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THE GROUP OF PL-HOMEOMORPHISMS OF A COMPACT PL-MANIFOLD IS AN l_2^f-MANIFOLD

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ABSTRACT. In this paper it is shown that if M is a compact PL-manifold and $H_{PL}(M)$ is the group of PL-homeomorphisms of M onto itself, then $H_{PL}(M)$ is an l_2^f-manifold. Here l_2 is the Hilbert space of all real-valued square-summable sequences and $l_2^f = \{(x_i) \in l_2: x_i = 0 \text{ for almost all } i\}$.

Introduction. Let M be a compact PL-manifold and $H_{PL}(M)$ be the group of PL-homeomorphisms of M with the compact-open topology. In this paper we show that $H_{PL}(M)$ is an l_2^f-manifold where $l_2^f = \{(x_i) \in l_2: x_i = 0$ for almost all $i\}$ where l_2 is the Hilbert space of all real-valued square-summable sequences. This result is related to the following question which remains unsolved. If M is a compact topological manifold and $H(M)$ is the group of homeomorphisms of M with the compact open topology, is $H(M)$ an l_2-manifold? This is known to be true for $M = S^1$ or I, the circle or interval, by a result of R. D. Anderson (see [10]). It is also known to be true for a 2-dimensional manifold by the results of R. Geoghegan [5], W. K. Mason [14], R. Luke and W. K. Mason [13], and H. Toruńczyk [18]. The proof that this is true for M a compact 2-dimensional manifold is as follows. By Geoghegan's result [5], $H(M)$ is homeomorphic to $H(M) \times l_2$. (This is true for M any manifold [5] and in fact for any metric space admitting a nondegenerate flow [10].) By Luke and Mason [13] and Mason [14], $H(M)$ is an absolute neighborhood retract for metric spaces. Toruńczyk has shown [18, Theorem 4.5] that any complete metric space X which is an ANR for metric spaces has the property that $X \times l_2(A)$ is an $l_2(A)$-manifold where $l_2(A) = \{(x_a): \Sigma_{a \in A} x_a^2 < \infty\}$ is Hilbert space and A has the cardinality of the weight of X. These results together imply that $H(M)$ is an l_2-manifold.

This argument shows that what needs to be shown for $H(M)$ to be an l_2-manifold for M an arbitrary compact manifold is to show that $H(M)$ is an absolute neighborhood retract for metric spaces. The proof in this paper showing that $H_{PL}(M)$ is an l_2^f-manifold for M a compact PL-manifold is patterned after the above proof.

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For M a compact PL-manifold, R. Geoghegan [7] has shown that $H_{PL}(M)$ is *σ-f.d. compact*, that is, $H_{PL}(M)$ is the countable union of compact finite-dimensional subsets. By the proofs of Černavskiĭ [3] it follows that $H_{PL}(M)$ is locally contractible. W. Haver [8] has shown that any σ-f.d. compact locally contractible metric space is an ANR. Thus $H_{PL}(M)$ is an ANR. By Toruńczyk [18, Proposition 4.6], $H_{PL}(M)$ is an l_2^f-manifold if and only if $H_{PL}(M)$ is homeomorphic to $H_{PL}(M) \times l_2^f$. Using the μ-parameterization of M. Morse [15] we show in this paper that $H_{PL}(M)$ is in fact homeomorphic to $H_{PL}(M) \times l_2^f$ and thus that $H_{PL}(M)$ is an l_2^f-manifold.

In the first section of the paper we quote the results that are needed from [4], [6], [8], and [18]. In the second section of the paper we show that $H_{PL}(M)$ is homeomorphic to $H_{PL}(M) \times l_2^f$ which will complete the proof that $H_{PL}(M)$ is an l_2^f-manifold.

1. Preliminary results. In this section we will state results that are needed for the proof of the fact that $H_{PL}(M)$ is an l_2^f-manifold. The results needed come from infinite-dimensional topology, the theory of function spaces, topological and piecewise-linear manifolds, and general topology. The theorems will be stated in a form which is most convenient for reference in this paper and not in the most general form in which they occur in the original papers.

