NOTES AND ERRATA, VOLUMES 10, 11.

VOLUME 10.

Page 315. W. B. FITE: *Irreducible homogeneous linear groups in an arbitrary domain.*

The results of this article are embodied in the theorem, *A necessary and sufficient condition that any group of finite order be simply isomorphic with an irreducible group in any domain is that its central be cyclic.* But that this condition is not sufficient is shown by the following example to which Professor BURNSIDE has kindly called my attention, namely the group defined by the operations A, P, Q, and R with the following relations:
$P^7 = 1$, $Q^7 = 1$, $R^n = 1$ (n any positive integer), $A^3 = 1$, $A^{-1}PA = P^2$, $A^{-1}QA = Q^2$, $A^{-1}RA = R$, and with P, Q, and R commutative among themselves.

The argument rests upon the supposed fact that $r_1 > 1$, and I can now see no reason why it might not equal unity.

I have been unable to determine what is both a sufficient and a necessary condition that a group of finite order may be simply isomorphic with an irreducible group. It is easy however to give sufficient conditions that are not necessary ones; for example, in order that a group of finite order be simply isomorphic with an irreducible group it is *sufficient* that its direct product with some group of finite order be simply isomorphic with an irreducible group.

VOLUME 11.

Page 489. G. E. WAHLIN: *On the base of a relative number field, with an application to the composition of fields.*

The assertion that $O_{\tau\tau}^{(1)}$ is prime to I_τ is not true for every ϑ. I wish therefore to replace lines 7–19 of p. 489 by the following.

Since $\omega_{\tau\tau}^{(1)} = \bar{\omega}_\tau / D_k(\vartheta)$ is an integer in K, $\bar{\omega}_\tau$ is divisible by $D_k(\vartheta)$, and consequently also by O_τ. That is,

$$\bar{\omega}_\tau = O_{1\tau}^{(1)} + O_{2\tau}^{(1)}\vartheta + \cdots + O_{\tau\tau}^{(1)}\vartheta^{\tau-1} \equiv 0\,(O_\tau).$$

From the fact,* that if J and J_1 are two ideals in a number field, then there exists an ideal J' such that JJ' is a principal ideal and J' is prime to J_1, we know that there exists in k an ideal J_τ, prime to $[D_k(\vartheta)]$ and such that

* DIRICHLET: *Vorlesungen über Zahlentheorie*, vierte Auflage, p. 559, Theorem XI.

$O_\tau \cdot J_\tau = (\alpha)$, a principal ideal. In the same way there exists an ideal J'_τ, prime to $[D_k(\vartheta)]$, such that $J_\tau \cdot J'_\tau = (\beta)$, a principal ideal, and since J_τ and J'_τ are both prime to $D_k(\vartheta)$, β is prime to $D_k(\vartheta)$.

Since $(\beta) = J_\tau \cdot J'_\tau$ and $(\alpha) = O_\tau \cdot J_\tau$ and since $\bar{\omega}_\tau$ is divisible by O_τ, it follows that $\beta\bar{\omega}_\tau$ is divisible by (α), and must therefore be a multiple of α, and

$$\frac{\beta\bar{\omega}_\tau}{\alpha} = \frac{\beta O_{1\tau}^{(1)} + \beta O_{2\tau}^{(1)}\vartheta + \cdots + \beta O_{\tau\tau}^{(1)}\vartheta^{\tau-1}}{\alpha}$$

is an integer in K.

But since $\bar{\omega}_\tau$ and $O_{\tau\tau}^{(1)}$ are both divisible by O_τ it follows that

$$X = O_{1\tau}^{(1)} + O_{2\tau}^{(1)}\vartheta + \cdots + O_{\tau-1,\tau}^{(1)}\vartheta^{\tau-2} = \bar{\omega}_\tau - O_{\tau\tau}^{(1)}\vartheta^{\tau-1}$$

must be divisible by O_τ and therefore, in the same way as above, we see that $\beta X/\alpha$ is an integer in K, and $R = \beta O_{\tau\tau}^{(1)}/\alpha$ an integer in k. We can now write

$$\frac{\beta\bar{\omega}_\tau}{\alpha} = \frac{\beta X + \beta O_{\tau\tau}^{(1)}\vartheta^{\tau-1}}{\alpha} = \frac{\beta X}{\alpha} + R\vartheta^{\tau-1}$$

and since O_τ is the highest common factor of $O_{\tau\tau}^{(1)}$ and $D_k(\vartheta)$, the ideals $O_{\tau\tau}^{(1)}/O_\tau$ and $I_\tau = D_k(\vartheta)/O_\tau$ are relatively prime, and since β is prime to I_τ, it follows that R is prime to I_τ and there exists in k an integer R' such that

$$R'R \equiv 1(I_\tau).$$

We have therefore

$$\frac{R'\beta\bar{\omega}_\tau}{\alpha} = \frac{R'\beta X}{\alpha} + R'R\vartheta^{\tau-1} \equiv \bar{\Omega}_\tau(I_\tau),$$

where

$$\bar{\Omega}_\tau = \frac{R'\beta O_{1\tau}^{(1)} + R'\beta O_{2\tau}^{(1)}\vartheta + \cdots + R'\beta O_{\tau-1\tau}^{(1)}\vartheta^{\tau-2}}{\alpha} + \vartheta^{\tau-1}.$$

Since $\bar{\omega}_\tau$ is divisible by $D_k(\vartheta)$, and α contains no other factor of $D_k(\vartheta)$ than O_τ, we see that the integer $\beta\bar{\omega}_\tau/\alpha$ must be divisible by I_τ and from the last congruence we have

$$\bar{\Omega}_\tau \equiv O(I_\tau).$$

While $\Omega_\tau = \bar{\Omega}_\tau/D_k(\vartheta)$ is not necessarily an integer in K, the products $\Omega_\tau \cdot O_{\tau\tau}^{(1)}$ and $\Omega_\tau \cdot D_k(\vartheta)$ are both integers in K. Moreover, since β is divisible by J_τ and since $D_k(\vartheta)$ and $O_{\tau\tau}^{(1)}$ are both divisible by O_τ, the coefficients $R'\beta O_{\tau\tau}^{(1)}$ when multiplied by $O_{\tau\tau}^{(1)}$ or $D_k(\vartheta)$ will be divisible by α and the integers $\Omega_\tau \cdot D_k(\vartheta)$ and $\Omega_\tau \cdot O_{\tau\tau}^{(1)}$ are therefore of the form $\omega_\tau^{(i)}$. The number Ω_τ thus found can therefore take the place of the Ω_τ in my original paper.