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LARGE DEVIATIONS FOR RANDOM EVOLUTIONS WITH INDEPENDENT INCREMENTS IN THE SCHEME OF THE POISSON APPROXIMATION

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Abstract. The asymptotic analysis of the large deviation problem for random evolutions with independent increments in the scheme of the Poisson approximation is performed. Large deviations for random evolutions in the scheme of the Poisson approximation are determined by the exponential generator for a jump process with independent increments.

1. Introduction

In the present paper, the asymptotic analysis of the large deviation problem is performed for random evolutions with independent increments in the scheme of the Poisson approximation (see [5, Chapter 7]).

Another problem is studied in the papers [6, 7] for random evolutions with locally independent increments in the scheme of the Poisson approximation.

A method based on the theory of convergent exponential (nonlinear) operators is developed in the monograph [2] to study the large deviation problem. As shown in [8, 9], the exponential operator in the scheme of series with a small parameter $\varepsilon \to 0$, $\varepsilon > 0$, is of the following form:

$$\mathbb{H}^\varepsilon \varphi(x) := e^{-\varphi(x)/\varepsilon} \varepsilon \mathbb{L}^\varepsilon e^{\varphi(x)/\varepsilon},$$

where the operators $\mathbb{L}^\varepsilon$, $\varepsilon > 0$, generate the Markov processes $x^\varepsilon(t)$, $t \geq 0$, $\varepsilon > 0$, in the scheme of the series.

Random evolutions with independent increments are defined by the following relation:

$$\xi(t) = \xi_0 + \int_0^t \eta(ds; x(s)), \qquad t \geq 0 \tag{1}$$

(see [5, Chapter 1]).

A Markov switching process $x(t)$, $t \geq 0$, in a standard phase space $(E, \mathcal{E})$ corresponds to the generator

$$Q\varphi(x) = q(x) \int_{\mathbb{E}} [\varphi(y) - \varphi(x)] P(x, dy), \qquad x \in E. \tag{2}$$

The following is the main assumption concerning the switching Markov process:

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C1: the Markov process $x(t)$, $t \ge 0$, is uniformly ergodic and has a stationary distribution $\pi(A)$, $A \in \mathcal{E}$.

Denote by Π the projector to the subspace of zeros of the reducible-invertible operator Q defined by equality (2):

$$\Pi\varphi(x) = \int_E \pi(dx)\varphi(x).$$

It is clear that

$$Q\Pi = \Pi Q = 0.$$

The potential R_0 is such that

$$QR_0 = R_0 Q = \Pi - I$$

(see [5, Chapter 1]).

Remark 1.1. The latter relation (known as the Poisson equation) implies that, under the solvability condition

$$\Pi\psi = 0,$$

the equation

$$Q\varphi = \psi$$

has a unique solution

$$\psi = R_0\varphi$$

if $\Pi\varphi = 0$.

The Markov processes with independent increments $\eta(t; x)$, $t \ge 0$, $x \in E$, are defined by their generators

$$\Gamma(x)\varphi(u) = \int_{\mathbf{R}} [\varphi(u+v) - \varphi(u)]\,\Gamma(dv; x), \qquad x \in E.$$

The random evolution (1) is characterized by the generator of the two component Markov processes $\xi(t)$, $x(t)$, $t \ge 0$,

$$\mathbb{L}\varphi(u, x) = Q\varphi(\cdot, x) + \Gamma(x)\varphi(u, \cdot)$$

(see [5, Chapter 2]).

Remark 1.2. Studies of limit properties of Markov processes are based on the martingale characterization. More precisely, consider the martingales

$$\mu_t = \varphi(x(t)) - \varphi(x(0)) - \int_0^t \mathbb{L}\varphi(x(s))\,ds, \tag{3}$$

where the generator $\mathbb{L}$ defining the Markov process $x(t)$, $t \ge 0$, in a standard phase space $(E, \mathcal{E})$ has a dense domain $\mathcal{D}(\mathbb{L}) \subseteq \mathcal{B}_E$ that contains the continuous functions as well as their derivatives. Here $\mathcal{B}_E$ is the Banach space of real valued bounded test functions $\varphi(x) \in E$ equipped with the norm $\|\varphi\| := \sup_{x\in E} |\varphi(x)|$.