The principal result used is one due to H. Toruńczyk [18, Theorem 4.5]. We state only a special case of this theorem.

1.1. Theorem (Toruńczyk [18]). *Let X be a separable metric space which is the countable union of compact finite-dimensional subspaces. Suppose that X is an ANR for metric spaces. Then $X \times l_2^f$ is an l_2^f-manifold.*

The next three results combine to show that if M is a compact PL-manifold, then $H_{PL}(M)$ is the countable union of compact finite-dimensional subspaces and an ANR for metric spaces. The first result is due to R. Geoghegan [6, Corollary 4.8].

1.2. Theorem (Geoghegan [6]). *Let K be a finite simplicial complex. Then $H_{PL}(K)$ is the countable union of compact finite-dimensional subspaces (i.e., σ-f.d. compact).*

W. Haver [8] proved the following result in order to show that $H_{PL}(M)$ is an ANR. It uses general topological techniques.

1.3. Theorem (Haver [8]). *Let X be a separable metric space which is the countable union of finite-dimensional compacta. If X is locally contractible, then X is an ANR.*

As pointed out in [8] R. Edwards has adapted the proofs in [3] to show the following.

1.4. **Theorem** (Černavskiĭ [3]). *Let M be a compact PL-manifold, then $H_{PL}(M)$ is locally contractible.*

The following corollary is an immediate consequence of Theorems 1.2, 1.3, and 1.4.

1.5. **Corollary.** *Let M be a compact PL-manifold. Then $H_{PL}(M)$ is an ANR and the countable union of finite-dimensional compacta.*

In order to show that $H_{PL}(M)$ is an l_2^f-manifold, we only need to show that $H_{PL}(M)$ is homeomorphic to $H_{PL}(M) \times l_2^f$. The result will then follow from Theorem 1.1 and Corollary 1.5.

2. **$H_{PL}(M)$ is homeomorphic to $H_{PL}(M) \times l_2^f$.** Assume that M is a compact PL-manifold with a given triangulation τ making M a PL-manifold. Let $x, y \in M$ be in the interior of an n-dimensional simplex σ of M where $n = \dim M$. Let A be a straight line in σ connecting x and y. Let $C_1, C_2, \cdots$ be a sequence of n-dimensional simplexes imbedded linearly in the interior of σ such that (1) the C_i's are pairwise disjoint; (2) $\operatorname{diam}(C_i) = \epsilon_i \longrightarrow 0$; (3) $C_i \longrightarrow x$ as $i \longrightarrow \infty$; and (4) for each i, a vertex $c_i \in C_i$ is on A and the barycenter d_i of the opposite $(n-1)$-dimensional face of C_i is also on A. Figure 1 illustrates the way the C_i's are assumed to be arranged.

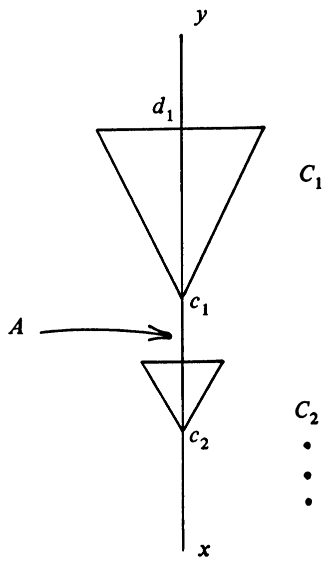

Figure 1

It is elementary to verify that such a sequence of C_i's exists and we will omit the argument. This construction will be used in the proof of Theorem 2.1.

2.1. **Theorem.** *If M is a compact PL-manifold, then $H_{PL}(M)$ is homeomorphic to $H_{PL}(M) \times l_2^f$.*

Before we proceed to the proof of Theorem 2.1 we will need to review the results concerning the μ-parameterization of curves developed by M. Morse [15]. This parameterization was used in [5] and [10].

2.2. **Definition.** Let I be the unit interval $[0, 1]$ and $C(I, X)$ be the continuous functions from I to X with X a metric space. Let $E(I, X) = \{f \in C(I, X)\colon f$ is an imbedding$\}$, and let $E_{PL}(I, X)$ denote the PL-imbeddings of I in X if X has a PL-structure. Endow these function spaces with the compact open topology.

