambda^*T}\Bigg(S_0\left(1-\nu^1\right)^{n+1}e^{\nu^1\lambda^*T}\big(\Phi(d_1(n+1)) - \Phi(d_1(n))\big) \\ & \qquad - Ke^{-rT}\Big(\Phi(d_2(n+1)) - \Phi(d_2(n))\Big)\Bigg) \\ & - KTe^{-rT}\sum_{n\geq 0}\frac{(\lambda^*T)^n}{n!}e^{-\lambda^*T}\big(1-\Phi(d_2(n))\big) \quad \text{if } \frac{\partial\lambda^*}{\partial r} \leq 0,\end{aligned} \tag{4.5}$$

and

$$\begin{aligned}\frac{\partial P}{\partial r} = {} & T\frac{\partial\lambda^*}{\partial r}\left(1-\nu^1\right) \\ & \times\sum_{n\geq 0}\frac{(\lambda^*T)^n}{n!}e^{-\lambda^*T}\Bigg(S_0\left(1-\nu^1\right)^n e^{\nu^1\lambda^*T}\big(\Phi(d_1(n+1)) - \Phi(d_1(n))\big) \\ & \qquad - Ke^{-rT}\big(\Phi(d_2(n+1)) - \Phi(d_2(n))\big)\Bigg) \\ & - \nu^1T\frac{\partial\lambda^*}{\partial r}Ke^{-rT}\sum_{n\geq 0}\frac{(\lambda^*T)^n}{n!}e^{-\lambda^*T}\big(\Phi(d_2(n+1)) - \Phi(d_2(n))\big) \\ & - KTe^{-rT}\sum_{n\geq 0}\frac{(\lambda^*T)^n}{n!}e^{-\lambda^*T}\big(1-\Phi(d_2(n))\big) \quad \text{if } \frac{\partial\lambda^*}{\partial r} \geq 0.\end{aligned} \tag{4.6}$$

1) If $\partial\lambda^*/\partial r \leq 0$, then from the sign of $A(n)$ we get $\partial P/\partial r < 0$.

2) Now if $\partial\lambda^*/\partial r > 0$, since $B(n) \le 0$ and $(1-\nu^1) > 0$, the first term of $\partial P/\partial r$ is negative. We only need to determine the sign of

$$-\nu^1 T \frac{\partial\lambda^*}{\partial r} K e^{-rT} \sum_{n\ge 0} \frac{(\lambda^* T)^n}{n!} e^{-\lambda^* T}\big(\Phi(d_2(n+1)) - \Phi(d_2(n))\big) - KTe^{-rT}\sum_{n\ge 0}\frac{(\lambda^* T)^n}{n!}e^{-\lambda^* T}\big(1-\Phi(d_2(n))\big). \tag{4.7}$$

As in the call case, we note that

$$\nu^1\big(\Phi(d_2(n+1)) - \Phi(d_2(n))\big) \le 0$$

and distinguish two cases $\nu^1 \ge 0$ and $\nu^1 \le 0$. Again we consider the problem of finding the sign of

$$-\nu^1\frac{\partial\lambda^*}{\partial r}\big(\Phi(d_2(n+1)) - \Phi(d_2(n))\big) - \big(1-\Phi(d_2(n))\big). \tag{4.8}$$

(a) For $\nu^1 \ge 0$, we rewrite the expression (4.8) as

$$\begin{aligned} Y &= \left(-\nu^1\frac{\partial\lambda^*}{\partial r}\big(\Psi(d_2(n)) - \Psi(d_2(n+1))\big) - \big(\Psi(d_2(n))\big)\right) \\ &= \left(\Psi(d_2(n))\left(-1-\nu^1\frac{\partial\lambda^*}{\partial r}\right) + \nu^1\frac{\partial\lambda^*}{\partial r}\Psi(d_2(n+1))\right), \end{aligned} \tag{4.9}$$

where $\Phi = 1-\Psi$.

The above expression is negative if an upper bound of Y is negative. But, for the same reason as in the call case (here Ψ is an increasing function of n),

$$Y < \left(\Psi(d_2(0))\left(-1-\nu^1\frac{\partial\lambda^*}{\partial r}\right) + \nu^1\frac{\partial\lambda^*}{\partial r}\right).$$

Hence for

$$\Psi(d_2(0)) = 1-\Phi(d_2(0)) > \frac{\nu^1\frac{\partial\lambda^*}{\partial r}}{1+\nu^1\frac{\partial\lambda^*}{\partial r}}$$

or

$$\Phi(d_2(0)) < \frac{1}{1+\nu^1\frac{\partial\lambda^*}{\partial r}}$$

$\partial P/\partial r$ is negative.

(b) For $\nu^1 \le 0$ we note that $\Phi(d_2(n))$ (resp. $\Psi(d_2(n))$) is a non-decreasing (resp. non-increasing) function of n and

$$Y \le -\Psi(d_2(n)).$$

Hence Y is negative and so is $\partial P/\partial r$. □

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