The theory of large deviations is based on the following result (also known as the martingale exponential characterization):

$$\widetilde{\mu}_t = \exp\left\{\varphi(x(t)) - \varphi(x(0)) - \int_0^t \mathbb{H}\varphi(x(s))\,ds\right\} \tag{4}$$

is a martingale (see [2, Chapter 1]).

Here the exponential nonlinear operator is given by

$$\mathbb{H}\varphi(x) := e^{-\varphi(x)}\mathbb{L}e^{\varphi(x)}, \qquad \varphi(x) \in \mathcal{B}_E.$$

The equivalence of relations (3) and (4) follows from the following assertion.

Proposition ([1, p. 66])**.** *The process*

$$\mu(t) = x(t) - \int_0^t y(s)\,ds$$

is a martingale if and only if

$$\widetilde{\mu}(t) = x(t) \exp\left\{-\int_0^t \frac{y(s)}{x(s)}\,ds\right\}$$

is a martingale.

Remark 1.3. In general, there are four steps to solving the large deviation problem (see [2, Chapter 2]), namely

(1) The evaluation of the limit exponential (nonlinear) operator defining the large deviations.
(2) The proof of the exponential compactness.
(3) The proof of the comparison principle for the limit operator.
(4) The construction of the variation representation of the action functional defining the large deviations.

Steps (2)–(4) for the exponential operator corresponding to the case of stochastic processes with independent increments are discussed in detail in the monograph [2].

Some of these four steps are presented in the monograph [3] where the large deviation problem is studied with the help of the cumulant of the process with independent increments. A relationship between the cumulant and exponential generator is described below.

The generator of a Markov process can be represented as follows:

$$\mathbb{L}\varphi(x) = \int_{\mathbf{R}} e^{\lambda x} a(\lambda)\overline{\varphi}(\lambda)\,d\lambda,$$

where $a(\lambda)$ is the cumulant of the process and where $\overline{\varphi}(\lambda) = \int_{\mathbf{R}} e^{\lambda x}\varphi(x)\,dx$.

The inverse transformation gives us

$$\int_{\mathbf{R}} e^{-\lambda x}\mathbb{L}\varphi(x)\,dx = a(\lambda)\overline{\varphi}(\lambda).$$

Rewriting the left-hand side of the latter equality as follows:

$$\int_{\mathbf{R}} e^{-\lambda x}\mathbb{L}\varphi(x)\,dx = \int_{\mathbf{R}} e^{-\lambda x} a(\lambda)\varphi(x)\,dx$$

and changing the variable

$$e^{-\lambda x}\varphi(x) = \widetilde{\varphi}(x),$$

we obtain

$$\int_{\mathbf{R}} e^{-\lambda x}\mathbb{L}e^{\lambda x}\widetilde{\varphi}(x)\,dx = \int_{\mathbf{R}} a(\lambda)\widetilde{\varphi}(x)\,dx.$$

Thus

$$e^{-\lambda x}\mathbb{L}e^{\lambda x} = a(\lambda)$$

or, using the exponential generator,

$$\mathbb{H}\varphi_0(x) = a(\lambda),$$

where $\varphi_0(x) = \lambda x$.

Remark 1.4. Koroliuk developed in [8, 9] a method to solve the singular perturbation problem in the framework of large deviations for random evolutions with independent increments in the scheme of a small diffusion.

The classical papers on the asymptotic analysis of the large deviation problem use, as a rule, a large parameter $n \to \infty$ of the scheme of series. Some other papers use several parameters of a scheme of series (see, for example, [10]).

