A CORRECTION OF SOME THEOREMS ON PARTITIONS

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Theorem 4 in [1] gives a convergent series representation for $p_a(n)$, the number of partitions of a positive integer n into positive summands of the form $mp \pm a_j$. Here p is an odd prime and a_j is an element of a set a consisting of r positive residues of p each of which is less than $p/2$. It is stated that the theorem holds for $n \geqq A/12p$, where $A = rp^2 - 6 \sum_{j=1}^{r} a_j(p - a_j)$. In the proof of this theorem the estimate $O(n^{1/3}k^{2/3+\varepsilon})$ is used for certain complicated exponential sums. The proof of this estimate given in Theorem 2 of [1] depends on the fact that $(A - 12pn, k) = O(n)$. This is clearly false (in general) if $A = 12pn$ since $(0, k) = k$. Thus, Theorem 4 of [1] has been established only if $n > A/12p$.

Similar remarks apply to Theorem 6 in [2] in which a convergent series is obtained for $q_a(n)$, the number of partitions of n into *distinct* positive summands of the form $mp \pm a_j$. Here it is asserted that the theorem holds for $n \geqq -A/12p$. However, the proof given is valid only if $n > -A/12p$. For the argument used to establish the required estimate $O(n^{1/3}k^{2/3+\varepsilon})$ for the exponential sums involved does not hold if $A = -12np$. Thus, until (if ever) the necessary estimates contained in Theorems 2 and 3 of [1] and Theorems 2 through 5 of [2] can be justified for $n = \pm A/12p$ we must exclude these values of n from consideration.

We conclude by giving a simple necessary condition for $A = \pm 12pn$. From the definition of A given above and the fact that either a_j or $p - a_j$ is even we see that if $A = \pm 12pn$ then $12 | r$.

References

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2. ———, *On a class of partitions with distinct summands*, Trans. Amer. Math. Soc. **112** (1964), 401–415.

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Received by the editors September 21, 1964.