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BEGINNING OF THE UKRAINIAN SCHOOL OF PROBABILITY THEORY: A HISTORICAL SKETCH

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Dedicated to the 90th birthday anniversary of Iosif Ill'ich Gikhman

May 26, 2008 marks the 90th birthday anniversary of Iosif Ill'ich Gikhman, a brilliant mathematician and teacher. Dedicated to this date, the conference "Modern Problems of Probability Theory and Related Questions" took place during May 24–26, 2008 in a small city of Uman' in the Cherkassy region, where Iosif Ill'ich Gikhman was born. In connection with this date, I would like to share some thoughts about the role of I. I. Gikhman in the formation of the Ukrainian School of Probability Theory.

It is hard to overestimate the role of Borys Volodymyrovych Gnedenko in the creation of the Ukrainian School of Probability Theory. Many students and young college professors became involved in the studies of contemporary probabilistic problems after his appearance in Ukraine, first in L'viv in 1945 and then in Kyiv in 1949. He had been a representative of the famous Moscow probabilistic school, headed at that time by the prominent mathematicians A. N. Kolmogorov and A. Ya. Khinchin, and had cultivated the spirit of the Moscow school among his young students in Ukraine. Moreover, the Ukrainian probabilistic school, which had been just emerging, had the opportunity to gather that spirit directly, when, in 1953, three of B. V. Gnedenko's students from Kyiv, namely V. S. Korolyuk, V. S. Mikhalevich, and A. V. Skorokhod, were sent to finish their studies at Moscow University due to Gnedenko's lengthy trip abroad. Even though each of these three students had a scientific problem formulated for his thesis by B. V. Gnedenko, the period in which they stayed in Moscow undoubtedly influenced their future scientific work and careers and, therefore, the work and careers of their future students.

Thus, as we can see, the creation of the Ukrainian probabilistic school had been largely influenced by the Moscow probabilistic school, and especially by one of its bright representatives, B. V. Gnedenko.

Similarly to a river that becomes stronger by taking in streams, a scientific school gets stronger by absorbing more and more different scientific ideas and methods. In this regard we have to recall B. V. Gnedenko's forerunners, M. P. Kravchuk and S. N. Bernstein, who had been working in Ukraine before World War II. Their influence on the development of probabilistic studies in our country still requires a thorough analysis.

My aim in writing this note is to describe shortly another line of development of the Ukrainian probabilistic school. This line starts from some papers by M. M. Bogolyubov and M. M. Krylov, who had been studying the limit behavior of a dynamical system under some perturbing factors that approached a "white noise" type stochastic process (see, e.g., the paper [1] of these authors). Bogolyubov and Krylov indicated in [1] that after passing to the limit such a system must be described by a Markov stochastic process whose

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I. I. Gikhman
(May 26, 1918–July 30, 1985)

transition probability density is a solution of the corresponding Fokker–Planck equation. The same equation was obtained in the 1930s by Bernstein [2] who constructed a certain recurrent scheme (which he called a stochastic differential equation) and considered the limiting distribution of its solution. Bernstein did not construct the limiting process but found its distribution at a fixed time.

The Bogolyubov and Krylov paper [1] does not contain a precise justification of the limiting procedure. The problem of providing such a justification was formulated in 1939 by Bogolyubov to his student I. I. Gikhman, who had just graduated from Kyiv University. I. I. Gikhman successfully accomplished this task and published two papers, [3, 4], that became a basis of his candidate's dissertation "On the influence of a stochastic process on a dynamical system". He defended the dissertation in Tashkent in 1942 during a short term leave from the army.

I. I. Gikhman did not confine himself to the proof of certain limit theorems for solutions of prelimit equations. He started working on the problem of constructing the limiting stochastic process. This research led him to the notion of a stochastic differential equation, which he first introduced in a short communication [5] and then considered in detail in a series of papers [6, 7, 8].

It is worth mentioning that the idea to construct stochastic processes from the trajectories of the basic processes, namely the Wiener and Poisson processes, was in the air, as one might say, during the 1940s. In this connection, one should recall the papers [9, 10, 11] by a Japanese mathematician Kiyosi Itô, who generalized the notion of the Wiener integral as well as the integral with respect to Poisson measure to a class of random integrands and, using this generalization, created the theory of stochastic differential equations describing a wide class of stochastic processes. Itô's approach turned out to be extremely useful. Now the notion of Itô's stochastic integral is the base for developing the theory of stochastic differential equations in nearly all monographs dealing with stochastic analysis.