When solving the large deviation problem in the scheme of the Poisson approximation with the help of a small parameter of series, the normalization of the random evolution (1) is given by

$$\xi_\varepsilon^\delta(t) = \xi_\varepsilon^\delta(0) + \int_0^t \eta_\varepsilon^\delta\left(ds; x\left(s/\varepsilon^2\right)\right), \qquad t \ge 0,$$

$$\eta_\varepsilon^\delta(t) = \varepsilon \eta^\delta\left(t/\varepsilon^2\right),$$

$$\Gamma_\varepsilon^\delta(x)\varphi(u) = \varepsilon^{-2}\int_{\mathbf{R}} [\varphi(u+\varepsilon v) - \varphi(u)]\, \Gamma^\delta(dv; x), \qquad x \in E,$$

where $\varepsilon, \delta \to 0$ in such a way that $\varepsilon^{-1}\delta \to 1$.

2. Main assumptions for the Poisson approximation

C2: *Poisson approximation.* The family of stochastic processes with independent increments $\eta^\delta(t; x)$, $x \in E$, $t \ge 0$, satisfy the following three assumptions for the Poisson approximation:

PA1: Approximation of averages:

$$a_\delta(x) = \int_{\mathbf{R}} v\, \Gamma^\delta(dv; x) = \delta\left[a(x) + \theta_a^\delta(x)\right]$$

and

$$c_\delta(x) = \int_{\mathbf{R}} v^2\, \Gamma^\delta(dv; x) = \delta\left[c(x) + \theta_c^\delta(x)\right],$$

where

$$\sup_{x\in E} |a(x)| \le a < +\infty, \qquad \sup_{x\in E} |c(x)| \le c < +\infty.$$

PA2: The kernel of intensities admits the asymptotic representation

$$\Gamma_g^\delta(x) = \int_{\mathbf{R}} g(v)\, \Gamma^\delta(dv; x) = \delta\left[\Gamma_g(x) + \theta_g^\delta(x)\right]$$

for all $g \in C_3(\mathbf{R})$, where $C_3(\mathbf{R})$ is a class of functions defining measure (see [4, Chapter 7]); here $\Gamma_g(x)$ is a bounded kernel:

$$|\Gamma_g(x)| \le \Gamma_g$$

(the constant Γ_g depends on g). The kernel $\Gamma^0(dv; x)$ is acting on functions of the class $C_3(\mathbf{R})$ as follows:

$$\Gamma_g(x) = \int_{\mathbf{R}} g(v)\, \Gamma^0(dv; x), \qquad g \in C_3(\mathbf{R}).$$

The negligible terms θ_a^δ, θ_c^δ, and θ_g^δ are such that

$$\sup_{x\in E} |\theta_\cdot^\delta(x)| \to 0, \qquad \delta \to 0.$$

PA3: The relation

$$c(x) := \int_{\mathbf{R}} v^2\, \Gamma^0(dv; x)$$

holds. This relation implies, in particular, that there is no diffusion term in the exponential generator that determines a solution of the large deviation problem.

C3: *Uniform square integrability*:

$$\lim_{c\to\infty} \sup_{x\in E} \int_{|v|>c} v^2\, \Gamma^0(dv; x) = 0.$$

C4: *Exponential boundedness*:

$$\int_{\mathbf{R}} e^{p|v|}\, \Gamma^\delta(dv; x) < \infty \quad \text{for all } p \in \mathbf{R}.$$

3. Main result

Theorem 3.1. *The solution of the large deviation problem for the random evolution*

$$\xi_\varepsilon^\delta(t) = \xi_\varepsilon^\delta(0) + \int_0^t \eta_\varepsilon^\delta\left(ds; x\left(s/\varepsilon^2\right)\right), \qquad t \geq 0,$$

corresponding to the generator of the two component Markov processes $\xi(t)$, $x(t)$, $t \geq 0$,

$$\mathbb{L}_\varepsilon^\delta \varphi(u, x) = \varepsilon^{-2} Q\varphi(\cdot, x) + \Gamma_\varepsilon^\delta(x)\varphi(u, \cdot), \tag{5}$$

