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SAMPLE CONTINUITY AND MODELING OF STOCHASTIC PROCESSES FROM THE SPACES $D_{V,W}$

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Abstract. Random sequences and stochastic processes belonging to the spaces $D_{V,W}$ are studied in the paper. Conditions for the sample continuity of such processes are found. The convergence of series of random variables belonging to the spaces $D_{V,W}$ are considered. Models of stochastic processes belonging to the spaces $D_{V,W}$ are studied. Several examples of models are given.

Introduction

The spaces $D_{V,W}$ introduced in the paper [1] are defined as pre-Banach spaces generated by certain pre-metrics, namely by

$$\|\xi\| = \sup_{x\geq 0} V(x)W^{(-1)}(\mathsf{P}\{|\xi| > x\}).$$

The basic properties of the spaces $D_{V,W}$, conditions for the convergence of series of random variables belonging to these spaces, and behavior of the supremum of stochastic processes in the spaces $D_{V,W}$ are considered in [1]. In this paper, we continue studies of the spaces $D_{V,W}$ and stochastic processes belonging to these spaces.

In Section 1, we give basic definitions and results concerning the spaces $D_{V,W}$. Section 2 contains several other results on the random variables and stochastic processes belonging to the spaces $D_{V,W}$ that will be used in the later sections. The sample continuity of stochastic processes is studied in Section 3. Some results concerning the models of stochastic processes in $D_{V,W}$ are obtained in Section 4; the models approximate the initial processes with a given reliability and accuracy. Examples of models for some stochastic processes are discussed in Section 5.

1. The spaces $D_{V,W}$

Let $\{\Omega, \mathcal{B}, \mathsf{P}\}$ be a standard probability space, $L_0(\Omega)$ the space of random variables defined on $\{\Omega, \mathcal{B}, \mathsf{P}\}$, and let $\mathcal{M} \subset L_0(\Omega)$ be some linear space.

Definition 1.1 ([2])**.** A function $\Theta = (\Theta(\xi), \xi \in \mathcal{M})$ is called a pre-norm if, for all random variables $\xi \in \mathcal{M}$,

1. $\Theta(\xi) \in [0, \infty)$;
2. $\Theta(0) = 0$;
3. $\Theta(-\xi) = \Theta(\xi)$.

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Definition 1.2 ([2])**.** If $\mathcal{M}$ is complete with respect to a pre-norm Θ, then it is called a pre-Banach space.

Definition 1.3. A pre-Banach space $\mathcal{M}$ is called a pre-K_σ-space if

a1) $\max(\xi,\eta) \in \mathcal{M}$ and $\min(\xi,\eta) \in \mathcal{M}$ for all $\xi,\eta \in \mathcal{M}$ (this, in particular, means that $|\xi| \in \mathcal{M}$);

a2) $|\xi| \in \mathcal{M}$ provided that $|\xi| \le |\eta|$ and $\eta \in \mathcal{M}$.

Definition 1.4 ([4])**.** Let every random variable $\xi \in \mathcal{M}$ correspond to a nonnegative number $\|\xi\|$ such that

1. $\|\xi\| = 0 \Leftrightarrow \xi = 0$ with probability one;
2. $\|\xi+\eta\| \le \|\xi\| + \|\eta\|$;
3. if $|\lambda| \le 1$, then $\|\lambda\xi\| \le \|\xi\|$.

Then the functional $\|\cdot\|$ is called a quasi-norm.

Definition 1.5. If $\mathcal{M}$ is complete with respect to a quasi-norm $\|\cdot\|$, then $\mathcal{M}$ is called a quasi-Banach space.

Remark 1.1. Every quasi-norm is a pre-norm. If we assume that $\|\lambda\xi\| = |\lambda| \cdot \|\xi\|$ instead of condition 3 in Definition 1.4, then a quasi-norm is a usual norm.

Definition 1.6 ([3])**.** A positive nondecreasing sequence $\mu(n)$, $n \ge 1$, is called a majorizing characteristics of a pre-Banach K_σ-space $\mathcal{M}$ if

$$\Theta(\max_{1\le k\le n} |\xi_k|) \le \mu(n) \max_{1\le k\le n} \Theta(\xi_k)$$

for all $\xi_k \in \mathcal{M}$, $k = 1,2,\dots,n$.

The notion of a characteristic is introduced in the papers [7] and [8] for Orlicz spaces, in [5] for K_σ-spaces, and in [6] for quasi-Banach K_σ-spaces.

Definition 1.7 ([3])**.** Let $J = J(\lambda)$ be a nondecreasing function such that $J(\lambda) \ge 0$ and $J(\lambda) \to 0$ as $\lambda \to 0$. If a pre-norm $\Theta(\cdot)$ defined in $\mathcal{M}$ is such that

$$\Theta(\lambda\xi) \le J(|\lambda|)\Theta(\xi),$$

then Θ is called a pre-norm subordinate to the function J.

Definition 1.8 ([2])**.** A continuous even convex function $U = (U(x), x \in \mathbf{R})$ is called a C-function if $U(0) = 0$ and $U(x)$ is increasing for $x > 0$.

Now we define the space $D_{V,W}(\Omega)$.

Definition 1.9 ([1])**.** Let $W = \{W(x), x \in \mathbf{R}\}$ and $V = \{V(x), x \in \mathbf{R}\}$ be two functions such that $W(0) = 0$, $W(x) > 0$, and $V(x) > 0$ for $x \ne 0$. Moreover, we assume that both functions are even, increasing, and continuous for $x > 0$. Let there exist a constant $C > 0$ and a continuous function $Z = \{Z(x), x > 0\}$ such that

$$W^{(-1)}(x+y) \le C\left(W^{(-1)}(x) + W^{(-1)}(y)\right),$$

$$V(ax) \le Z(a)V(x)$$

for $x > 0$ and for all constants $a > 0$, and

$$0 < Z(x) < \infty$$

for $|x| < \infty$. We say that a random variable ξ belongs to the space $D_{V,W}(\Omega)$ if

$$\sup_{x\ge 0} V(x) W^{(-1)}(\mathsf{P}\{|\xi| > x\}) < \infty. \tag{1}$$

Examples of functions W and V with the above properties are $W(x) = |x|^a$ or $W(x) = \exp\{|x|^a\} - 1$, $a > 0$, and $V(x) = |x|^b$, $b > 0$.

