TRANSACTIONS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 162, December 1971

BERGMAN MINIMAL DOMAINS IN SEVERAL COMPLEX VARIABLES

BY
SHIGEO OZAKI AND SADAO KATŌ

Abstract. K. T. Hahn has obtained the inequality between the Jacobians of a biholomorphic mapping and a holomorphic automorphism of a Bergman minimal domain. This paper extends Hahn's result. Some inequalities concerning Jacobians of the mappings of minimal domains onto another minimal domain are considered, and an example is given.

1. **Introduction.** Let D be any bounded schlicht domain in C^n of n complex variables $z\equiv(z_1,\dots,z_n)'$, where symbol $'$ denotes the transpose. Let $w=f(z)\equiv(f_1(z),\dots,f_n(z))'$ be a biholomorphic mapping of D onto a domain in C^n, that is, $f_i(z)$ are holomorphic functions on D with Jacobian $J_f(z)=\det(df(z)/dz)\neq 0$, where $df/dz\equiv(\partial/\partial z_1,\dots,\partial/\partial z_n)\times f$ [10] and the sign $\times$ denotes the Kronecker product. We consider the class $\mathfrak{S}_a(t_0)$ of biholomorphic mappings $w=f(z)$, which are restricted at t_0 ($\in D$) by the condition $J_f(t_0)=a$ for a constant a. A domain D is called the *Bergman minimal domain* (hereafter merely called *minimal domain*) with center at a point t_0 ($\in D$) if any $w=f(z)$ in $\mathfrak{S}_1(t_0)$ maps D onto a domain B such that $v(D)\leqq v(B)$, where $v(D)$ denotes the Euclidean volume of D [2], [9], [11].

$L^2(D)$ denotes the class of square integrable holomorphic functions of D and $L^2_a(D)$ the class of functions $w=f(z)$ in $L^2(D)$ satisfying $f(t_0)=a$ at fixed $t_0\in D$ for a given constant a. The Bergman kernel function $k_D(z,\bar t)$ of D is a holomorphic function of z and $\bar t$, and belongs to $L^2(D)$. Moreover, $k_D(t,\bar z)=k_D(z,\bar t)^-$, $k_D(z,\bar z)>0$ and the reproducing property

$$f(t)=\int_D k_D(t,\bar z)f(z)\,dv_D \tag{1.1}$$

holds for any $f(z)$ in $L^2(D)$, where dv_D denotes the Euclidean volume element on D [1]. Let $T_D(z,\bar t)\equiv\partial^2\log k_D(z,\bar t)/\partial t^*\,\partial z$, where symbol $*$ denotes the transposed conjugate, then .

$$T_D(z,\bar t)=(df(t)/dt)^*T_B(f(z),f(t)^-)(df(z)/dz)$$

under any biholomorphic mapping $w=f(z)$ of D onto B and $T_D(z,\bar z)$ is positive definite. Therefore the Bergman metric $ds_D^2\equiv dz^*T_D(z,\bar z)\,dz$ is absolutely invariant

Received by the editors September 9, 1970.

AMS 1969 *subject classifications.* Primary 3235; Secondary 3290.

Key words and phrases. Bergman kernel function, minimal domain, biholomorphic mapping, holomorphic automorphism, relative invariant.

under any biholomorphic mapping [3]. The Bergman kernel $k_D(z, \bar{t})$ is a *relative invariant* of D, that is,

$$(1.2) \qquad k_D(z, \bar{t}) = J_f(t)^- k_B(f(z), f(t)^-) J_f(z), \qquad z, t \in D,$$

and also $\det T_D(z, \bar{t})$, so that $I_D(z, \bar{t}) \equiv k_D(z, \bar{t})/\det T_D(z, \bar{t})$ is an *absolute invariant* $I_D(z, \bar{t}) = I_B(f(z), f(t)^-)$, under any biholomorphic mapping $w = f(z)$ of D onto B [7], [11].

The following theorem, which is the fundamental theorem relating to minimal domains, is known as the Bergman minimal problem [3], [6]. We state it without proof.

