ON THE PRODUCT OF F-SPACES(1)

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1. **Introduction.** The results contained in this paper deal with the following question, first proposed by Gillman in 1960: When is the product of two spaces an F-space? (Definitions appear in §2.) It is probably correct to say that the status of this problem, thus far, has consisted of a remark made by Rudin, and improved by P. C. Curtis [C] and Henriksen, and of a counterexample found by Gillman [G] (see §3). As a result, it has been known that for $X \times Y$ to be an F-space (X, Y being topological spaces) it is necessary that either X or Y is a P-space, and that both are F-spaces; thus in attempting to solve the problem posed above we may assume that X is a P-space and Y is an F-space. The main result in this paper states that if Y is in addition compact, then $X \times Y$ is an F-space (6.1). An immediate, and somewhat intriguing, consequence is the following characterization of P-spaces: X is a P-space when and only when $X \times \beta X$ is an F-space (6.5).

The paper is organized as follows: The first three sections include the statement of definitions, known facts about the problem, and the main results of this paper. In §4, the necessary lemmas are developed, especially 4.6. In §5, we consider Baire sets in F-spaces; it is proved that a Baire set of a compact F-space is C^*-embedded (5.3). In §6, the main theorems appear (6.1, 6.3 and 6.5). In §7, some examples are considered; in particular, for every P-space (of nonmeasurable cardinal) we construct an extremally disconnected space Y such that $X \times Y$ is not an F-space (7.3).

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2. **Definitions.** All topological spaces will be assumed to be completely regular Hausdorff spaces. For a space X, we denote by βX its Stone-Čech compactification, and by $C(X)$ ($C^*(X)$) the ring of all real-valued continuous (bounded) functions on X. A subset A of X is said to be C-embedded (C^*-embedded) in X if every element in $C(A)$ ($C^*(A)$) is the restriction of an element in $C(X)$ ($C^*(X)$). A zero-set of X is the set where a real-valued continuous function on X vanishes; the complement of a zero-set is a cozero-set. A space is an F-space if every cozero-set is C^*-embedded; equivalently, X is an F-space if any pair of disjoint cozero-sets can be separated by an element in $C^*(X)$, i.e. if there is some f in $C^*(X)$ which is equal to zero on one cozero-set and to one on the other. A space is a P-space if every zero-set is open; it is basically disconnected if the closure of every cozero-set is open; it is extremally disconnected if the closure of every open set is open. The

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following implications hold:

$$\begin{tikzcd}[column sep=small] & \text{extremally disconnected} \arrow[dr, Rightarrow] & & \\ \text{discrete} \arrow[ur, Rightarrow] \arrow[dr, Rightarrow] & & \text{basically disconnected} \arrow[r, Rightarrow] & F\text{-spaces} \\ & P\text{-spaces} \arrow[ur, Rightarrow] & & \end{tikzcd}$$

The required detailed information on these classes of spaces is to be found in [GH] and [GJ]. The reader is also referred to [GJ, 8] for a background on realcompact spaces, which are mentioned in §7.

3. **Known results.** The following facts are known in relation to the problem considered.

3.1 The product of two P-spaces is a P-space (see [GJ, 4K]).

3.2 If X, Y are infinite compact spaces, then $X\times Y$ is not an F-space (Rudin, see [C]).

3.3 If $X\times Y$ is an F-space, then either X or Y is a P-space (Curtis [C] and Henriksen). Since a compact P-space is a finite set [GJ, 4K], it follows that 3.3 implies 3.2.

3.4 There is a P-space X and an F-space (in fact, an extremally disconnected space) Y, such that $X\times Y$ is not an F-space (Gillman [G]).

4. The proof of the main theorems (given in §6) will require a sequence of lemmas. Let π_1, π_2 denote the usual projections, e.g. $\pi_1\colon X\times K\to X$. The proof of the first lemma will be left to the reader.