Theorem 1.1 ([1]). *The space $D_{V,W}(\Omega)$ is a pre-K_σ-space with respect to the following pre-norm:*

$$\|\xi\|_{V,W} = \left(\sup_{x>0} V(x) W^{-1}(\mathsf{P}\{|\xi| > x\})\right)^{1/2}.$$

If $\|\xi_n - \xi_m\|_{V,W} \to 0$ as $n, m \to \infty$ and $\sup_n \|\xi_n\|_{V,W} < \infty$, then there exists a random variable $\xi \in D_{V,W}(\Omega)$ such that $\|\xi_n - \xi\|_{V,W} \to 0$ as $n \to \infty$. Moreover, the pre-norm $\|\cdot\|_{V,W}$ is subordinate to the function $J(\lambda) = (Z(\lambda))^{1/2}$.

Let $W(x)$ be an Orlicz C-function and let $V(x)$ be the inverse to an Orlicz C-function. Then the functional $\|\cdot\|$ is a quasi-norm and the space is complete with respect to this quasi-norm.

Finally,

$$\mathsf{P}\{|\xi| > x\} \le W\left(\frac{\|\xi\|^2_{V,W}}{V(x)}\right) \tag{2}$$

for all $x > 0$.

Theorem 1.2 ([1]). *The sequence*

$$\mu(n) = \sup_{0<t<1/n} \left(\frac{W^{(-1)}(tn)}{W^{(-1)}(t)}\right)^{1/2}$$

is a majorizing characteristic of the space $D_{V,W}(\Omega)$.

2. Properties of series of random variables and stochastic processes belonging to the spaces $D_{V,W}$

Theorem 2.1 ([1]). *Let ξ_k be random variables belonging to $D_{V,W}(\Omega)$, $\|\cdot\|$ be a pre-norm such that $\|\xi_k\| > 0$, $f(x) = xV(W(x))$, $x > 0$, and let $f^{(-1)}(x)$ be the inverse to the function $f(x)$. The series*

$$\sum_{k=1}^{\infty} \xi_k \tag{3}$$

converges in probability if the series

$$\sum_{k=1}^{\infty} \alpha_k^* \tag{4}$$

converges, where

$$\alpha_k^* = V^{(-1)}\left(\frac{\|\xi_k\|^2}{f^{(-1)}(\|\xi_k\|^2)}\right).$$

Moreover, if

$$x \ge \mu = \sum_{k=1}^{\infty} V^{(-1)}\left(\frac{\|\xi_k\|^2}{f^{(-1)}(\|\xi_k\|^2)}\right),$$

then

$$\mathsf{P}\left\{\left|\sum_{k=1}^{\infty} \xi_k\right| \ge x\right\} \le \sum_{k=1}^{\infty} W\left(\frac{\|\xi_k\|^2}{V(\frac{\alpha_k^* x}{\mu})}\right), \tag{5}$$

where the series on the right hand side of (5) *converges for $x \ge \mu$.*

Remark 2.1. The function $x/f^{(-1)}(x)$ increases, since the function $f(x)/x = V(W(x))$ increases.

Theorem 2.2 ([1]). *Let $W(x) = |x|^a$, $a > 0$, and let $V(x) = |x|^b$, $b > 0$. Then series* (3) *converges in probability if the series*

$$\mu = \sum_{k=1}^{\infty} \|\xi_k\|^{2a/(ab+1)}$$

converges. Moreover,

$$\mathsf{P}\left\{\left|\sum_{k=1}^{\infty} \xi_k\right| > x\right\} \leq \frac{1}{x^{ab}} \left(\sum_{k=1}^{\infty} \|\xi_k\|^{2a/(ab+1)}\right)^{ab+1}$$

for $x \geq \mu$, that is, $\sum_{k=1}^{\infty} \xi_k$ belongs to the space $D_{V,W}$, and

$$\left\|\sum_{k=1}^{\infty} \xi_k\right\| \leq \left(\sum_{k=1}^{\infty} \|\xi_k\|^{2a/(ab+1)}\right)^{(ab+1)/(2a)}.$$

Definition 2.1 ([1]). We say that a stochastic process $X(t) = \{X(t), t \in T\}$ belongs to the space $D_{V,W}$ if $X(t) \in D_{V,W}$ for all t.

The processes represented in the form

$$\xi(t) = \sum_{k=1}^{\infty} \xi_k \phi_k(t), \qquad t \in T, \tag{6}$$

are examples of stochastic processes belonging to the space $D_{V,W}$ if $\xi_k \in D_{V,W}$ and if the latter series converges in the space $D_{V,W}$.

Conditions for the convergence of series in (6) are presented in [1].

Definition 2.2 ([2]). A function $\rho(t,s)$, $t,s \in T$, is called a quasi–metric if $\rho(t,s) \in [0,\infty)$, $\rho(t,t) = 0$, and $\rho(t,s) = \rho(s,t)$.

Let $X = \{X(t), t \in T\}$ be a stochastic process belonging to the space $D_{V,W}$. Then $\rho_X(t,s) = \|X(s) - X(t)\|$ is called the pre-metric generated by the process X.

Let a process X be such that

- **(A1)** $\sup_{t\in T} \|X(t)\| < \infty$;
- **(A2)** the space (T, ρ_X) is separable and X is a separable process in (T, ρ_X).

Put $\varepsilon_0 = \sup_{t,s\in T} \rho_X(t,s)$. Condition **(A1)** implies that $\varepsilon_0 < \infty$. Let $\theta \in (0,1)$, $\varepsilon_k = \varepsilon_0 \theta^k$, and let $N(\varepsilon)$ be the metric capacity of the space (T,ρ), that is, $N(\varepsilon)$ is the minimum number of closed balls covering (T,ρ).