THEOREM 1.1. *Let $w = M_D^a(z, t_0)$ be the function in $L_a^2(D)$ which minimizes $\int_D |w|^2 \, dv_D$. Then the minimizing function exists uniquely and is expressed as follows: $M_D^a(z, t_0) = a k_D(z, \bar{t}_0)/k_D(t_0, \bar{t}_0)$, and the corresponding minimum value is $|a|^2/k_D(t_0, \bar{t}_0)$.*

In §2 it is shown that a minimal domain D with center at t_0 is equivalent to $t_0 \in c(D)$ (Theorem 2.2), where

$$(1.3) \qquad c(D) \equiv \{t; t \in D, k_D(t, \bar{t}) = 1/v(D)\} \qquad [5].$$

Moreover, the inequalities concerning Jacobians of the biholomorphic mapping which maps a minimal domain onto another minimal domain are obtained (Theorem 2.8). In §3 various applications are given from the properties of absolute and relative invariants. Theorem 3.1 is the main result which implies Hahn's theorem [5, Theorem 3.2], and we follow his procedure wherever possible.

2. **Minimal domains and the mappings onto them.** Throughout this paper we shall deal with only bounded schlicht domains in C^n for which the kernel functions become infinite everywhere on the boundary. First we consider the mapping $w = \varphi(z)$ in $\mathfrak{S}_1(t_0)$ which maps any domain D onto a minimal domain Δ. Theorem 1.1 shows that if such a mapping exists then $J_\varphi(z) = M_D^1(z, t_0) = k_D(z, \bar{t}_0)/k_D(t_0, \bar{t}_0)$ and $v(\Delta) = 1/k_D(t_0, \bar{t}_0)$. Existence of such a mapping is given by the following theorem.

THEOREM 2.1. *There exists a mapping $w = \varphi(z)$ in $\mathfrak{S}_1(t_0)$, $t_0 \in D$, which maps any bounded schlicht domain D in C^n onto a minimal domain* [9].

We remark that such mappings need not be unique.

It is well known that a necessary and sufficient condition for a bounded schlicht domain D in C^n to be a minimal domain with center at t_0 is

$$(2.1) \qquad k_D(z, \bar{t}_0) \equiv 1/v(D)$$

for all $z \in D$ [9, Theorems 3.1, 3.2]. Thus, for the set $c(D)$ defined by (1.3), we have

THEOREM 2.2. *A bounded schlicht domain D in C^n is a minimal domain with center at t_0 if and only if $t_0 \in c(D)$.*

Thus "minimal domain D with center at t_0" may be replaced by "$t_0 \in c(D)$".

COROLLARY 2.1. *Suppose that D is a bounded schlicht domain in C^n. If $t_0 \in c(D)$, then the volume of the image domain Δ of D under f in $\mathfrak{S}_1(t_0)$ is not smaller than the volume of the original domain D, and conversely* [5, *Theorem* 2.2].

THEOREM 2.3. *A bounded schlicht minimal domain D in C^n cannot have more than one center* [9, *Theorem* 4.2].

Proof. Let s_0, t_0 be two distinct centers of D. Then, since (2.1) holds and $k_D(s_0, \bar{t}_0)\,(=k_D(t_0, \bar{t}_0))$ is real, $k_D(z, \bar{s}_0)=k_D(t_0, \bar{s}_0)=k_D(s_0, \bar{t}_0)^-=k_D(s_0, \bar{t}_0)=k_D(z, \bar{t}_0)$ for all $z \in D$. If we consider $f(z)\equiv z-s_0$ in $L^2(D)$, then it follows from (1.1) that

$$0 = f(s_0) = \int_D k_D(s_0, \bar{z})f(z)\, dv_D = \int_D k_D(t_0, \bar{z})f(z)\, dv_D = f(t_0) \neq 0,$$

which is a contradiction.

THEOREM 2.4. *A product domain D in C^n is a minimal domain if and only if its components are minimal domains* [9, *Theorem* 4.3].

Since the ring domain $D\colon 0<r<|z|<1$ in a complex plane is not a minimal domain, we have

EXAMPLE 1. The domain $R\equiv R_1\times\cdots\times R_n$ cannot become a minimal domain, where $R_i\equiv\{z_i;\, 0<r_i<|z_i|<1\}$, $i=1,\ldots,n$ (see [5, Theorem 2.4]).

COROLLARY 2.2. *The set $c(D)$ consists of at most one point of D* [5, *Theorem* 2.3].

Now, we consider the mappings of a minimal domain onto a minimal domain.