2.3. **Definition.** Let $n \geq 2$ be an integer and let $A_n = \{(t_1, \cdots, t_n)\colon 0 \leq t_1 \leq \cdots \leq t_n \leq 1\}$. Let $f \in C(I, X)$ and define $\delta(f, t_1, \cdots, t_n) = \min\{d(f(t_i), f(t_{i+1}))\colon 1 \leq i \leq n-1\}$. Let $\mu_n(f) = \sup\{\delta(f, t_1, \cdots, t_n)\colon (t_1, \cdots, t_n) \in A_n\}$. Then let

$$\mu(f) = \sum_{n=2}^{\infty} 2^{1-n}\mu_n(f).$$

Now let $k_t\colon I \longrightarrow I$ be defined by $k_t(s) = t \cdot s$ for $0 \leq t \leq 1$. Let $f \in C(I, X)$ and define $f_t = f \circ k_t$. Then $\mu(f_t)$ ranges monotonically between 0 and $\mu(f)$. If $0 \leq s \leq \mu(f)$, then there is a t_s such that $\mu(f_{t_s}) = s$. If $\mu(f) \neq 0$, let f_μ be defined by $f(s/\mu(f)) = f(t_s)$ for $0 \leq s \leq \mu(f)$. Then f_μ is well defined and continuous. If $\mu(f) = 0$, that is, if f is constant, then let $f_\mu = f$ by definition. The function f_μ is the *μ-parameterization* of f.

2.4. **Proposition.** *Let $\psi\colon C(I, X) \longrightarrow C(I, X)$ be defined by $\psi(f) = f_\mu$. Then ψ is a continuous retraction.*

This is Theorem 1.12 in [5]. We note at this point that if X has a PL-structure, then the image under ψ of $E_{PL}(I, X)$ is generally not contained in $E_{PL}(I, X)$ even when the metric on X is linear. We will not give an example of this, since it is somewhat tedious and not pertinent to the main results of the paper.

Let $M_0(I) = \{f \in C(I, I)\colon f$ is monotone nondecreasing and onto$\}$. Let $H_0(I) = \{f \in M_0(I)\colon f$ is a homeomorphism$\}$. Let $C'(I, X) = \{f \in C(I, X)\colon \mu(f) \neq 0\}$. Let $r\colon C'(I, X) \longrightarrow M_0(I)$ be defined by $r(f)(t) = \mu(f_t)/\mu(f)$. Then $\psi(f) \circ r(f) = f$.

2.5. **Proposition.** *The map $r\colon C'(I, X) \longrightarrow M_0(I)$ is continuous. The image of $E(I, X)$ under r is $H_0(I)$.*

This is Theorem 1.12 in [5]. We note that if X has a PL-structure, then the image of $E_{PL}(I, X)$ under r is not generally contained in $H_{PL}(I)$ even with a linear metric on X.

We now state the proposition that we will need. Let $p\colon E(I, X) \longrightarrow I$ be defined by $p(f) = r(f)^{-1}(½)$. Then $0 < p(f) < 1$ and $f(p(f)) = f_\mu(½)$, since $f_\mu(½) =$

$f_\mu(r(f) \circ r(f)^{-1}(\frac{1}{2})) = f(r(f)^{-1}(\frac{1}{2})) = f(p(f))$. Clearly p is continuous, since r is continuous. Thus we have shown the following.

2.6. **Proposition.** *There is a continuous* p: $E(I, X) \to (0, 1)$ *such that* $f_\mu(\frac{1}{2}) = f(p(f))$ *for all* $f \in E(I, X)$.

The following lemma is the last property of the μ-parameterization that we will need. It can be routinely verified from the definition and we omit its proof.

2.7. **Lemma.** *If* $f \in E(I, R^m)$ *is a linear imbedding of* I *into* R^m *and* R^m *has the usual norm metric, then* $f_\mu = f$.

We observe another caveat for the reader. If $f \in E_{PL}(I, R^m)$, then f_μ may not be piecewise linear in general, even with the usual norm metric on R^m.