The following result contains conditions for $\sup_{t\in T} X(t) < \infty$ with probability one as well as estimates for the distribution of this supremum.

Theorem 2.3 ([1]). *Let a stochastic process X satisfy conditions* **(A1)** *and* **(A2)**. *If the series*

$$\sum_{n=1}^{\infty} V^{(-1)}\left(\frac{\mu(N(\varepsilon_n))^2 \varepsilon_{n-1}^2}{f^{(-1)}(\mu(N(\varepsilon_n))^2 \varepsilon_{n-1}^2)}\right)$$

converges, where f is defined in Theorem 2.1, *then*

$$\mathsf{P}\left\{\sup_{t\in T} |X(t)| \geq x\right\} \leq W\left(\frac{\inf_{t\in T} \|X(t)\|^2}{V(\psi_0 x)}\right) + \sum_{k=1}^{\infty} W\left(\frac{\mu(N(\varepsilon_k))^2 \varepsilon_{k-1}^2}{V(\psi_k x)}\right), \tag{7}$$

where

$$\psi_0 = \frac{1}{\Psi} V^{(-1)} \left(\frac{\inf_{t\in T} \|X(t)\|^2}{f^{(-1)}(\inf_{t\in T} \|X(t)\|^2)} \right), \qquad \psi_k = \frac{1}{\Psi} V^{(-1)} \left(\frac{\mu(N(\varepsilon_k))^2 \varepsilon_{k-1}^2}{f^{(-1)}(\mu(N(\varepsilon_k))^2 \varepsilon_{k-1}^2)} \right),$$

$$\Psi = \sum_{k=0}^{\infty} \psi_k, \qquad x > \Psi.$$

Theorem 2.4 ([1]). *Let a stochastic process $X = \{X(t), t \in T\}$ be such that $X \in D_{V,W}$ and let $W(x) = |x|^a$, $a > 0$, and $V(x) = |x|^b$, $b > 0$. Assume that X satisfies conditions* **(A1)** *and* **(A2)**.

If

$$\int_0^{\Delta_0 p} \left(N(u^{(ab+1)/(2a)}) \right)^{1/(ab+1)} du < \infty, \tag{8}$$

where $p = \theta^{2a/(ab+1)}$, $0 < \theta < 1$, $\Delta_0 = \varepsilon_0^{2a/(ab+1)}$, and $\varepsilon_0 = \sup_{t,s\in T} \rho_X(t,s)$, then

$$\sup_{t\in T} |X(t)| \in D_{V,W}$$

and, moreover,

$$\mathsf{P}\left\{ \sup_{t\in T} |X(t)| \geq x \right\} \leq \frac{1}{x^{ab}} \left(\inf_{t\in T} \|X(t)\|^{2a/(ab+1)} + \frac{1}{p(1-p)} \int_0^{\Delta_0 p} \left(N\left(u^{(ab+1)/(2a)} \right) \right)^{1/(ab+1)} du \right).$$

3. The continuity of stochastic processes belonging to the spaces $D_{V,W}$

Let X be a stochastic process belonging to the space $D_{V,W}$ such that

$$\sup_{t\in T} \|X(t)\| < \infty.$$

Let $\rho_X(t,s) = \|X(t) - X(s)\|$ be the quasi-metric generated by the process X. Also let (T, ρ_X) be a separable space and X be a separable process in (T, ρ_X).

Let $\theta \in (0,1)$ and $\varepsilon_k = \varepsilon_0 \theta^k$, $k \geq 1$, where

$$\varepsilon_0 = \sup_{t,s\in T} \|X(t) - X(s)\|.$$

By V_{ε_k}, we denote the set of centers of closed balls of radius ε_k that form a minimal covering of the space (T, ρ). The cardinality of the set V_{ε_k} is equal to $N(\varepsilon_k)$. Let $t, s \in T$ be some points such that $\rho(t,s) < \varepsilon$ for $0 < \varepsilon < \varepsilon_0$.

Now we find k such that $\varepsilon_k < \varepsilon < \varepsilon_{k-1}$. Then

$$V_k = \bigcup_{j=k}^{\infty} V_{\varepsilon_j}$$

is the set of separability of the process $X(t)$, since $X(t)$ is continuous in probability.

By S_n, we denote the minimal ε_n-net of the set T with respect to the pseudo-metric ρ_x. Put $S = \bigcup_{n=0}^{\infty} S_n$.

Definition 3.1 ([2]). A family of mappings $\alpha_k(t)$, $k = 0, 1, \ldots$, is called an α-procedure if every point of S corresponds to a unique point α_k of S_k such that $\rho(t, \alpha_k(t)) \leq \varepsilon_k$.

Theorem 3.1. *Assume that a stochastic process X satisfies all the above conditions. If the following two series*

$$\sum_{l=k}^{\infty} V^{(-1)}\left(\frac{\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2}{f^{(-1)}\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2)}\right)$$

and

$$\sum_{l=k}^{\infty} V^{(-1)}\left(\frac{\varepsilon_{l-1}^2}{f^{(-1)}(\varepsilon_{l-1}^2)}\right)$$

converge and $x \geq \Psi$, where

$$\Psi = V^{(-1)}\left(\frac{\mu^2(N^2(\varepsilon_k))\hat{\varepsilon}^2}{f^{(-1)}(\mu^2(N^2(\varepsilon_k))\hat{\varepsilon}^2)}\right) + \sum_{l=k}^{\infty} V^{(-1)}\left(\frac{\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2}{f^{(-1)}(\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2)}\right),$$

then

$$\begin{aligned}&\mathsf{P}\left\{\sup_{\rho(t,s)<\varepsilon} |X(t) - X(s)| \geq x\right\}\\ &\qquad \leq W\left(\frac{\mu^2(N^2(\varepsilon_k))\hat{\varepsilon}_k^2}{V(\psi_0 x)}\right) + \sum_{l=k}^{\infty} W\left(\frac{\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2}{V(\psi_l x)}\right),\end{aligned}$$

where

$$\begin{aligned}\psi_0 &= \frac{1}{\Psi} V^{(-1)}\left(\frac{\mu^2(N^2(\varepsilon_k))\hat{\varepsilon}^2}{f^{(-1)}(\mu^2(N^2(\varepsilon_k))\hat{\varepsilon}^2)}\right),\\ \psi_l &= \frac{1}{\Psi} V^{(-1)}\left(\frac{\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2}{f^{(-1)}(\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2}\right),\end{aligned}$$

and

$$\hat{\varepsilon} = \varepsilon \frac{5 - 3\theta}{1 - \theta}.$$

Moreover $X(t)$ is a sample-continuous stochastic process in the space (T, ρ).