LEMMA 2.1. *Let D be a bounded schlicht minimal domain in C^n with center at t_0 and B the image domain of D under a mapping $w=f(z)$ in $\mathfrak{S}_1(t_0)$, then $v(D)\leqq v(B)$. The equality occurs if and only if $J_f(z)\equiv 1$ on D (see also* [8, *Theorem* 1.2]).

Proof. The first part of this theorem is obvious from the definition of minimal domain. To prove the last part we use the following relation:

$$v(B) = \int_B dv_B = \int_D |J_f(z)|^2\, dv_D \geqq \int_D |M_D^1(z, t_0)|^2\, dv_D,$$

where $J_f(z)$ is holomorphic on D, so that $J_f(z) \in L_1^2(D)$. Theorem 1.1 asserts in this inequality that the equality occurs if and only if $J_f(z)=M_D^1(z, t_0)$. On the other hand, since D is a minimal domain having t_0 as center, $M_D^1(z, t_0)=k_D(z, \bar{t}_0)/k_D(t_0, \bar{t}_0)\equiv 1$ on D, which implies $\int_D |M_D^1(z, t_0)|^2\, dv_D=v(D)$. Thus the conclusion follows.

THEOREM 2.5. *Let D be a bounded schlicht minimal domain in C^n with center at t_0 and Δ the image domain of D under a mapping $w=f(z)$ in $\mathfrak{S}_1(t_0)$. Then Δ is a minimal domain with center at τ_0 $(\equiv f(t_0))$ if and only if $J_f(z)\equiv 1$ on D.*

Proof. Let Δ be a minimal domain with center at τ_0 then Δ is one of the equivalent class of D, and hence $v(\Delta)=v(D)$. Therefore $J_f(z)\equiv 1$. The converse is trivial.

THEOREM 2.6. *Let Δ, Δ_1 be the images of a bounded schlicht domain D in C^n under the mappings f, f_1 in $\mathfrak{S}_a(t_0)$, and the domains B, B_1 the images of D under $f/a^{1/n}$, $f_1/a^{1/n}$, respectively. If $v(\Delta) \leqq v(\Delta_1)$ then $v(B) \leqq v(B_1)$, and conversely.*

Proof. Suppose that $v(B) > v(B_1)$. Since $v(\Delta) = \int_\Delta dv_\Delta = \int_B |J_f/J_g|^2 \, dv_B = |a|^2 v(B)$, where $g \equiv f/a^{1/n}$, similarly $v(\Delta_1) = |a|^2 v(B_1)$, we have $v(\Delta) > v(\Delta_1)$. This contradicts the hypothesis. Q.E.D.

Let $w = f(z)$ be a biholomorphic mapping and $f(t_0) = \tau_0$. Noting that the mapping $F(z) \equiv (f(z) - \tau_0)/J_f(t_0)^{1/n} + \tau_0$ normalized at $t_0 \in D$, belongs to $\mathfrak{S}_1(t_0)$, we have

COROLLARY 2.3. *Let D be a bounded schlicht minimal domain in C^n with center at t_0 and Δ the image domain of D under a biholomorphic mapping $w = f(z)$. Then Δ is a minimal domain with center at τ_0 $(\equiv f(t_0))$ if and only if $J_f(z) \equiv J_f(t_0)$ on D* [9, *Theorem* 5.1].

COROLLARY 2.4. *Let D be a bounded schlicht domain in C^n with $c(D) = \{t_0\}$ and Δ the image domain of D under the biholomorphic mapping $w = f(z)$. Then $c(\Delta) = \{\tau_0\}$ for $\tau_0 = f(t_0)$ if and only if the Jacobian $J_f(z)$ of the mapping is identically constant on D* [5, *Theorem* 3.1(a)].

The following lemma is useful to study the properties relating to minimal domains.

LEMMA 2.2. *If a domain D in C^n is a bounded schlicht minimal domain with center at t_0, then*

$$k_D(t_0, \bar{t}_0) < k_D(\zeta, \bar{\zeta}) \tag{2.2}$$

for any ζ $(\neq t_0) \in D$ [9, *Lemma*].

Proof. Consider a minimal domain Δ with respect to $\zeta \in D$ under the mapping $w = \varphi(z)$ in $\mathfrak{S}_1(\zeta)$. Then, since a minimal domain D has only one center by Theorem 2.3, we have $1/k_D(t_0, \bar{t}_0) = v(D) > v(\Delta) = 1/k_D(\zeta, \bar{\zeta})$, which implies $k_D(t_0, \bar{t}_0) < k_D(\zeta, \bar{\zeta})$. Q.E.D.