Proof of Theorem 2.1. Let M be a compact n-dimensional PL-manifold with triangulation τ and assume that M has been imbedded linearly in some Euclidean space R^m. Let R^m have the usual norm metric and let M be given the metric induced by this imbedding. Let $\{C_i\}_{i=1}^\infty$ be the sequence of n-dimensional simplexes imbedded in the interior of the n-simplex σ in M described in the beginning of this section. For each i let A_i be the portion of the arc A joining c_i and d_i in C_i. Observe that A is a straight line in R^m, since M was imbedded linearly and A was a straight line in the simplex σ. Also A_i is a straight line in R^m for all i. Let g_i: $I \to A_i$ be linear with $g_i(0) = d_i$ and $g_i(1) = c_i$. Now let $t \in (0, 1)$ and i be a positive integer. Define $h_{t,i} \in H_{PL}(M)$ in the following manner. Let $h_{t,i}$ be the identity on M minus the interior of C_i and let $h_{t,i}$ be defined in such a way that $g_i^{-1} \circ h_{t,i} \circ g_i(\frac{1}{2}) = t$. Then extend $h_{t,i}$ by mapping the straight line joining the point $x \in \operatorname{Bd}(C_i)$ and the point $g_i(\frac{1}{2})$ linearly onto the straight line joining x and the point $g_i(t)$. Figure 2 shows how $h_{t,i}$ is defined on C_i.

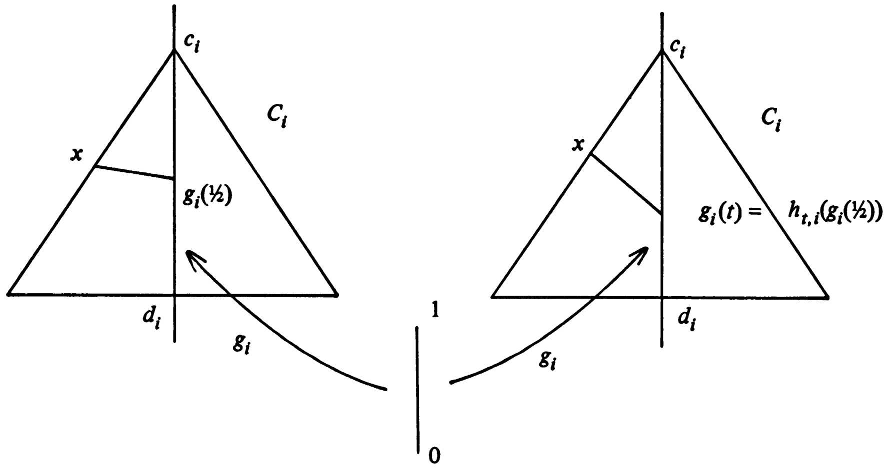

Figure 2

Clearly $h_{t,i}$ defined in this way is a PL-homeomorphism of M onto itself. Note that $h_{t,i}$ takes A_i onto itself. For each i the assignment $t \longrightarrow h_{t,i}$ is a continuous map from $(0, 1)$ into $H_{PL}(M)$.

Now let $h \in H_{PL}(M)$ and define $H_i(h) = p(h \circ g_i) \in (0, 1)$. Then $H_i(h)$ is the $t \in (0, 1)$ such that $h \circ g_i(t) = \psi(h \circ g_i)(\frac{1}{2})$. Let $H: H_{PL}(M) \longrightarrow (0, 1)^\infty$ be defined by $H(h) = (H_i(h))$. Now we will show that for $h \in H_{PL}(M)$, $H_i(h) = \frac{1}{2}$ for almost all i.

Claim 1. For $h \in H_{PL}(M)$, there is an n such that for $i \geq n$, $H_i(h) = \frac{1}{2}$.

