A SECOND CORRECTION

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In my paper in the present volume these Transactions (vol. 35, pp. 274–304 and 557–558), the Example 5 on page 304 is erroneous and Postulate 5 on page 301 is redundant. That is, Postulates 1, 2, 3, 4, 6, 7 (without 5) form a set of independent postulates for the “informal” system of *Principia Mathematica*.

The proof of 5 from 1, 2, 3, 4, 6, 7 is as follows.*

6a. If \( a+b \) is in \( T \) and \( a \) not in \( T \), then \( b \) is in \( T \). (From 6.)
6b. If \( a \) not in \( T \) and \( b \) not in \( T \), then \( a+b \) not in \( T \). (From 6.)
7a. If \( a \) is in \( T \), then \( a' \) is not in \( T \). (From 7.)
3a. If \( b \) is in \( T \), then \( a+b \) is in \( T \).

For, by \( 3a \), \( b' \) is not in \( T \). But by \( 3 \), \( b'+(a+b) \) is in \( T \). Hence by \( 6a \), \( a+b \) is in \( T \).

4a. If \( b \) is in \( T \), then \( b+a \) is in \( T \).

For, by \( 3a \), \( a+b \) is in \( T \), whence by \( 7a \), \( (a+b)' \) is not in \( T \). But by \( 4 \), \( (a+b)'+(b+a) \) is in \( T \). Hence by \( 6a \), \( b+a \) is in \( T \).

5a. If \( a \) is not in \( T \), then \( a' \) is in \( T \).

For, suppose \( a' \) not in \( T \). Then by \( 6b \), \( a'+a \) not in \( T \), whence by \( 6b \), \( a'+(a'+a) \) not in \( T \), contrary to \( 3 \).

5. If \( a, b, etc. are in K \), then \( (b'+c)'+[(a+b)'+(a+c)] \) is in \( T \).

Case 1: \( a \) in \( T \). By \( 4a \), \( a+c \) is in \( T \). Hence the theorem, by \( 3a \) (twice).

Case 2: \( b \) in \( T \). By \( 7a \), \( b' \) not in \( T \). If \( c \) is in \( T \), then by \( 3a \), \( a+c \) is in \( T \), whence the theorem, by \( 3a \) (twice). If \( c \) is not in \( T \), then by \( 6b \), \( b'+c \) not in \( T \), whence by \( 5a \), \( (b'+c)' \) is in \( T \), whence the theorem, by \( 4a \).

Case 3: \( a \) not in \( T \) and \( b \) not in \( T \). By \( 6b \), \( a+b \) not in \( T \), whence by \( 5a \), \( (a+b)' \) is in \( T \). Hence the theorem, by \( 4a \) and \( 3a \).

The proof is thus complete. It can also be shown that \( 1, 2, 3a, 4a, 5a, 6a, 7 \) form a set of independent postulates equivalent to the set \( 1, 2, 3, 4, 6, 7 \).

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