CORRECTION AND ADDITION TO SOME THEOREMS
CONCERNING PARTITIONS

BY

EMIL GROSSWALD

Theorem 2 of the above mentioned paper is valid only provided that the set \{a\} of smallest positive residues does not consist of the single element \(a = q\). Indeed, if it is required that all summands of a partition be divisible by the prime \(q\), then, clearly, \(p_n(q) = p_n(q, l) = 0\) for \(n \not\equiv 0 \pmod{q}\), while, for \(n = qn_1, p_{qn_1} = p_{n_1}(1)\) and \(p_{qn_1} = p_{n_1}(1, l)\). Here \(p_{n_1}(1)\) and \(p_{n_1}(1, l)\) are the corresponding partitions without congruence restrictions; they may be obtained from the formulae of Theorem 2, by setting formally \(m = q - 1\). \(p_n(1)\) is, of course, the number of unrestricted partitions and is well-known (see [2] and [10]). It is easy to see that this is actually the only case in which the statement of Theorem 2 needs a modification. Indeed, Theorem 2 is an immediate consequence of the lemma. The proof that conditions (b) and (c) of the lemma hold for the generating functions \(F(x)\) and \(H(x)\) does not depend on the set \{a\}. But the verification of condition (a) makes essential use of the fact that (see text on top of p. 124 and on p. 120, after (11)) \(L/k^2 \leq t\Lambda\), with \(t = \max_a B/q^2 = 1 - 6(q - 1)/q^2 < 1\). In case \(k = q = a\), however, \(A = B = q^2\) and, if \(m = 1, \Lambda = \pi^2/6q, L = \pi^2q/6\) so that \(L/k^2 = L/q^2 = \Lambda > t\Lambda\). This simply reflects the fact, evident from \(F_0(x) = \prod_{n=1}^\infty (1 - x^{q^n})^{-1}\), that if \(x = \exp \{\log r + 2\pi i h/q\}\), then \(|F_0(x)|\) takes on the same value, for every integer \(h\); the situation for \(H(x)\) is similar. If, however, \(m > 1\), then \(L = (\pi^2/6q) \sum_{a \in [a]} B \leq (\pi^2/6q) [(m - 1)q + q^2] = \pi^2mqt_1/6\), with \(t_1 = [(m - 1)t + 1]/m < 1\) and the argument of the test goes through with \(t_1 < 1\) instead of \(t\).

University of Pennsylvania,
Philadelphia, Pennsylvania

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