Gikhman did not possess a notion of Itô type stochastic integrals; however, his definition of a stochastic differential equation was quite rigorous and can be applied for describing an even wider class of stochastic processes as compared to Itô's approach. This can be explained by two weaker assumptions used by Gikhman. First, in Gikhman's theory, a stochastic process that locally determined the process to be constructed was not necessarily a process with independent increments (as was the case in Itô's theory), but rather a square-integrable martingale of a certain type. Second, the condition imposed on the drift coefficient was somewhat weaker than the Lipschitz condition with respect to the spatial argument; instead, Gikhman's condition took into account the direction of this coefficient (a similar condition was used by S. N. Bernstein as well).

Having proved a theorem on the existence and uniqueness of a solution of a stochastic differential equation, Gikhman showed that this solution was a differentiable function of the initial data in the case of smooth coefficients. This result allowed him to derive the backward Kolmogorov equation for the mathematical expectation of the corresponding solution in the case of processes without aftereffect (that is, in the case where the above-mentioned martingale was a Wiener process). As a consequence, he obtained the proof of a theorem on the existence of a solution to the Cauchy problem for parabolic equations with smooth coefficients without any assumption on the matrix of coefficients of the second derivatives to be nondegenerate (the role of this assumption is well known in the analytic theory of parabolic equations).

This was a revolutionary result, indeed. Gikhman's approach differs significantly from the devices used at that time by specialists in the field of partial differential equations. His method opened the path for the penetration of purely probabilistic methods into the theory of partial differential equations of parabolic and elliptic types. Moreover, this was the only way to analyze such equations in infinite-dimensional spaces, namely through the corresponding stochastic differential equation (developed by Yu. L. Daletsky and his students).

A. V. Skorokhod (undoubtedly influenced by I. I. Gikhman) was among the first who realized the power of the methods of the theory of stochastic differential equations. His doctoral dissertation "Studies in the theory of random processes", published in 1961 by Kyiv University, dealt solely with the theory of stochastic differential equations. While Gikhman and Itô have been constructing solutions of stochastic differential equations by the method of successive approximations, starting from given coefficients and a given Wiener process (as well as from Poisson measure or some martingale field as in the case of Gikhman), Skorokhod developed a new approach based on the compactness principle for the set of measures generated by stochastic processes. This together with his earlier method of a common probability space allowed him to construct the solutions of equations under wider conditions imposed on the coefficients, namely that the coefficients should be only continuous functions instead of a local Lipschitz condition with respect to the spatial argument as assumed by Gikhman and Itô.

In general, the Skorokhod monograph mentioned above is extremely rich in new approaches and ideas. We find there, for example, a comparison theorem for solutions of a pair of stochastic differential equations, a theorem on the uniqueness of solutions (in the one-dimensional case) proved under much weaker conditions than the Lipschitz condition, a theorem on the absolute continuity of measures corresponding to solutions of a pair of stochastic differential equations, a theorem on embedding sums of independent random variables into the trajectories of a Brownian motion, and a series of limit theorems for Markov processes (using his topology in the space of functions without discontinuities of the second kind). Moreover, Skorokhod pioneered in creating a theory of stochastic differential equations for processes in regions with boundaries. For these regions his results stimulated much interesting research in different probabilistic centers of the world, namely, in the USA (Stroock and Varadhan), in Japan (Ikeda, Watanabe), and many others.

Thus, at the beginning of the 1960s, due to the efforts of Gikhman and Skorokhod, the Ukrainian probabilistic school had taken a position of a world leader in the development of the theory of stochastic differential equations, a theory that became one of the greatest achievements of mathematics in the second half of the 20th century.

An account of the results of the first stage of development of the theory of stochastic differential equations was given in the Gikhman and Skorokhod monograph "Stochastic differential equations", published in 1968 in Kyiv by "Naukova Dumka". Later this monograph was translated into German (in 1971) and also into English (in 1972).

This monograph was followed by other books published together by Gikhman and Skorokhod. Both Gikhman and Skorokhod have carried out this research very intensively and have combined their research with teaching at the University.

In conclusion, I would like to emphasize that I. I. Gikhman's scientific activity was not limited to the theory of stochastic differential equations; rather, it had been expanded to various fields of mathematical statistics, information theory, etc., where he was considered as a leading expert, as well. However the main achievement of his scientific activity, as for me, remains the creation of the theory of stochastic differential equations and bringing to A. V. Skorokhod's attention the problems of this scientific field.

I think that all people who have been lucky enough to study or work with Iosif Ill'ich carry in their hearts the bright image of this outstanding man.

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