THEOREM 2.7. *Let D be a bounded schlicht minimal domain in C^n with center at t_0 and Δ the image domain of D under a biholomorphic mapping $w = f(z)$ such that $f(\zeta) = \tau_0$ for $\zeta \neq t_0$. If Δ is a minimal domain having τ_0 as center, then*

$$|J_f(t_0)|^2 = k_D(t_0, \bar{t}_0)/k_\Delta(s, \bar{s}) < k_D(t_0, \bar{t}_0)/k_\Delta(\tau_0, \bar{\tau}_0) = |J_f(t_0)| \, |J_f(\zeta)|, \tag{2.3}$$

where $s \equiv f(t_0)$. In particular, $|J_f(t_0)| < |J_f(\zeta)|$.

Proof. It is trivial from (1.2) for the left side of (2.3). Also from (2.2) the central inequality follows. Since

$$k_D(t_0, \bar{t}_0) = k_D(\zeta, \bar{t}_0) = J_f(t_0)^- k_\Delta(\tau_0, \bar{s}) J_f(\zeta) = J_f(t_0)^- k_\Delta(\tau_0, \bar{\tau}_0) J_f(\zeta),$$

we have the right side. Q.E.D.

COROLLARY 2.5. *Let D be a bounded schlicht domain in C^n with $c(D)=\{t_0\}$ and Δ the image domain of D under the biholomorphic mapping $w=f(z)$. If $c(\Delta)=\{\tau_0\}$ for $\tau_0\neq f(t_0)$, then $|J_f(t_0)|<|J_f(\zeta)|$, where $f(\zeta)=\tau_0$* [5, *Theorem* 3.1(b)].

THEOREM 2.8. *Let D be a bounded schlicht minimal domain in C^n with center at t_0 and Δ the image domain of D under a biholomorphic mapping $w=f(z)$ such that $f(\zeta)=\tau_0$ for $\zeta\neq t_0$. If Δ is a minimal domain having τ_0 as center and $v(\Delta)=v(D)$, then*

$$|J_f(t_0)| < 1 < |J_f(\zeta)|. \tag{2.4}$$

Proof. In (2.3), if $v(\Delta)=v(D)$ then $|J_f(t_0)|^2<1=|J_f(t_0)|\,|J_f(\zeta)|$, since $k_\Delta(\tau_0,\bar\tau_0)=1/v(\Delta)=1/v(D)=k_D(t_0,\bar t_0)$. Therefore, it follows that $|J_f(t_0)|<1$ and hence $|J_f(\zeta)|=1/|J_f(t_0)|>1$. Q.E.D.

REMARK 1. The right half of (2.4) has been obtained by M. Maschler [9, Theorem 5.2].

THEOREM 2.9. *Let D be a bounded schlicht minimal domain in C^n with center at t_0. If there exists a holomorphic automorphism $w=h(z)$ which maps ζ ($\neq t_0$) into t_0, then $|J_h(t_0)|<1<|J_h(\zeta)|$.*

Considering a case of $\Delta\equiv D$ in Theorem 2.8, the above theorem is obtained and also the following corollary.

COROLLARY 2.6. *Let D be a bounded schlicht minimal domain in C^n with center at t_0 and $w=h(z)$ any holomorphic automorphism of D which maps ζ into t_0. If $I_D(z,\bar z)\leqq 1$ on D, then*

$$I_D(z,\bar z) \leqq |J_h(\zeta)| \tag{2.5}$$

for all $z\in D$.

REMARK 2. Since $1<|J_h(\zeta)|$, (2.5) implies $I_D(z,\bar z)\leqq|J_h(\zeta)|^2$. Therefore, this corollary contains [5, Theorem 3.3] and its corollary.