where

$$\Gamma_\varepsilon^\delta(x)\varphi(u) = \varepsilon^{-2} \int_{\mathbf{R}} [\varphi(u + \varepsilon v) - \varphi(u)]\, \Gamma^\delta(dv; x), \qquad x \in E, \tag{6}$$

is determined by the exponential generator

$$H^0\varphi(u) = \widetilde{a}\varphi'(u) + \int_{\mathbf{R}} \left[e^{v\varphi'(u)} - 1 - v\varphi'(u)\right] \widetilde{\Gamma}^0(dv), \tag{7}$$

$$\widetilde{a} = \Pi a(x) = \int_E \pi(dx) a(x), \qquad \widetilde{\Gamma}^0(v) = \Pi\Gamma^0(v; x) = \int_E \pi(dx)\Gamma^0(v; x).$$

The averaging above corresponds to the stationary measure of the switching Markov process.

Remark 3.1. Large deviations for random evolutions in the scheme of the Poisson approximation are defined by the exponential generator for the jump process with independent increments. An exhaustive study of the large deviation problem for a jump process with independent increments is given in the monograph [3].

We need the following auxiliary result to prove the theorem.

Lemma 3.1. *The exponential generator in the scheme of the series*

$$H_\Gamma^{\varepsilon,\delta}(x)\varphi(u) = e^{-\varphi/\varepsilon} \varepsilon \Gamma_\varepsilon^\delta(x) e^{\varphi/\varepsilon} \tag{8}$$

admits the following asymptotic representation:

$$H_\Gamma^{\varepsilon,\delta}(x)\varphi(u) = H_\Gamma(x)\varphi(u) + \theta_\Gamma^{\varepsilon,\delta}(x),$$

where $\sup_{x\in E} |\theta_\Gamma^{\varepsilon,\delta}(x)| \to 0$ *as* $\varepsilon, \delta \to 0$ *and*

$$H_\Gamma(x)\varphi(u) = a(x)\varphi'(u) + \int_{\mathbf{R}} \left[e^{v\varphi'(u)} - 1 - v\varphi'(u)\right] \Gamma^0(dv; x).$$

Proof of Lemma 3.1. We rewrite equality (8) by taking into account the form of the generator (6). We have

$$H_\Gamma^{\varepsilon,\delta}(x)\varphi(u) = \varepsilon^{-1}\int_{\mathbf{R}}\left[e^{\Delta_\varepsilon\varphi(u)} - 1\right]\Gamma^\delta(dv;x),$$

where

$$\Delta_\varepsilon\varphi(u) := \varepsilon^{-1}[\varphi(u+\varepsilon v) - \varphi(u)].$$

Now we rewrite the expression for the generator as follows:

$$\begin{aligned} H_\Gamma^{\varepsilon,\delta}(x)\varphi(u) &= \varepsilon^{-1}\int_{\mathbf{R}}\left[e^{\Delta_\varepsilon\varphi(u)} - 1 - \Delta_\varepsilon\varphi(u) - \frac{1}{2}(\Delta_\varepsilon\varphi(u))^2\right]\Gamma^\delta(dv;x) \\ &\quad + \varepsilon^{-1}\int_{\mathbf{R}}\left[\Delta_\varepsilon\varphi(u) + \frac{1}{2}(\Delta_\varepsilon\varphi(u))^2\right]\Gamma^\delta(dv;x). \end{aligned}$$

It is easy to see that the function

$$\psi_u^\varepsilon(v) = e^{\Delta_\varepsilon\varphi(u)} - 1 - \Delta_\varepsilon\varphi(u) - \frac{1}{2}(\Delta_\varepsilon\varphi(u))^2$$

belongs to the class $C_3(\mathbf{R})$. Indeed,

$$\frac{\psi_u^\varepsilon(v)}{v^2} \to 0, \qquad v \to 0.$$

Moreover, this function is continuous and bounded for all ε if the function $\varphi(u)$ is bounded. In addition, the function $\psi_u^\varepsilon(v)$ is uniformly bounded with respect to u under the assumptions **C3** and **C4** if the derivative $\varphi'(u)$ is bounded.