Proof of Claim 1. We may suppose that h is a simplicial homeomorphism of M onto itself for some triangulation τ' which refines τ. We may assume that x is a vertex of τ' and that there is a vertex y' of τ' with $x \neq y' \in A$ such that the line A' joining x and y' is a 1-simplex in τ'. There is an n such that, for $i \geq n$, $A_i \subset A'$. Then for $i \geq n$, $h \circ g_i$ is a *linear* map of I into R^m. Now by Lemma 2.7 $\psi(h \circ g_i) = h \circ g_i$ for $i \geq n$ and thus $h \circ g_i(\frac{1}{2}) = \psi(h \circ g_i)(\frac{1}{2})$. Thus $H_i(h) = \frac{1}{2}$ for all $i \geq n$ and the claim is proved.

Claim 2. The image of $H_{PL}(M)$ under the map H in $(0, 1)^\infty$ is homeomorphic to l_2^f.

Proof of Claim 2. Map $(0, 1)$ onto R by any convenient homeomorphism in such a way that the image of $\frac{1}{2}$ in $(0, 1)$ is 0 in R. Then use this homeomorphism to map $(0, 1)^\infty$ onto R^∞ and call this homeomorphism $F: (0, 1)^\infty \longrightarrow R^\infty$. Then the image of $H_{PL}(M)$ under $F \circ H$ is precisely the set of points $(x_i) \in R^\infty$ such that almost all x_i's are 0. By [1, Theorem 5.1] this implies that $F \circ H[H_{PL}(M)]$ is homeomorphic to l_2^f. Thus $H[H_{PL}(M)]$ is homeomorphic to l_2^f since F is a homeomorphism.

We now resume the proof of Theorem 2.1. Let

$$H^0_{PL}(M) = \{h \in H_{PL}(M): \psi(h \circ g_i)(\tfrac{1}{2}) = h \circ g_i(\tfrac{1}{2}) \text{ for } all\ i\}.$$

We will show that $H_{PL}(M)$ is homeomorphic to $H^0_{PL}(M) \times l_2^f$.

Claim 3. $H_{PL}(M)$ is homeomorphic to $H^0_{PL}(M) \times l_2^f$.

Proof of Claim 3. We define a continuous function $G: H_{PL}(M) \longrightarrow H^0_{PL}(M)$ in the following manner. Let $h \in H_{PL}(M)$ be given and let n be such that for $i \geq n$, $h \circ g_i(\frac{1}{2}) = \psi(h \circ g_i)(\frac{1}{2})$. Note that for $i \geq n$, $H_i(h) = \frac{1}{2}$ and thus that $h_{H_i(h),i}$ is the identity on M for $i \geq n$. Define $G(h) = h \circ h_{H_1(h),1} \circ \cdots \circ h_{H_n(h),n}$. Note that the definition of $G(h)$ does not depend on the choice of n. Note also that $G(h) \circ g_i = h \circ h_{H_i(h),i} \circ g_i$ for all i including $i > n$ since for $i > n$, $h_{H_i(h),i}$ is the identity on M. Now from the definition of $h_{H_i(h),i}$ we have that

$$G(h) \circ g_i(\tfrac{1}{2}) = h \circ h_{H_i(h),i} \circ g_i(\tfrac{1}{2}) = h \circ g_i(p(h \circ g_i)) = \psi(h \circ g_i)(\tfrac{1}{2}).$$

Now since ψ is a retraction of $E(I, M)$ into itself and $\psi(f \circ g) = \psi(f)$ for $f \in E(I, M)$ and $g \in H_0(I)$, it is clear that $\psi(h \circ g_i) = \psi(h \circ h_{H_i(h),i} \circ g_i) = \psi(G(h) \circ g_i)$. Thus $G(h) \circ g_i(\frac{1}{2}) = \psi(G(h) \circ g_i)(\frac{1}{2})$. Thus $G(H)$ is an element of $H^0_{PL}(M)$. We leave it to the reader to verify that G is continuous. Clearly G is a retraction.

