NOTES AND ERRATA, VOLUMES 10, 11.

VOLUME 10.


The results of this article are embodied in the theorem, A necessary and sufficient condition that any group of finite order be simply isomorphic with an irreducible group in any domain is that its central be cyclic. But that this condition is not sufficient is shown by the following example to which Professor Burnside has kindly called my attention, namely the group defined by the operations $A, P, Q,$ and $R$ with the following relations:

$P^n = 1, \quad Q^7 = 1, \quad R^n = 1 \quad (n \text{ any positive integer}), \quad A^3 = 1, \quad A^{-1}PA = P^2, \quad A^{-1}QA = Q^2, \quad A^{-1}RA = R,$ and with $P, Q,$ and $R$ commutative among themselves.

The argument rests upon the supposed fact that $r_1 > 1,$ and I can now see no reason why it might not equal unity.

I have been unable to determine what is both a sufficient and a necessary condition that a group of finite order may be simply isomorphic with an irreducible group. It is easy however to give sufficient conditions that are not necessary ones; for example, in order that a group of finite order be simply isomorphic with an irreducible group it is sufficient that its direct product with some group of finite order be simply isomorphic with an irreducible group.

VOLUME 11.

Page 489. G. E. WAHLIN: On the base of a relative number field, with an application to the composition of fields.

The assertion that $O_{\tau}^{(1)}$ is prime to $I_\tau$ is not true for every $\theta$. I wish therefore to replace lines 7–19 of p. 489 by the following.

Since $\omega_{\tau}^{(1)} = \overline{\omega}_\tau / D_k(\theta)$ is an integer in $K$, $\overline{\omega}_\tau$ is divisible by $D_k(\theta)$, and consequently also by $O_{\tau}$. That is,

$$\overline{\omega}_\tau = O_{\tau}^{(1)} + O_{\tau}^{(1)}\theta + \cdots + O_{\tau}^{(1)}\theta^{r-1} = O(\omega_{\tau}).$$

From the fact,* that if $J$ and $J_1$ are two ideals in a number field, then there exists an ideal $J'$ such that $J_1J'$ is a principal ideal and $J'$ is prime to $J_1$, we know that there exists in $k$ an ideal $J_\tau$, prime to $[D_k(\theta)]$ and such that

* DIRICHLET: Vorlesungen über Zahlentheorie, vierte Auflage, p. 559, Theorem XI.
\( O_r \cdot J_r = (\alpha) \), a principal ideal. In the same way there exists an ideal \( J'_r \), prime to \( \left[ D_k(\mathcal{H}) \right] \), such that \( J_r \cdot J'_r = (\beta) \), a principal ideal, and since \( J_r \) and \( J'_r \) are both prime to \( D_k(\mathcal{H}) \), \( \beta \) is prime to \( D_k(\mathcal{H}) \).

Since \((\beta) = J_r \cdot J'_r \) and \((\alpha) = O_r \cdot J_r \) and since \( \bar{\omega}_r \) is divisible by \( O_r \), it follows that \( \beta \bar{\omega}_r \) is divisible by \( (\alpha) \), and must therefore be a multiple of \( \alpha \), and

\[
\frac{\beta \bar{\omega}_r}{\alpha} = \frac{\beta O_r^{(1)} + \beta O_r^{(1)} \delta + \cdots + \beta O_r^{(1)} \delta^{r-1}}{\alpha}
\]

is an integer in \( K \).

But since \( \bar{\omega}_r \) and \( O_r^{(1)} \) are both divisible by \( O_r \), it follows that

\[
X = O_r^{(1)} + O_r^{(2)} \delta + \cdots + O_r^{(r-1)} \delta^{r-2} = \bar{\omega}_r - O_r^{(1)} \delta^{r-1}
\]

must be divisible by \( O_r \), and therefore, in the same way as above, we see that \( \beta X/\alpha \) is an integer in \( K \), and \( R = \beta O_r^{(1)}/\alpha \) an integer in \( k \). We can now write

\[
\frac{\beta \bar{\omega}_r}{\alpha} = \frac{\beta X + \beta O_r^{(1)} \delta^{r-1}}{\alpha} = \frac{\beta X}{\alpha} + R \delta^{r-1}
\]

and since \( O_r \) is the highest common factor of \( O_r^{(1)} \) and \( D_k(\mathcal{H}) \), the ideals \( O_r^{(1)}/O_r \) and \( I_r = D_k(\mathcal{H})/O_r \) are relatively prime, and since \( \beta \) is prime to \( I_r \), it follows that \( R \) is prime to \( I_r \) and there exists in \( k \) an integer \( R' \) such that

\[
RR' \equiv 1 (I_r).
\]

We have therefore

\[
\frac{R' \beta \bar{\omega}_r}{\alpha} = \frac{R' \beta X}{\alpha} + R' R \delta^{r-1} \equiv \bar{\Omega}_r (I_r),
\]

where

\[
\bar{\Omega}_r = \frac{R' \beta O_r^{(1)} + R' \beta O_r^{(2)} \delta + \cdots + R' \beta O_r^{(r-1)} \delta^{r-2}}{\alpha} + \delta^{r-1}.
\]

Since \( \bar{\omega}_r \) is divisible by \( D_k(\mathcal{H}) \), and \( \alpha \) contains no other factor of \( D_k(\mathcal{H}) \) than \( O_r \), we see that the integer \( \beta \bar{\omega}_r/\alpha \) must be divisible by \( I_r \) and from the last congruence we have

\[
\bar{\Omega}_r \equiv O(I_r).
\]

While \( \bar{\Omega}_r = \bar{\Omega}_r / D_k(\mathcal{H}) \) is not necessarily an integer in \( K \), the products \( \Omega_r \cdot O_r^{(1)} \) and \( \Omega_r \cdot D_k(\mathcal{H}) \) are both integers in \( K \). Moreover, since \( \beta \) is divisible by \( J_r \) and since \( D_k(\mathcal{H}) \) and \( O_r^{(1)} \) are both divisible by \( O_r \), the coefficients \( R' \beta O_r^{(1)} \) when multiplied by \( O_r^{(1)} \) or \( D_k(\mathcal{H}) \) will be divisible by \( \alpha \) and the integers \( \Omega_r \cdot D_k(\mathcal{H}) \) and \( \Omega_r \cdot O_r^{(1)} \) are therefore of the form \( \omega_r^{(1)} \). The number \( \Omega_r \), thus found can therefore take the place of the \( \Omega_r \); in my original paper.