Hence

$$\begin{aligned} H_\Gamma^{\varepsilon,\delta}(x)\varphi(u) &= \varepsilon^{-1}\delta\int_{\mathbf{R}}\left[e^{\Delta_\varepsilon\varphi(u)} - 1 - \Delta_\varepsilon\varphi(u) - \frac{1}{2}(\Delta_\varepsilon\varphi(u))^2\right]\Gamma^0(dv;x) \\ &\quad + \varepsilon^{-1}\int_{\mathbf{R}}\left[\Delta_\varepsilon\varphi(u) - v\varphi'(u) - \varepsilon\frac{v^2}{2}\varphi''(u)\right]\Gamma^\delta(dv;x) \\ &\quad + \varepsilon^{-1}\delta a(x)\varphi'(u) + \delta c(x)\varphi''(u) \\ &\quad + \varepsilon^{-1}\int_{\mathbf{R}}\left[\frac{1}{2}(\Delta_\varepsilon\varphi(u))^2 - \frac{v^2}{2}(\varphi'(u))^2\right]\Gamma^\delta(dv;x) + \varepsilon^{-1}\delta\frac{1}{2}c(x)(\varphi'(u))^2. \end{aligned}$$

Applying the Taylor formula to the test function $\varphi(u) \in C^3(\mathbf{R})$ we derive from assumption **PA2** that

$$\begin{aligned} H_\Gamma^{\varepsilon,\delta}(x)\varphi(u) &= \varepsilon^{-1}\delta\int_{\mathbf{R}}\left[e^{v\varphi'(u)} - 1 - v\varphi'(u) - \frac{v^2}{2}(\varphi'(u))^2\right]\Gamma^0(dv;x) \\ &\quad + \varepsilon^{-1}\delta\int_{\mathbf{R}}\left(e^{v\varphi'(u)}\varepsilon\frac{v^2}{2}\varphi''(\widetilde{u}) - \varepsilon\frac{v^2}{2}\varphi''(\widetilde{u}) - \varepsilon^2\frac{v^4}{8}(\varphi''(\widetilde{u}))^2\right)\Gamma^0(dv;x) \\ &\quad + \varepsilon^{-1}\delta\int_{\mathbf{R}}\varepsilon^2\frac{v^3}{3!}\varphi'''(\widetilde{u})\,\Gamma^0(dv;x) + \varepsilon^{-1}\delta a(x)\varphi'(u) + \delta c(x)\varphi''(u) \\ &\quad + \varepsilon^{-1}\delta\int_{\mathbf{R}}\varepsilon^2\frac{v^4}{4}(\varphi''(\widetilde{u}))^2\,\Gamma^0(dv;x) + \varepsilon^{-1}\delta\frac{1}{2}c(x)(\varphi'(u))^2. \end{aligned}$$

Taking into account assumption **PA3** and the limit condition $\varepsilon^{-1}\delta \to 1$, we finally get

$$H_\Gamma^{\varepsilon,\delta}(x)\varphi(u) = H_\Gamma(x)\varphi(u) + \theta_\Gamma^{\varepsilon,\delta}(x),$$

where $\sup_{x\in E}|\theta_\Gamma^{\varepsilon,\delta}(x)| \to 0$ as $\varepsilon, \delta \to 0$.

Lemma 3.1 is proved. □

Proof of Theorem 3.1. One can pass to the limit in the exponential nonlinear generator of the random evolution for the test functions

$$\varphi_\varepsilon^\delta(u,x) = \varphi(u) + \varepsilon \ln[1 + \delta\varphi_1(u,x)].$$

Thus

$$\mathbb{H}_\varepsilon^\delta \varphi^\varepsilon = e^{-\varphi^\varepsilon/\varepsilon}\varepsilon\mathbb{L}_\varepsilon^\delta e^{\varphi^\varepsilon/\varepsilon} = e^{-\varphi/\varepsilon}[1+\delta\varphi_1]^{-1}\varepsilon\mathbb{L}_\varepsilon^\delta e^{\varphi/\varepsilon}[1+\delta\varphi_1].$$

The asymptotic behavior of the latter exponential generator as $\varepsilon, \delta \to 0$ is described in the following result.