Proof. Let $m > k$ be an arbitrary number. Consider the points

$$t_m = \alpha_m(t), \quad t_{m-1} = \alpha_{m-1}(t_m), \quad \ldots, \quad t_k = \alpha_k(t_{k+1})$$

and

$$s_m = \alpha_m(t), \quad s_{m-1} = \alpha_{m-1}(s_m), \quad \ldots, \quad s_k = \alpha_k(s_{k+1}),$$

where $\alpha_k(t)$ is an α-procedure. Then

$$\begin{aligned}&X(t) - X(s)\\ &\quad = \big(X(t) - X(\alpha_m(t))\big) + \big(X(s) - X(\alpha_m(s))\big)\\ &\qquad + \sum_{l=k}^{m-1}(X(t_{l+1}) - X(t_l)) + \sum_{l=k}^{m-1}(X(s_{l+1}) - X(s_l)) + (X(t_k) - X(s_k)).\end{aligned} \tag{9}$$

This implies that

$$
\begin{aligned}
&\mathsf{P}\{|X(t_k)-X(s_k)|\ge x\}\\
&\quad\le \mathsf{P}\{|X(t)-X(\alpha_m(t))|>x\psi_1'\}+\mathsf{P}\{|X(s)-X(\alpha_m(s))|>x\psi_1''\}\\
&\qquad+\sum_{l=k}^{m-1}\mathsf{P}\{|X(t_{l+1})-X(t_l)|>x\psi_l'\}+\sum_{l=k}^{m-1}\mathsf{P}\{|X(s_{l+1})-X(s_l)|>x\psi_l''\}\\
&\qquad+\mathsf{P}\{|X(t)-X(s)|>x\psi_0\}\\
&\quad\le W\left(\frac{\|X(t)-X(\alpha_m(t))\|^2}{V(x\psi_1')}\right)+W\left(\frac{\|X(s)-X(\alpha_m(s))\|^2}{V(x\psi_1'')}\right)\\
&\qquad+\sum_{l=k}^{m-1}W\left(\frac{\|X(t_l)-X(\alpha_{l-1}(t_l))\|^2}{V(x\psi_l')}\right)\\
&\qquad+\sum_{l=k}^{m-1}W\left(\frac{\|X(s_l)-X(\alpha_{l-1}(s_l))\|^2}{V(x\psi_l'')}\right)\\
&\qquad+W\left(\frac{\|X(t)-X(s)\|^2}{V(x\psi_0)}\right)\\
&\quad\le 2W\left(\frac{\varepsilon_{m-1}^2}{V(x\psi_1)}\right)+2\sum_{l=k}^{m-1}W\left(\frac{\varepsilon_{l-1}^2}{V(x\psi_l)}\right)+W\left(\frac{\varepsilon_0^2}{V(x\psi_0)}\right),
\end{aligned}
\tag{10}
$$

since $\|X(t)-X(\alpha_{n-1}(t))\|\le\varepsilon_{n-1}$.

Then equality (9) implies that

$$
\begin{aligned}
|X(t_k)-X(s_k)| &\le |X(t)-X(\alpha_m(t))|+|X(s)-X(\alpha_m(s))|\\
&\quad+\sum_{l=k}^{m-1}|X(t_{l+1})-X(t_l)|+\sum_{l=k}^{m-1}|X(s_{l+1})-X(s_l)|+|X(t)-X(s)|\\
&\le 2\sum_{l=k}^{m-1}\max_{u\in V_{\varepsilon_l}}|X(u)-X(\alpha_l(u))|+|X(t)-X(\alpha_k(t))|\\
&\quad+|X(s)-X(\alpha_k(s))|+|X(t)-X(s)|\\
&\le 2\sum_{l=k}^{m-1}\varepsilon_l+2\varepsilon_k+\varepsilon\le\hat{\varepsilon},
\end{aligned}
$$

where

$$
\hat{\varepsilon}=\varepsilon\frac{5-3\theta}{1-\theta}.
$$

Passing to the limit in (9) as $m\to\infty$ we get

$$
\begin{aligned}
\sup_{\rho(t,s)\le\varepsilon}|X(t)-X(s)| &= \sup_{|t-s|\le\varepsilon,t,s,\in V}|X(t)-X(s)|\\
&\le \max_{v,w\in V_k}|X(v)-X(w)|+2\sum_{l=k}^{\infty}\max_{u\in V_{l+1}}|X(u)-X(\alpha_l(u))|
\end{aligned}
$$

provided inequality (10) holds.

After some transformations we obtain

$$\mathsf{P}\left\{\sup_{\rho(t,s)\le\varepsilon}|X(t)-X(s)|\ge x\right\}\le\mathsf{P}\left\{\max_{v,w\in V_k}|X(v)-X(w)|\ge\psi_0 x\right\}+\sum_{l=k}^{\infty}\mathsf{P}\left\{\max_{u\in V_{l+1}}|X(u)-X(\alpha_l(u))|\ge\psi_l x\right\}.$$