4.1 LEMMA. *Let X be a P-space, K a compact space, and U a cozero set of $X\times K$. For $x\in X$, we set $U_x=U\cap\pi_1^{-1}(x)$. Then* (i) *$\pi_1(U)$ is open-and-closed in X, and* (ii) *for every $x\in X$ there is an open neighborhood $V(x)$ of x such that $V(x)\times\pi_2(U_x)\subset U$.*

4.2 LEMMA ([CN, THEOREMS 3.1, 3.2]). *If $\pi_1\colon X\times Y\to X$ is a closed mapping, then $X\times Y$ is C^*-embedded in $X\times\beta Y$; in particular, if X is a P-space and Y is a Lindelöf space, then $X\times Y$ is C^*-embedded in $X\times\beta Y$.*

For a compact space K, we topologize $C(K)$ with the uniform topology, which coincides with the compact-open topology. We will use the exponential law for mapping spaces in the form $C(X\times K)=C(X, C(K))$. This is a special case of a standard theorem on the compact-open topology (here $C(X, Y)$ denotes the set of all continuous functions from X to Y endowed with the compact-open topology); see [H, III, 9.9].

4.3 LEMMA. *Let F be a closed subset of the compact space K, and let $r\colon C(K)\to C(F)$ be the restriction mapping. Then there is a continuous section of r, say $P\colon C(F)\to C(K)$ (i.e. $r\cdot P=\mathrm{id}_{C(F)}$).*

Proof. Clearly, r is an onto mapping. Let $\omega: C(F)\times F\to R$ be the evaluation mapping defined by $\omega(f,p)=f(p)$. On account of the compactness of F, ω is continuous (cf. [H, III, J]). Further, the space $C(F)\times K$ is paracompact—being the product of a metric with a compact space—and $C(F)\times F$ is a closed subset of it, so that there is a continuous extension $\bar{\omega}: C(F)\times K\to R$ of ω. We let $P: C(F)\to C(K)$ be given by $(P(f))(p)=\bar{\omega}(f,p)$ for $f\in C(F), p\in K$. The exponential law assures the continuity of P, and it is clear that $r\cdot P=\mathrm{id}_{C(F)}$.

4.4 Remark. The section P in Lemma 4.3 cannot in general be chosen to be linear. E.g., let X be a locally compact space. The restriction $C(\beta X)\to C(\beta X-X)$ admits a section by the above lemma. If in addition, we choose X to be non-pseudocompact, then the section is never linear, as it has been recently shown by Conway (*Projections and retractions*, Proc. Amer. Math. Soc. **17** (1966), 843–847).

We also state without proof the dual of Lemma 4.3, which can be shown in a similar way. It will not be used in the sequel.

4.3a Proposition. *Let E and X be compact spaces, and $\varphi: E\to X$ a continuous, onto mapping. Let $\varphi^*: C(X)\to C(E)$ be given by $\varphi^*(f)=f\cdot\varphi$. Then there is a retraction $r: C(E)\to C(X)$ for φ^*, i.e. $r\cdot\varphi^*=\mathrm{id}_{C(X)}$.*

4.4a Remark. In 4.3a, r cannot in general be chosen to be linear. E.g., consider X a nonextremally disconnected space, and E its Gleason "projective cover" with $\varphi: E\to X$ the natural mapping. Then it can be proved that r cannot be chosen to be linear.

4.5 Lemma. *Let F be a closed subset of the compact space K. For any space X, the set $X\times F$ is C-embedded in $X\times K$.*

Proof. Let $f\in C(X\times F)$; by the exponential law for mapping spaces, the mapping $f^*: X\to C(F)$ given by $(f^*(x))(y)=f(x,y)$ for $(x,y)\in X\times F$ is continuous. By 4.3, there is a continuous section $P: C(F)\to C(K)$. Let $g^*=P\cdot f^*: X\to C(K)$. The mapping $g: X\times K\to R$ given by $g(x,k)=(g^*(x))(k)$ is continuous by the exponential law. Clearly, g is an extension of f.

We recall the notation used in 4.1: if U is a cozero-set of $X\times K$, we set $U_x=U\cap\pi_1^{-1}(x)$ for every $x\in X$. The following lemma will be crucial for the proof of the main theorems.