Lemma 3.2. *The following asymptotic representation*

$$\mathbb{H}_\varepsilon^\delta \varphi^\varepsilon = Q\varphi_1 + H_\Gamma^{\varepsilon,\delta}(x)\varphi(u) + \theta^{\varepsilon,\delta}(x)$$

holds, where $\sup_{x\in E}|\theta^{\varepsilon,\delta}(x)| \to 0$ *as* $\varepsilon, \delta \to 0$.

Proof. It follows from (5) that

$$\begin{aligned}
\mathbb{H}_\varepsilon^\delta \varphi^\varepsilon &= e^{-\varphi/\varepsilon}\left[1 - \delta\varphi_1 + \frac{\delta^2\varphi_1^2}{1+\delta\varphi_1}\right]\left\{\varepsilon^{-1}Q + \varepsilon\Gamma_\varepsilon^\delta(x)\right\}e^{\varphi/\varepsilon}[1+\delta\varphi_1] \\
&= e^{-\varphi/\varepsilon}\left[1 - \delta\varphi_1 + \frac{\delta^2\varphi_1^2}{1+\delta\varphi_1}\right]\left\{\varepsilon^{-1}\delta e^{\varphi/\varepsilon}Q\varphi_1 + \varepsilon\Gamma_\varepsilon^\delta(x)e^{\varphi/\varepsilon} + \varepsilon\delta\Gamma_\varepsilon^\delta(x)e^{\varphi/\varepsilon}\varphi_1\right\} \\
&= Q\varphi_1 + H_\Gamma^{\varepsilon,\delta}(x)\varphi(u) + \theta^{\varepsilon,\delta}(x),
\end{aligned}$$

where

$$\theta^{\varepsilon,\delta}(x) = \delta\left[\frac{\varepsilon}{1+\delta\varphi_1}e^{-\varphi/\varepsilon}\Gamma^\delta(x)e^{\varphi/\varepsilon}\varphi_1 - \frac{\varepsilon^{-1}\delta\varphi_1}{1+\delta\varphi_1}Q\varphi_1 - \frac{\delta\varphi_1}{1+\delta\varphi_1}e^{-\varphi/\varepsilon}\Gamma^\delta(x)e^{\varphi/\varepsilon}\right].$$

The lemma is proved. ☐

Using Lemma 3.1 we get

$$\mathbb{H}^{\varepsilon,\delta}\varphi^\varepsilon = Q\varphi_1 + H_\Gamma(x)\varphi(u) + h^{\varepsilon,\delta}(x),$$

where $h^{\varepsilon,\delta}(x) = \theta^{\varepsilon,\delta}(x) + \theta_\Gamma^{\varepsilon,\delta}(x)$.

Now we use the solution of the singular perturbation problem for the reducible-invertible operator Q (see [5, Chapter 1]). The solvability condition implies that

$$Q\varphi_1 + H_\Gamma(x)\varphi(u) = H^0\varphi(u),$$

where

$$H^0\varphi(u) = \Pi Q\Pi\varphi_1 + \Pi H_\Gamma(x)\Pi\varphi(u).$$

Now equality (6) follows, since

$$H^0\varphi(u) = \widetilde{a}\varphi'(u) + \int_{\mathbf{R}}\left[e^{v\varphi'(u)} - 1 - v\varphi'(u)\right]\widetilde{\Gamma}^0(dv).$$

The remainder term $h^{\varepsilon,\delta}(x)$ can be obtained explicitly by using the solution of the Poisson equation:

$$\varphi_1(u,x) = R_0\widetilde{H}(x)\varphi(u), \qquad \widetilde{H}(x) := H_\Gamma(x) - H^0$$

(see Remark 1.1 above; more detail is given in [5]).

The theorem is proved. ☐

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