Reasoning similarly to the proof of Theorem 2.3 (see [1])) we get

$$\mathsf{P}\left\{\sup_{\rho(t,s)\le\varepsilon}|X(t)-X(s)|\le x\right\}\le W\left(\frac{\mu^2(N^2(\varepsilon_k))\hat{\varepsilon}_k^2}{V(\psi_0 x)}\right)+\sum_{l=k}^{\infty}W\left(\frac{\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2}{V(\psi_l x)}\right),$$

where

$$\psi_0=\frac{1}{\Psi}V^{(-1)}\left(\frac{\mu^2(N^2(\varepsilon_k))\hat{\varepsilon}^2}{f^{(-1)}(\mu^2(N^2(\varepsilon_k))\hat{\varepsilon}^2)}\right),$$

$$\psi_l=\frac{1}{\Psi}V^{(-1)}\left(\frac{\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2}{f^{(-1)}(\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2}\right),$$

$$\Psi=V^{(-1)}\left(\frac{\mu^2(N^2(\varepsilon_k))\hat{\varepsilon}^2}{f^{(-1)}(\mu^2(N^2(\varepsilon_k))\hat{\varepsilon}^2)}\right)+\sum_{l=k}^{\infty}V^{(-1)}\left(\frac{\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2}{f^{(-1)}(\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2}\right).$$

Since $W(x)$ increases for $x>0$,

$$W\left(\frac{\mu^2(N^2(\varepsilon_k))\hat{\varepsilon}^2}{V(\psi_0 x)}\right)\to 0$$

if x is fixed. Since the series

$$\sum_{l=k}^{\infty}W\left(\frac{\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2}{V(\psi_l x)}\right)$$

converges, we pass to the limit as $k\to\infty$ and obtain

$$W\left(\frac{\mu^2(N^2(\varepsilon_k))\hat{\varepsilon}^2}{V(\psi_0 x)}\right)+\sum_{l=k}^{\infty}W\left(\frac{\mu^2(N^2(\varepsilon_l))\varepsilon_{l-1}^2}{V(\psi_l x)}\right)\to 0,$$

whence

$$\mathsf{P}\left\{\sup_{\rho(t,s)<\varepsilon}|X(t)-X(s)|\ge x\right\}\to 0$$

as $k\to\infty$.

This implies that the process is sample-continuous in (T,ρ). □

Theorem 3.2. *Let* $W(x)=x^a$, $a>1$, *and* $V(x)=x^b$, $0<b<1$. *If*

$$\int_0^{\Delta_0 p^{k+1}} N\left(u^{(ab+1)/(2a)}\right)^{2/(ab+1)}du<\infty,$$

then

$$
\begin{aligned}
&\mathsf{P}\left\{\sup_{\rho(t,s)<\varepsilon}|X(s)-X(t)|\geq x\right\} \\
&\leq \frac{1}{x^{ab}p(1-p)}\left(\hat{C}\int_{\Delta_0 p^{k+1}}^{\Delta_0 p^k} N\left(u^{(ab+1)/(2a)}\right)^{2/(ab+1)} du\right. \\
&\qquad\left. + 2\int_0^{\Delta_0 p^{k+1}} N\left(u^{(ab+1)/(2a)}\right)^{2/(ab+1)} du\right),
\end{aligned}
\tag{11}
$$

where

$$
\hat{C} = \left(\frac{5-3\theta}{1-\theta}\right)^{2/(ab+1)}. \tag{12}
$$

Moreover, $X(t)$ is a sample-continuous stochastic process in (T,ρ).

Proof. Reasoning as in the proof of Theorem 2.3 (see [1]) we get

$$
\begin{aligned}
&\mathsf{P}\left\{\sup_{\rho(t,s)<\varepsilon}|X(s)-X(t)|\geq x\right\} \\
&\leq \frac{1}{x^{ab}p(1-p)}\left(\left(\mu\left(N^2(\varepsilon_k)\right)\hat{\varepsilon}\right)^{2a/(ab+1)} + 2\sum_{l=k+1}^{\infty}\left(\mu\left(N^2(\varepsilon_l)\right)\varepsilon_{l-1}\right)^{2a/(ab+1)}\right) \\
&\leq \frac{1}{x^{ab}p(1-p)}\left(\int_{\Delta_0 p^{k+1}}^{\Delta_0 p^k}\mu\left(N^2\left(u^{(ab+1)/(2a)}\right)\left(\frac{5-3\theta}{1-\theta}\right)^2\right)^{2a/(ab+1)} du\right. \\
&\qquad\left. +2\int_0^{\Delta_0 p^{k+1}}\mu\left(N^2\left(u^{(ab+1)/(2a)}\right)\right)^{2a/(ab+1)} du\right),
\end{aligned}
$$

where the numbers Δ_0 and p are defined in the proof of Theorem 2.3 in [1].

Theorem 1.2 implies that

$$
\mu(n) = \sup_{0<t<1/n}\left(\frac{W^{(-1)}(tn)}{W^{(-1)}(t)}\right)^{1/2} = n^{1/2a}.
$$

Then

$$
\begin{aligned}
&\mathsf{P}\left\{\sup_{\rho(t,s)<\varepsilon}|X(s)-X(t)|\geq x\right\} \\
&\leq \frac{1}{x^{ab}p(1-p)}\left(\hat{C}\int_{\Delta_0 p^{k+1}}^{\Delta_0 p^k}\left(N\left(u^{(ab+1)/(2a)}\right)\right)^{2/(ab+1)} du\right. \\
&\qquad\left. + 2\int_0^{\Delta_0 p^{k+1}}\left(N^2\left(u^{(ab+1)/(2a)}\right)\right)^{2/(ab+1)} du\right) \\
&\leq \frac{C}{x^{ab}p(1-p)}\int_0^{\Delta_0 p^{k+1}} N\left(u^{(ab+1)/(2a)}\right)^{2/(ab+1)} du.
\end{aligned}
$$

□

Theorem 3.3. *Let $X=\{X(t), t\in[0,T]\}$ be a stochastic process such that $X\in D_{V,W}$. Assume that X is a separable process in $[0,T]$. Let $W(x)=|x|^a$, $a>0$, and $V(x)=|x|^b$, $b>0$. If*

$$
\sup_{|t-s|\leq h}\|X(t)-X(s)\|\leq Dh^{\zeta}
$$

for some $D>0$ and $\zeta>1/a$, then

$$\sup_{t\in[0,T]}|X(t)|\in D_{V,W}$$

and

$$\mathsf{P}\left\{\sup_{\rho(t,s)\le\varepsilon}|X(s)-X(t)|\ge x\right\}$$
$$\le\frac{1}{x^{ab}p(1-p)}\left(\hat{C}\int_{\Delta_0p^{k+1}}^{\Delta_0p^k}\left(\frac{DT}{2u^{(ab+1)/(2a\zeta)}}+1\right)^{2/(ab+1)}du\right.$$
$$\left.+2\int_0^{\Delta_0p^{k+1}}\left(\frac{DT}{2u^{(ab+1)/(2a\zeta)}}+1\right)^{2/(ab+1)}du\right)$$

for all $x>0$, where $\hat{C}$ is defined in (12).