4.6 Lemma. *Let X be a P-space, K a compact space, and U a cozero-set in $X\times K$. Then, for any $x\in X$, there is an open neighborhood V_x of x, such that $V_x\times\pi_2(U_x)=U\cap(V_x\times K)$.*

Proof. By 4.1(ii) there is an open neighborhood $V_x^{(1)}$ of x such that $V_x^{(1)}\times\pi_2(U_x)\subset U$. We notice that the set $K-\pi_2(U_x)$ is closed and that $U\cap(X\times(K-\pi_2(U_x)))$ is a cozero-set of $X\times(K-\pi_2(U_x))$. Let

$$\tilde{\pi}_1: X\times(K-\pi_2(U_x))\to X$$

be the usual projection. By 4.1(i) the set $\tilde{\pi}_1(U \cap (X \times (K - \pi_2(U_x))))$ is open-and-closed in X, and it does not contain x. Let now $V_x^{(2)} = X - \tilde{\pi}_1(U \cap (X \times (K - \pi_2(U_x))))$. This set is an open neighborhood of x, and further, $(U \cap (V_x^{(2)} \times K)) - (V_x^{(2)} \times \pi_2(U_x)) = \varnothing$ by definition. Set $V_x = V_x^{(1)} \cap V_x^{(2)}$; clearly V_x satisfies the conditions of our lemma.

We now introduce the following notation, where X, K and U retain the meaning as in 4.6. For any subset C of K we let $X_C = \{x \in X : \pi_2(U_x) = C\}$.

4.7 LEMMA. *Let X be a P-space, K a compact space, and U a cozero-set of $X \times K$. If C is a cozero-set of K, then X_C is open-and-closed in X.*

Proof. Lemma 4.6 clearly implies that X_C is an open subset of X. To show that X_C is closed, let x_λ be a net of points in X_C such that $x_\lambda \to x$. We thus have that $\pi_2(U_{x_\lambda}) = C$ for all λ. Let now $(x, k) \in U_x$ be such that $k \notin C$. Notice that $(x_\lambda, k) \to (x, k)$. We choose $\varphi \in C(X \times K)$, $0 \leqq \varphi \leqq 1$, such that

$$U = \{(y, l) \in X \times K : \varphi(y, l) > 0\}.$$

Since $(x_\lambda, k) \notin U$, it follows that $\varphi(x_\lambda, k) = 0$. By continuity, $\varphi(x, k) = 0$, i.e. $(x, k) \notin U_x$. We have shown that $\pi_2(U_x) \subset C$. Conversely, let $k \in C$; then $(x_\lambda, k) \in U$, and hence $(x_\lambda, k) > 0$. The space $X \times (k)$ is a P-space, and hence the set

$$\{(y, k) : \varphi(y, k) > 0\}$$

(with k fixed) must be closed in $X \times (k)$. As a consequence, $\varphi(x, k) > 0$, i.e. $k \in \pi_2(U_x)$. Thus we have shown that $\pi_2(U_x) = C$, i.e. that $x \in X_C$.

5. **Baire sets in compact F-spaces.** A Baire set is defined to be an element of the σ-field generated by the family of all cozero-sets. It will be shown in this section that every Baire set in a compact F-space is C^*-embedded. The proof will require the following theorem, proved independently by Frolík [F] and by the author [N]. A mapping will be called compact if it is onto, continuous, closed, and such that every compact subset of the range has a compact total preimage.

5.1 THEOREM. *A space X is a Baire set in its Stone-Čech compactification when and only when there is a separable metric Borel space M and a compact mapping $\varphi: X \to M$. In particular, if X is a Baire set in a compact space, then X has the Lindelöf property.*

We now prove the following set-theoretic fact about F-spaces.

5.2 THEOREM. *Every Lindelöf subspace of an F-space is C^*-embedded.*

Proof. Let C be a subspace of the F-space X having the Lindelöf property. The proof will be via the Urysohn extension theorem [GJ, 1.17]. Let A and B be two (closed) subsets of C which are completely separated in C, then we must prove that they are completely separated in X (two sets are completely separated in X if they

