THE DEPENDENCE OF AN AXIOM OF LUKASIEWICZ(1)

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In this note we prove that axiom scheme A5 (cf. §10 of the preceding paper by Rose and Rosser, [1]) is derivable from the axiom schemes A1–A4. We use the notation of [1], and the formulas of [1] are referred to by their numbers. We shall use formulas only from the first three sections, which depend only on axion schemes A1–A4. Indeed, except for the result (3.51), we use formulas only from the first two sections, which depend only on axiom schemes A1–A3. Thus we actually show that axiom scheme A5 can be derived from (3.51) and axiom schemes A1–A3. This gives added point to the question raised by Rose and Rosser in [1] whether (3.51) can be proved from A1–A3 alone.

Our proof is the one cited on p. 51 of [2].

Putting CPQ, CQP, and P respectively for P, Q, and R in A2 gives

$$CCCPQCQPCAQPCCPQP.$$

By the commutative law for A, expressed in (2.2),

$$CCCPQCQPCAPQCCPQP.$$

Putting $CCPQCQP$, $CAPQCCPQP$, and $CCQCPQCQP$ respectively for P, Q, and R in A2, and using the formula just above gives

$$CCCAPQCCPQPCCQCPQCQPCCCPQCQPCCQCPQCQP.$$

In (3.51), put Q, CPQ, and P respectively for P, Q, and R, and use the formula just above. This gives $CCCPQCQPCCQCPQCQP$. By using (2.6) and A1 with this, we get $ACPQCQP$ which is A5.

References

1. Alan Rose and J. Barkley Rosser, *Fragments of many-valued statement calculi*, Trans. Amer. Math. Soc. vol. 87 (1957) pp. 1–53.

2. A. Tarski, *Logic, semantics, metamathematics*, Oxford University Press, 1956.

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(1) *Editor's Note.* This proof was discovered by Meredith before 1955 and so precedes the alternate proof of A5 given by Chang in the next note, since Chang's proof was discovered in 1956. The original proof by Meredith was carried out in ignorance of the results in [1], so that Meredith had to prove the subsidiary results used, such as (3.51) of [1]. When Meredith heard of [1], he showed his proof to Rose, who produced the present curtailed version by making use of the results of [1].