Moreover, $X(t)$ is a sample-continuous stochastic process in (T,ρ).

Proof. The assumptions of the theorem imply that

$$N(\varepsilon)\le\frac{DT}{2\varepsilon^{1/\zeta}}+1.$$

Inequality (11) can be used to obtain the following estimates:

$$\mathsf{P}\left\{\sup_{\rho(t,s)<\varepsilon}|X(s)-X(t)|\ge x\right\}$$
$$\le\frac{1}{x^{ab}p(1-p)}\left(\hat{C}\int_{\Delta_0p^{k+1}}^{\Delta_0p^k}\left(N\left(u^{(ab+1)/(2a)}\right)\right)^{2/(ab+1)}du\right.$$
$$\left.+2\int_0^{\Delta_0p^{k+1}}\left(N\left(u^{(ab+1)/(2a)}\right)\right)^{2/(ab+1)}du\right)$$
$$\le\frac{1}{x^{ab}p(1-p)}\left(\hat{C}\int_{\Delta_0p^{k+1}}^{\Delta_0p^k}\left(\frac{DT}{2u^{(ab+1)/(2a\zeta)}}+1\right)^{2/(ab+1)}du\right.$$
$$\left.+2\int_0^{\Delta_0p^{k+1}}\left(\frac{DT}{2u^{(ab+1)/(2a\zeta)}}+1\right)^{2/(ab+1)}du\right).$$

The two latter integrals converge if so does the integral

$$\int_0^{\Delta_0p^{k+1}}\frac{1}{u^{1/(a\zeta)}}\,du,$$

which is the case if $\zeta>1/a$.

The integrals

$$\int_0^{\Delta_0p^{k+1}}\left(\frac{DT}{2u^{(ab+1)/(2a\zeta)}}+1\right)^{2/(ab+1)}du$$

and

$$\int_{\Delta_0p^{k+1}}^{\Delta_0p^k}\left(\frac{DT}{2u^{(ab+1)/(2a\zeta)}}+1\right)^{2/(ab+1)}du$$

can be estimated via the hypergeometric function. □

4. Models of stochastic processes belonging to the spaces $D_{V,W}$

Consider a stochastic process X represented in the following form:

$$X(t) = \sum_{k=1}^{\infty} \xi_k \phi_k(t) \tag{13}$$

for $t \in [0,T]$. We also consider another process X_N given by

$$X_N(t) = \sum_{k=1}^{N} \xi_k \phi_k(t).$$

Then $X_N(t)$ is called a model of the process X.

Put

$$\tilde{X}_N(t) := \sum_{k=N+1}^{\infty} \xi_k \phi_k(t) = X(t) - X_N(t). \tag{14}$$

Theorem 4.1. *Let $X = \{X(t), t \in [0,T]\}$ be a stochastic process represented in the form of (13) and such that $\xi_k \in D_{V,W}$. Let $W(x) = |x|^a$, $a > 1$, and $V(x) = |x|^b$, $0 < b < 1$. We assume that conditions* **(A1)** *and* **(A2)** *hold for X. We further assume that*

$$\sup_{|t-s|<h} |\phi_k(s) - \phi_k(t)| \le C_k |h|^{\zeta}.$$

If

$$\sum_{k=1}^{\infty} \|\xi_k\|^{2a/(ab+1)} C_k^{ab/(ab+1)} < \infty$$

and

$$\zeta > \frac{1}{ab},$$

then $\sup_{t\in T} |\tilde{X}_N(t)| \in D_{V,W}$. Moreover,

$$\begin{aligned}\mathsf{P}\left\{\sup_{t\in[0,T]} |\tilde{X}_N(t)| > x\right\} \\ \le \frac{1}{x^{ab}} \Bigg(\sum_{k=N+1}^{\infty} \|\xi_k\|^{2a/(ab+1)} \inf_{t\in[0,T]} \left|\phi_k^{ab/(ab+1)}(t)\right| \\ + \frac{T^{1/(ab+1)} \left(\sum_{k=N+1}^{\infty} C_k^{ab/(ab+1)} \|\xi_k\|^{2a/(ab+1)}\right)^{1/(ab\zeta)}}{2^{ab/(ab+1)} p(1-p)} \frac{ab\zeta(\Delta_N p)^{1-1/(ab\zeta)}}{ab\zeta - 1} \\ + \frac{\Delta_N}{1-p} \Bigg),\end{aligned}$$

where

$$\Delta_N = \left(\sup_{t,s\in[0,T]} (X_N(t) - X_N(s)) \right)^{2a/(ab+1)}.$$

Proof. Let

$$\sup_{|t-s|<h} |\phi_k(t) - \phi_k(s)| \le C_k \cdot |h|^{\zeta}.$$

Then

$$\begin{aligned}\sup_{|t-s|<h}\|\tilde X_N(t)-\tilde X_N(s)\| &= \sup_{|t-s|<h}\left\|\sum_{k=N+1}^{\infty}\xi_k(\phi_k(t)-\phi_k(s))\right\|\\ &\le \left(\sum_{k=N+1}^{\infty}\|\xi_k\|^{2a/(ab+1)}\sup_{|t-s|<h}J^{2a/(ab+1)}\left(\phi_k(t)-\phi_k(s)\right)\right)^{(ab+1)/(2a)}\\ &\le \left(\sum_{k=1}^{\infty}\|\xi_k\|^{2a/(ab+1)}\left(C_k^{b/2}h^{b\zeta/2}\right)^{2a/(ab+1)}\right)^{(ab+1)/(2a)}\\ &= h^{b\zeta/2}\left(\sum_{k=1}^{\infty}\|\xi_k\|^{2a/(ab+1)}C_k^{ab/(ab+1)}\right)^{(ab+1)/(2a)},\end{aligned}$$

since $J(z)=z^{b/2}$ and

$$\left\|\sum_{k=N+1}^{\infty}\xi_k\right\| \le \left(\sum_{k=N+1}^{\infty}\|\xi_k\|^{2a/(ab+1)}\right)^{(ab+1)/(2a)}.$$