can be separated by an element of $C^*(X)$—cf. §2, above). Notice that $\bar{A} \cap B = A \cap \bar{B} = \varnothing$ (the bar denotes closure in X). For every $p \in A$, let $U(p)$ be a cozero-set neighborhood of p, and $Z(p)$ a zero-set of X, such that $Z(p) \supset U(p)$ and $Z(p) \cap \bar{B} = \varnothing$; and for every $q \in B$, let $V(q)$ be a cozero-set neighborhood of q, and $H(q)$ a zero-set of X such that $H(q) \supset V(q)$ and $H(q) \cap \bar{A} = \varnothing$. Since both A and B are Lindelöf spaces, there are countable subcovers, say $\{U_n\}$, $\{V_n\}$ of the covers $\{U(q)\}$ and $\{V(q)\}$, respectively. Let $\{Z_n\}$, $\{H_n\}$ be the corresponding zero-sets. Inductively, define $\tilde{U}_1 = U_1$, $\tilde{V}_1 = V_1 - Z_1$, $\tilde{U}_n = U_n - (H_1 \cup \cdots \cup H_{n-1})$, $\tilde{V}_n = V_n - (Z_1 \cup \cdots \cup Z_n)$ for $n \geqq 2$; notice that $\tilde{U}_n$, $\tilde{V}_n$, $n \geqq 1$, are cozero-sets. Finally, we set $\tilde{U} = \bigcup_n U_n$, $\tilde{V} = \bigcup_n V_n$, then $\tilde{U}$, $\tilde{V}$ are disjoint cozero-sets in X, with $\tilde{U} \supset A$, $\tilde{V} \supset B$. Since X is an F-space, $\tilde{U}$ and $\tilde{V}$ are completely separated; hence A, B are completely separated in X.

5.3 Corollary. *In a compact F-space every Baire set is C^*-embedded.*

A special case of 5.3 for F-spaces of the form $\beta Y - Y$ for Y locally compact, σ-compact has been proved earlier by the author [N].

5.4 Example. An F-space with a zero-set which is not C^*-embedded. The space Π defined in [GJ, 6Q] is extremally disconnected (in fact $N \subset \Pi \subset \beta N$), but the zero-set $\Pi - N$ is not C^*-embedded in Π.

On the other hand, compactness is not necessary; e.g., an F-space which is a Baire set in its Stone-Čech compactification satisfies 5.3.

We include here an easy consequence of 4.7, which will be needed in the proof of 6.1, below.

5.5 Lemma. *Let X be a P-space, K a compact space, and B a Baire set of $X \times K$. Then, there is a decomposition of the space X in open-and-closed sets, say $\{V_\alpha\}$, and there are Baire sets of K, $\{B_\alpha\}$, such that $B = \bigcup_\alpha (V_\alpha \times B_\alpha)$.*

Proof. We simply verify that the sets B of the form above constitute a σ-field (since X is a P-space), containing (by 4.7) the family of cozero-sets of $X \times K$.

6. **The main theorems.** The proofs of our main theorems can now be given with no difficulty.

6.1 Theorem. *Let X be a P-space and K a compact F-space. Then, every Baire set of $X \times K$ is C^*-embedded. In particular, $X \times K$ is an F-space.*

Proof. If B is a Baire set of $X \times K$, we must show it is C^*-embedded in $X \times K$. Indeed, let $f \in C^*(B)$. By 5.5, there is a decomposition of the space X into open and closed sets, say $\{V_\alpha\}$, and there are Baire sets of K, $\{B_\alpha\}$, such that $B = \bigcup_\alpha (V_\alpha \times B_\alpha)$. Clearly then, we only need to extend $f_\alpha = f \mid (V_\alpha \times B_\alpha)$ to some continuous $\bar{f}_\alpha \colon V_\alpha \times K \to R$. By 5.1, each B_α is a Lindelöf space, and thus we may use 4.2, in conjunction with 5.3, to extend f_α to some continuous $\tilde{f}_\alpha \colon V_\alpha \times \bar{B}_\alpha \to R$; by Lemma 4.5, we may extend $\tilde{f}_\alpha$ to $\bar{f}_\alpha$, as required. This completes the proof of the theorem.

We now draw an immediate generalization of the weaker statement in 6.1.

6.2 COROLLARY. *If X is a P-space, Y an F-space and $\pi_1: X\times Y\to X$ a closed mapping, then $X\times Y$ is an F-space. In particular, the conclusion holds if X is a P-space and Y a Lindelöf F-space.*

Proof. It is known that βY is an F-space (e.g. see [GJ, 14.25]), and hence 6.1 implies that $X\times\beta Y$ is an F-space; Lemma 4.2 shows that $X\times Y$ is an F-space.