EXAMPLE 2. Let D: $|z|<1$ be a unit-hypersphere in C^n. Then there exists a holomorphic automorphism $w=h(z)\equiv U\Gamma(\zeta)(z-\zeta/1-\zeta^*z)$ which maps ζ ($\neq 0$) into 0, where U is any unitary matrix and $\Gamma(\zeta)\equiv(1-|\zeta|^2)^{1/2}(E_n-\zeta\zeta^*)^{-1/2}$ [6]. A formal calculation shows that

$$dh(z)/dz = U\Gamma(\zeta)\{(1-\zeta^*z)E_n+(z-\zeta)\zeta^*\}/(1-\zeta^*z)^2.$$

Using $|\det U|=1$ and $|\det\Gamma(\zeta)|=(1-|\zeta|^2)^{(n-1)/2}$ which is obtained by a simple computation, we have

$$\text{(i)}\quad \begin{aligned}|J_h(\zeta)| &= |\det(dh(\zeta)/dz)| = |\det U|\,|\det\Gamma(\zeta)|\,|\det(E_n/(1-|\zeta|^2))| \\ &= 1/(1-|\zeta|^2)^{(n+1)/2} > 1,\end{aligned}$$

$$\text{(ii)}\quad |J_h(0)| = |\det U|\,|\det\Gamma(\zeta)|\,|\det(E_n-\zeta\zeta^*)| = (1-|\zeta|^2)^{(n+1)/2} < 1,$$

and thus $|J_h(0)|\,|J_h(\zeta)|=1$.

3. **Applications.** In this section we study the properties concerning the mappings of minimal domains into themselves.

LEMMA 3.1. *Let D be any bounded domain in C^n and $w=g(z)$ any biholomorphic mapping of D into itself. Then*

$$k_D(w, \bar{w})|J_g(z)|^2 \leqq k_D(z, \bar{z}) \tag{3.1}$$

for all $z \in D$.

Proof. Let G ($\subset D$) be the image of D under $w=g(z)$, then $k_D(w, \bar{w}) \leqq k_G(w, \bar{w})$ [2]. Since the Bergman kernel is a relative invariant, that is, $k_D(z, \bar{z}) = k_G(w, \bar{w})|J_g(z)|^2$, the desired conclusion is obtained.

COROLLARY 3.1. *Let D be any bounded domain in C^n and $w=g(z)$ any biholomorphic mapping of D into itself. Then the invariant*

$$I_D(z, \bar{z}) \ (\equiv k_D(z, \bar{z})/\det T_D(z, \bar{z})) \leqq 1$$

on D if and only if $k_D(w, \bar{w})|J_g(z)|^2 \leqq \det T_D(z, \bar{z})$ for $z \in D$ ([5, *Lemma*] *and* [4, *Theorem* 4]).

In fact, it is obvious from Lemma 3.1 for necessary condition. The converse is immediate if $w=g(z)$ is the identity mapping.

THEOREM 3.1. *Let D be any bounded schlicht minimal domain in C^n with center at t_0 and $w=g(z)$ any biholomorphic mapping of D into itself. If there exists a holomorphic automorphism $w=h(z)$ of D which maps ζ into t_0, then*

$$|J_g(\zeta)| \leqq |J_h(\zeta)|. \tag{3.2}$$

Proof. From Lemma 3.1, $|J_g(\zeta)|^2 \leqq k_D(\zeta, \bar{\zeta})/k_D(g(\zeta), g(\zeta)^-)$ follows replacing z by ζ in (3.1). Since t_0 is the center of minimal domain D, it follows from (2.2) that $k_D(g(\zeta), g(\zeta)^-) \geqq k_D(t_0, \bar{t}_0)$. Therefore $k_D(\zeta, \bar{\zeta})/k_D(g(\zeta), g(\zeta)^-) \leqq k_D(\zeta, \bar{\zeta})/k_D(t_0, \bar{t}_0)$. On the other hand, from (1.2), $k_D(\zeta, \bar{\zeta})/k_D(t_0, \bar{t}_0) = |J_h(\zeta)|^2$ follows. Thus our conclusion is obtained.

REMARK 3. If $I_D(z, \bar{z}) \leqq 1$ is added to Theorem 3.1, then it is trivial from (3.2) that $|J_g(\zeta)|^2 \leqq |J_h(\zeta)|^2/I_D(z, \bar{z})$ for all $z \in D$. In particular, if $I_D(t_0, \bar{t}_0) \leqq 1$ then $|J_g(\zeta)|^2 \leqq |J_h(\zeta)|^2/I_D(t_0, \bar{t}_0)$ (see [5, Theorem 3.2]).