Since $t\in[0,T]$, we have

$$N(\varepsilon)\le \frac{T}{2\delta^{(-1)}(h)}+1,$$

where $\delta(h)$ can be chosen such that

$$\delta(h)=h^{b\zeta/2}\left(\sum_{k=1}^{\infty}\|\xi_k\|^{2a/(ab+1)}C_k^{ab/(ab+1)}\right)^{(ab+1)/(2a)}$$

if the series

$$\sum_{k=N+1}^{\infty}\|\xi_k\|^{2a/(ab+1)}C_k^{ab/(ab+1)}$$

converges.

By Theorem 2.4,

$$\begin{aligned}&\mathsf{P}\left\{\sup_{t\in[0,T]}|\tilde X_N(t)|>x\right\}\\ &\quad\le \frac{1}{x^{ab}}\left(\inf_{t\in[0,T]}\|\tilde X_N(t)\|^{2a/(ab+1)}+\frac{1}{p(1-p)}\int_0^{\Delta_N p}\left(N\left(u^{(ab+1)/(2a)}\right)\right)^{1/(ab+1)}du\right)\\ &\quad\le \frac{1}{x^{ab}}\Bigg(\inf_{t\in[0,T]}\|\tilde X_N(t)\|^{2a/(ab+1)}\\ &\qquad\qquad+\frac{1}{p(1-p)}\int_0^{\Delta_N p}\left(\frac{T}{2\delta^{(-1)}\left(u^{(ab+1)/(2a)}\right)}+1\right)^{1/(ab+1)}du\Bigg)\end{aligned}$$

$$\le \frac{1}{x^{ab}} \Bigg(\inf_{t\in[0,T]} \|\tilde{X}_N(t)\|^{2a/(ab+1)} + \frac{1}{p(1-p)} \times \int_0^{\Delta_N p} \left(\frac{T\Big(\sum_{k=N+1}^{\infty} C_k^{ab/(ab+1)} \|\xi_k\|^{2a/(ab+1)}\Big)^{(ab+1)/(ab\zeta)}}{2u^{(ab+1)/(ab\zeta)}} + 1 \right)^{1/(ab+1)} du \Bigg)$$

$$\le \frac{1}{x^{ab}} \Bigg(\inf_{t\in[0,T]} \|\tilde{X}_N(t)\|^{2a/(ab+1)} + \frac{T^{1/2a}\left(\sum_{k=N+1}^{\infty} C_k^{ab/(ab+1)} \|\xi_k\|^{2a/(ab+1)}\right)^{(ab+1)/(2a^2b\zeta)}}{p(1-p)2^{ab/(ab+1)}} \times \int_0^{\Delta_N p} \frac{du}{u^{(ab+1)/(2a^2b\zeta)}} + \frac{\Delta_0}{1-p} \Bigg)$$

$$\le \frac{1}{x^{ab}} \Bigg(\sum_{k=N+1}^{\infty} \|\xi_k\|^{2a/(ab+1)} \inf_{t\in[0,T]} \left|\phi_k^{ab/(ab+1)}(t)\right| + \frac{T^{1/(ab+1)}\left(\sum_{k=N+1}^{\infty} C_k^{ab/(ab+1)} \|\xi_k\|^{2a/(ab+1)}\right)^{1/(ab\zeta)}}{p(1-p)2^{ab/(ab+1)}} \times \frac{ab\zeta(\Delta_N p)^{1-1/(ab\zeta)}}{ab\zeta - 1} + \frac{\Delta_N}{1-p} \Bigg)$$

if the integral

$$\int_0^{\Delta_N p} \frac{1}{u^{1/(ab\zeta)}}\, du$$

is finite, which is the case for

$$\zeta > \frac{1}{ab}. \qquad \square$$

Corollary 4.1. *Assume that*

$$\sup_{|t-s|<h} |\phi_k(s) - \phi_k(t)| \le C_k |h|^{\zeta}.$$

A model $X_N(t)$ approximates a process $X(t)$ for $t \in [0,T]$ in the space $D_{V,W}(\Omega)$ with given accuracy æ > 0 and reliability $1-\nu$, $0<\nu<1$, which means that

$$\mathsf{P}\left\{ \sup_{t\in[0,T]} |\widetilde{X}_N(t)| > æ \right\} \le \nu$$

if

$$\nu \geq \frac{1}{æ^{ab}}\left(\sum_{k=N+1}^{\infty}\|\xi_k\|^{2a/(ab+1)} \inf_{t\in[0,T]}\left|\phi_k^{ab/(ab+1)}(t)\right| + \frac{T^{1/(ab+1)}\left(\sum_{k=N+1}^{\infty} C_k^{ab/(ab+1)}\|\xi_k\|^{2a/(ab+1)}\right)^{1/(ab\zeta)}}{2^{ab/(ab+1)}p(1-p)}\frac{ab\zeta(\Delta_N p)^{1-1/(ab\zeta)}}{ab\zeta-1} + \frac{\Delta_N}{1-p}\right),$$

$$\sum_{k=1}^{\infty}|C_k|^{ab/(ab+1)}\|\xi_k\|^{2a/(ab+1)} < \infty,$$

and

$$\zeta > \frac{1}{ab},$$

where

$$\Delta_N = \left(\sup_{s,t\in[0,T]}(X_N(t)-X_N(s))\right)^{2a/(ab+1)}.$$

5. Examples of models of stochastic processes belonging to the spaces $D_{V,W}$

In this section we consider stochastic processes X represented in the interval $[0,T]$ in the form of the following series:

$$X(t)=\sum_{k=1}^{\infty}\xi_k\varphi_k(t),$$

where $\xi_k \in D_{V,W}$.