6.3 THEOREM. *If X is a P-space and K a compact basically disconnected space, then $X\times K$ is a basically disconnected space.*

Proof. We must prove that two disjoint open sets, one of which is a cozero-set, have disjoint closures. Let U_1 be a cozero-set disjoint from the open set U_2 in $X\times K$. We use Lemma 4.7 to find a decomposition of X into open-and-closed sets $\{V_\alpha\}$, and cozero-sets of K, $\{C_\alpha\}$, such that $U_1=\bigcup_\alpha (V_\alpha\times C_\alpha)$; evidently, there is no loss of generality to assume that $U_1=X\times C$ for some cozero-set C of K. Then $\pi_2(U_2)\cap\pi_2(U_1)=\pi_2(U_2)\cap C=\varnothing$. Since K is basically disconnected, we have $\mathrm{cl}\,(\pi_2(U_2))\cap\mathrm{cl}\,(C)=\varnothing$ (cl=closure), and hence $\tilde{U}_1\cap\tilde{U}_2=\varnothing$.

We also state the analogue of 6.2 for basically disconnected spaces.

6.4 COROLLARY. *If X is a P-space, Y a basically disconnected space, and $\pi_1: X\times Y\to X$ a closed mapping, then $X\times Y$ is basically disconnected. In particular, the conclusion holds if X is a P-space and Y a Lindelöf basically disconnected space.*

The following theorem provides an interesting characterization of a P-space X in terms of the space $X\times\beta X$.

6.5 THEOREM. *The following conditions are equivalent on a space X.*

(1) *X is a P-space.*

(2) *$X\times\beta X$ is an F-space.*

(3) *Every Baire subset of $X\times\beta X$ is C^*-embedded.*

(4) *$X\times\beta X$ is basically disconnected.*

Proof. (1) implies (4): If X is a P-space, then βX is basically disconnected. By 6.3, (4) holds.

(4) implies (2): Trivial.

(2) implies (1): By the result mentioned in 3.3, either X or βX must be a P-space. A compact P-space is finite; hence, in any case, X is a P-space.

(3) implies (2): Trivial.

(2) implies (3): If $X\times\beta X$ is an F-space, then βX is an F-space, and the result follows from 6.1.

7. This section contains examples which show the limitations of some of the theorems proved above, as well as the necessity of some of our assumptions.

7.1 EXAMPLE. It will be noticed that 6.2 and 6.4 are the most general results offered in this paper toward the solution of the problems: "When is the product of two spaces an F-space (respectively, a basically disconnected space)?" We now

produce a simple example which shows that the conditions of 6.2 and 6.4 are not necessary. In [CN, Example 4.5] two P-spaces X and Y are constructed such that $X\times Y$ is not C^*-embedded in $X\times\beta Y$. Let now Z be the discrete union of X and Y. Clearly, Z is a P-space, such that $Z\times Z$ is not C^*-embedded in $Z\times\beta Z$.

On the other hand, one may consider the following rather trivial statement as a weak converse to Theorem 6.2.

7.2 PROPOSITION. *If X is a P-space and Y a realcompact F-space, whose product is not C^*-embedded in $X\times\beta Y$, then there is $p\in\beta Y-Y$ such that the space $X\times\{Y\cup(p)\}$ is not an F-space.*

Proof. Indeed, there is $p\in\beta Y-Y$ such that $X\times Y$ is not C^*-embedded in $X\times\{Y\cup(p)\}$ (cf. [GJ, 6H]). The fact that Y is realcompact implies that $X\times Y$ is a cozero-set of $X\times\{Y\cup(p)\}$; hence, $X\times\{Y\cup(p)\}$ is not an F-space.

In connection with this proposition, the question arises whether the condition of realcompactness on Y is necessary. More specifically, is $X\times Y$ an F-space when and only when $X\times vY$ is an F-space? I do not know the answer even for Y pseudocompact.

As it has been mentioned in 3.4, Gillman [G] was the first to construct an example of a P-space and an extremally disconnected space whose product is not an F-space. We generalize this procedure to prove the following result.