COROLLARY 3.2. *Let D be any bounded schlicht homogeneous minimal domain in C^n with center at t_0 and $w=g(z)$ any biholomorphic mapping of D into itself. Then, for all $z \in D$ we have $|J_g(z)| \leqq |J_h(z)|$, where $w=h(z)$ is a holomorphic automorphism of D which maps z into t_0.*

THEOREM 3.2. *Let D be a bounded schlicht minimal domain in C^n with center at t_0 and $w=g(z)$ a biholomorphic mapping of D into itself. Then*

$$|J_g(z)|^2 \leqq v(D) k_D(z, \bar{z}). \tag{3.3}$$

Also

$$(3.4) \qquad v(g(B)) \leqq v(D)U_D(B)$$

for any measurable $B \subset D$, *where* $U_D(B) \equiv \int_B k_D(z, \bar{z})\, dv_B$.

Proof. Since D is a bounded schlicht minimal domain, $k_D(z, \bar{z})$ attains its minimum at $z = t_0$ by (2.2) and the minimum value is $1/v(D)$. Then the conclusions of this theorem follow from Lemma 3.1.

Since $U_D(B) = U_{g(D)}(g(B))$, we have

COROLLARY 3.3. *Under the hypothesis of Theorem* 3.2, $v(g(B))/U_{g(D)}(g(B)) \leqq v(D)$ *for any measurable set* $B \subset D$ *with nonzero measure. In particular if* $w = g(z)$ *is the identity mapping, then* $v(B)/U_D(B) \leqq v(D)$.

REMARK 4. Let $k_D(z, \bar{z}) \leqq \det T_D(z, \bar{z})$ on D then $U_D(B) \leqq V_D(B)$, where $V_D(B) \equiv \int_B \det T_D(z, \bar{z})\, dv_B$. Thus, if $I_D(z, \bar{z}) \leqq 1$ is added to Theorem 3.2, then it follows from (3.3), (3.4) that $|J_g(z)|^2 \leqq v(D) \det T_D(z, \bar{z})$; also $v(g(B)) \leqq v(D) V_D(B)$. Similarly, if $I_D(z, \bar{z}) \leqq 1$ is added to Corollary 3.3, then it follows from $v(g(B))/U_{g(D)}(g(B)) \leqq v(D)$ and $v(B)/U_D(B) \leqq v(D)$ that $v(g(B))/V_{g(D)}(g(B)) \leqq v(D)$ and $v(B)/V_D(B) \leqq v(D)$, respectively (see [4, Theorem 5 and its corollary]).

REFERENCES

1. S. Bergman, *Sur les fonctions orthogonales de plusieurs variables complexes avec les applications à la théorie des fonctions analytiques*, Mémor. Sci. Math., no. 106, Gauthier-Villars, Paris, 1947. MR **11**, 344.

2. ———, *Sur la fonction-noyau d'un domaine et ses applications dans la théorie des transformations pseudo-conformes*, Mémor. Sci. Math., no. 108, Gauthier-Villars, Paris, 1948. MR **11**, 344.

3. ———, *The kernel function and conformal mapping*, Math. Surveys, no. 5, Amer. Math. Soc., Providence, R. I., 1950. MR **12**, 402.

4. K. T. Hahn and J. Mitchell, *Generalization of Schwarz-Pick lemma to invariant volume in a Kähler manifold*, Trans. Amer. Math. Soc. **128** (1967), 221–231. MR **39** #4444.

5. K. T. Hahn, *Some properties of relative invariants on bounded domains*, Duke Math. J. **34** (1967), 325–332. MR **35** #5658.

6. S. Katō, *Theorems in several complex variables*, Rep. Fac. Tech. Kanagawa Univ. **4** (1965), 7–27.

7. ———, *Canonical domains in several complex variables*, Pacific J. Math. **21** (1967), 279–291. MR **35** #5659.

8. ———, *Mitchell minimal domains in several complex variables*, Duke Math. J. **37** (1970), 139–143. MR **40** #7475.

9. M. Maschler, *Minimal domains and their Bergman kernel function*, Pacific J. Math. **6** (1956), 501–516. MR **18**, 473.

10. S. Ozaki and I. Ono, *Analytic functions of several complex variables*, Sci. Rep. Tokyo Bunrika Daigaku A. **4** (1953), 262–270. MR **15**, 117.

11. T. Tsuboi, *Bergman representative domains and minimal domains*, Japan. J. Math. **29** (1959), 141–148. MR **22** #12242.

SCIENCE UNIVERSITY OF TOKYO, TOKYO, JAPAN

YAMANASHI UNIVERSITY, KŌFU, JAPAN