To complete the proof of Claim 3 we will show that the map $P\colon H_{PL}(M) \to H^0_{PL}(M) \times (0, 1)^\infty$ defined by $P(h) = (G(h), H(h))$ is a homeomorphism onto its image and that its image is homeomorphic to $H^0_{PL}(M) \times l^f_2$. Call the image of $H_{PL}(M)$ under H, $Z \subset (0, 1)^\infty$. By Claim 2, Z is homeomorphic to l^f_2. We will now define a map Q from $H^0_{PL}(M) \times Z$ to $H_{PL}(M)$ such that Q is continuous with $P \circ Q$ the identity on $H^0_{PL}(M) \times Z$ and $Q \circ P$ the identity on $H_{PL}(M)$. Let $(h, (x_i)) \in H^0_{PL}(M) \times Z$. Then there is an n such that for $i \geq n$, $x_i = \frac{1}{2}$. Let

$$Q(h, (x_i)) = h \circ h^{-1}_{x_n, n} \circ \cdots \circ h^{-1}_{x_1, 1}.$$

Then $Q(h, (x_i)) \in H_{PL}(M)$. It is easily verified that $P \circ Q(h, (x_i)) = (h, (x_i))$. The continuity of Q is also left to the reader to verify. Now let $h \in H_{PL}(M)$ and consider $Q \circ P(h) = Q(G(h), H(h))$. Now $G(h) = h \circ h_{H_1(h),1} \circ \cdots \circ h_{H_n(h),n}$ where n is such that for $i \geq n$, $H_i(h) = \frac{1}{2}$. Also $H(h) = (H_i(h))$ and $H_i(h) = \frac{1}{2}$ for $i \geq n$. But by the definition of Q,

$$\begin{aligned} Q(G(h), H(h)) &= G(h) \circ h^{-1}_{H_n(h),n} \circ \cdots \circ h^{-1}_{H_1(h),1} \\ &= (h \circ h_{H_1(h),1} \circ \cdots \circ h_{H_n(h),n}) \circ h^{-1}_{H_n(h),n} \circ \cdots \circ h^{-1}_{H_1(h),1} = h. \end{aligned}$$

Thus $Q \circ P$ is the identity on $H_{PL}(M)$. Thus we have shown that $H_{PL}(M)$ is homeomorphic to $H^0_{PL}(M) \times Z$ and thus $H_{PL}(M) \times l^f_2$ as asserted. The proof of Claim 3 is now complete.

The proof of Theorem 2.1 now follows from the observation that $l^f_2 \times l^f_2$ is homeomorphic to l^f_2. Thus $H_{PL}(M) \times l^f_2$ is homeomorphic to $(H^0_{PL}(M) \times l^f_2) \times l^f_2$ which is homeomorphic to $H^0_{PL}(M) \times l^f_2$ which is homeomorphic to $H_{PL}(M)$. The proof of Theorem 2.1 is now complete.

We now have completely proved the main result of this paper.

2.8. **Theorem.** *Let M be a compact PL-manifold and $H_{PL}(M)$ be the group of PL-homeomorphisms of M onto itself. Then $H_{PL}(M)$ is an l^f_2-manifold.*

The following corollary follows from the proofs given above together with a standard result from infinite-dimensional topology.

2.9. **Corollary.** *Let σ^n be the n-simplex and let $H^0_{PL}(\sigma^n)$ be the PL-homeomorphisms of σ^n onto itself which are the identity on the boundary of σ^n. Then $H^0_{PL}(\sigma^n)$ is homeomorphic to l^f_2.*

Proof. The standard Alexander isotopy gives us a contraction of $H^0_{PL}(\sigma^n)$ in itself to the identity function e on σ^n. Since $H^0_{PL}(\sigma^n)$ is a contractible topological group, it is locally contractible. Since $H^0_{PL}(\sigma^n)$ is a closed subset of $H_{PL}(\sigma^n)$, $H^0_{PL}(\sigma^n)$ must be σ-f.d. compact. Since it is locally contractible, it must be an ANR by Theorem 1.3. The proof of Theorem 2.1 can be slightly modified to show that $H^0_{PL}(\sigma^n)$ is homeomorphic to $H^0_{PL}(\sigma^n) \times l^f_2$. Thus by Theorem 1.1, $H^0_{PL}(\sigma^n)$ is an l^f_2-manifold. Therefore $H^0_{PL}(\sigma^n)$ is a contractible l^f_2-manifold. A contractible l^f_2-manifold is homeomorphic to l^f_2 by [9] and the corollary is proved.

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