7.3 THEOREM. *For every nondiscrete P-space X of nonmeasurable cardinal, there is an extremally disconnected space Y, whose product $X\times Y$ is not an F-space.*

Proof. Let p be a nonisolated point of X. Let D be the set $X-(p)$ with the discrete topology, and choose $q\in\beta D-D$ such that the ultrafilter on D converging to q is mapped under the identity mapping $D\to X-(p)$ to a z-ultrafilter on $X-(p)$ that converges to p. Such a choice is possible by [GJ, 14F.1]. Let $Y=D\cup(q)$ with the topology inherited from βD; thus Y is extremally disconnected. We claim that $X\times Y$ is not an F-space. Notice that D is a realcompact space (since $|D|$ is nonmeasurable), and hence $X\times D$ is a cozero-set in $X\times Y$. We will establish our claim, if we prove that $X\times D$ is not C^*-embedded in $X\times Y$. Let $C=(p)\times D$, then C is C-embedded in $X\times D$. For every $d\in D$, choose a zero-set Z_d in X, such that $d\in Z_d$ but $p\notin Z_d$, and define $\Delta=\bigcup_{d\in D}Z_d\times(d)$. Thus, Δ is a zero-set of $X\times D$, disjoint from C; according to [GJ, 1.18] Δ and C are completely separated. We prove that $(p,q)\in\bar{\Delta}\cap\bar{C}$ (the bar denotes closure in $X\times Y$); this will show that $X\times D$ is not C^*-embedded in $X\times Y$. Clearly, $(p,q)\in\bar{C}$. On the other hand, a basic neighborhood of (p,q) has the form $U\times\{V\cup(q)\}$ where U is an open set of X containing p, and V is an element of the ultrafilter converging to q. By our choice of q, $V\cap U\neq\varnothing$, i.e. $\{U\times\{V\cup(q)\}\}\cap\Delta\neq\varnothing$. This completes the proof.

7.4 EXAMPLE. In view of 6.2 and 6.4, and the fact that a P-space is a basically disconnected space (and hence an F-space), it is reasonable to ask if the product of an extremally disconnected P-space and a compact extremally disconnected space is always extremally disconnected. For all practical purposes, the question is

rendered trivial (and true) on account of the following fact, due to Isbell (see [GJ, 12H]): An extremally disconnected P-space of nonmeasurable cardinal is discrete. This, however, need not be the case if we admit the existence of measurable cardinals.

Let D be a discrete set of a measurable cardinal; then D is an extremally disconnected nondiscrete P-space, and βD is a compact extremally disconnected space. A brief consideration of the continuous mapping defined on $D \times \beta D$ to be equal to zero off the "diagonal", and equal to one on the "diagonal" shows that $D \times \beta D$ is not C^*-embedded in $vD \times \beta D$. (The same mapping has been used in [CN] to show that $v(D \times \beta D) = vD \times \beta D$ fails.) Since every dense (or alternatively, every open) subset of an extremally disconnected space is known to be C^*-embedded, it follows that $vD \times \beta D$ is not extremally disconnected. Of course, by 6.4, it is basically disconnected.

7.5 Example. If X is a Lindelöf F-space, then according to 5.2, βX is the only compactification which is an F-space. For general spaces we cannot make a statement of this sort. E.g. let D be an uncountable discrete set and let E_1 be the set of points of $\beta D - D$, which are not in the closure of any countable subset of D. Let K be the quotient space of βD, by identifying E_1 to a point, say $\{\alpha\}$. Then it is easy to see that K is basically disconnected. (Indeed the dense P-space $D \cup \{\alpha\}$ is C^*-embedded in K.) A more subtle example can be given, satisfying an additional condition. We say that K is an r-compactification of X if for every $p \in K - X$, there is a zero-set Z of K such that $p \in Z \subset K - X$. It is easily seen that every compactification of a Lindelöf space is an r-compactification. It would seem plausible that the only r-compactification of a (realcompact) F-space X, which is itself an F-space, is βX. The following is a counterexample. Let X, Y be realcompact P-spaces such that $X \times Y$ is not C^*-embedded in $X \times \beta Y$ (see 7.1). Then $\beta(X \times \beta Y)$ is an r-compactification of $X \times Y$, which in fact is basically disconnected by 6.3.

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