Example 5.1. Assume that a process $X(t)$ is represented as follows:

$$X(t)=\sum_{k=1}^{\infty}\sqrt{2}\xi_k\sin(\pi kt). \tag{15}$$

For this process,

$$\sup_{|t-s|<h}|\phi_k(t)-\phi_k(s)| = \sup_{|t-s|<h}|\sin(\pi kt)-\sin(\pi ks)| \leq 2\left|\sin\left(\frac{\pi kh}{2}\right)\right| \leq \pi kh,$$

that is, $C_k=\pi k$ and $\zeta=1$ if

$$a\in\left(\frac{1}{b},+\infty\right).$$

At the same time,

$$\sum_{k=N+1}^{\infty}\|\xi_k\|^{2a/(ab+1)}\inf_{t\in[0,T]}|\sin(\pi kt)|^{ab/(ab+1)}=0$$

and

$$\begin{aligned}\varepsilon_N &= \sup_{t,s\in[0,T]} \|X_N(t) - X_N(s)\| \\ &\le \sup_{t,s\in[0,T]} \left(\sum_{k=N}^{\infty} \left\|\sqrt{2}\xi_k(\sin(\pi kt) - \sin(\pi ks))\right\|^{2a/(ab+1)}\right)^{(ab+1)/(2a)} \\ &\le 2^{3b/4} \left(\sum_{k=N}^{\infty} \|\xi_k\|^{2a/(ab+1)}\right)^{(ab+1)/(2a)},\end{aligned}$$

$$\Delta_N = 2^{3ab/(2ab+2)} \sum_{k=N}^{\infty} \|\xi_k\|^{2a/(ab+1)}.$$

The value of N is chosen for the given accuracy æ, reliability $1-\nu$, and constant θ. The inequality

$$\begin{aligned}\nu \ge \frac{1}{æ^{ab}} &\left(\frac{abT^{1/(ab+1)}\left(\theta^{2a/(ab+1)}2^{3ab/(2ab+2)}\sum_{k=1}^{\infty}\|\xi_k\|^{2a/(ab+1)}\right)^{1-1/(ab)}}{\theta^{2a/(ab+1)}(1-\theta^{2a/(ab+1)})(ab-1)}\right. \\ &\times \left.\left(\sum_{k=N+1}^{\infty} \|\xi_k\|^{2a/(ab+1)}(\pi k)^{ab/(ab+1)}\right)^{1/(ab)} + \frac{2^{3ab/(2ab+2)}\sum_{k=1}^{\infty}\|\xi_k\|^{2a/(ab+1)}}{1-\theta^{2a/(ab+1)}}\right)\end{aligned}$$

can be used to choose N provided that

$$\sum_{k=1}^{\infty} (\pi k)^{ab/(ab+1)} \|\xi_k\|^{2a/(ab+1)} < \infty.$$

The same conditions can be used to study the process

$$X(t) = \sum_{k=1}^{\infty} \sqrt{2}\xi_k \cos(\pi kt). \tag{16}$$

Moreover, the same constants can be used for processes (16) as in the case of processes (15).

Example 5.2. Let a stochastic process $X(t)$ be represented in the following form:

$$X(t) = \sum_{k=0}^{\infty} \xi_k (A_k \sin(B_k t) + C_k \cos(D_k t)),$$

where $A_k > 0$ and $C_k > 0$. In this case,

$$\begin{aligned}&\sup_{|t-s|<h} \left|(A_k \sin(B_k t) + C_k \cos(D_k t)) - (A_k \sin(B_k s) + C_k \cos(D_k s))\right| \\ &\quad = \sup_{|t-s|<h} \left|2A_k \sin\left(B_k \frac{t-s}{2}\right) \sin\left(B_k \frac{t+s}{2}\right) - 2C_k \sin\left(D_k \frac{t-s}{2}\right) \sin\left(D_k \frac{t+s}{2}\right)\right|.\end{aligned}$$

Since $\sin x \le x^\alpha$, $0 < \alpha \le 1$,

$$\begin{aligned}&\sup_{|t-s|<h} \left|2A_k \sin\left(B_k \frac{t-s}{2}\right) \sin\left(B_k \frac{t+s}{2}\right) - 2C_k \sin\left(D_k \frac{t-s}{2}\right) \sin\left(D_k \frac{t+s}{2}\right)\right| \\ &\qquad \le 2A_k |\sin(B_k h/2)| + 2C_k |\sin(D_k h/2)| \le 2^{1-\alpha}(A_k B_k^\alpha + C_k D_k^\alpha) h^\alpha.\end{aligned}$$

The corresponding integral converges if

$$a \in \left(\frac{1}{\alpha b}, +\infty\right).$$

Put $E_k := 2^{1-\alpha}|A_k B_k^\alpha + C_k D_k^\alpha|$. In this case, $\inf_{t\in[0,T]} \|\tilde{X}_N\|$ and Δ_0 do not depend on the coefficients and cannot be evaluated explicitly. Thus we first choose the reliability $1-\nu$, accuracy æ, and the constant θ. Then we use the inequality

$$\nu \geq \frac{1}{æ^{ab}}\Bigg(\inf_{t\in[0,T]} \|\tilde{X}_N(t)\|^{2a/(ab+1)} + \frac{\alpha ab T^{1/(ab+1)} \left(\sum_{k=N+1}^{\infty} \|\xi_k\|^{2a/(ab+1)} E_k^{ab/(ab+1)}\right)^{1/(ab\alpha)}}{\theta^{2a/(ab+1)}\left(1-\theta^{2a/(ab+1)}\right)} \times \frac{(\Delta_N p)^{1-1/(ab\alpha)}}{ab\alpha - 1} + \frac{\Delta_N p}{1-\theta^{2a/(ab+1)}}\Bigg)$$

to evaluate the number N under the assumption that

$$\sum_{k=1}^{\infty} E_k^{ab/(ab+1)} \|\xi_k\|^{2a/(ab+1)} < \infty.$$

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