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CORRECTIONS AND ADDITIONS TO "ON THE DEGREES AND RATIONALITY OF CERTAIN CHARACTERS OF FINITE CHEVALLEY GROUPS"

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ABSTRACT. Some theorems in the Benson-Curtis paper [1] were stated subject to possible exceptions in type E_7, corresponding to the two irreducible characters of the Weyl group of degree 512. An argument due to T. A. Springer(1) shows that these cases actually are exceptions to the theorems, and also that there are four exceptional cases in type E_8 (whose possible existence was overlooked in the original version of the paper), corresponding to the characters of the Weyl group of degree 4096.

CORRECTIONS AND ADDITIONS.

p. 251, line 12 of abstract; p. 254, lines 3 and 4 of the statement of Theorem (2.8); p. 254, line 7 from bottom; p. 255, line 10 of the statement of Theorem (2.9); p. 270, line 3. For "with the possible exceptions of two irreducible characters of degree 512, in case (W, R) is of type E_7" (or the equivalent), read "with the definite exception of two irreducible characters of degree 512, in case (W, R) is of type E_7, and four irreducible characters of degree 4096, in case (W, R) is of type E_8" (or the equivalent).

p. 254, line 10 from bottom. Add: "In case (W, R) is of type E_8, the four characters of W of degree 4096 fall into two pairs of characters which cannot be distinguished by the methods of Theorem (2.7)".

p. 256, line 2. For "except possibly for W of type E_7" read "except for W of type E_7 or E_8".

p. 256, line 6. For "possible exception of E_7" read "exception of E_7 and E_8".

p. 270. After line 7, insert the following paragraphs:

The fact that the characters of degree 512, in type E_7, and 4096, in type E_8, actually are exceptions to Theorem (2.8), is proved by the following argument, due to T. A. Springer. Let (W, R) be a Coxeter system of type E_7 or

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E_8, and let w_0 be the element of W of maximal length. Let $\{a_{wf}\}_{w\in W}$ be the standard basis for $A_f \cong H(G(q), B(q))$, as in §2, for the specialization f: $u \longrightarrow q$. The fact that $w_0 r = r w_0$, for all $r \in R$, implies that

$$a_{w_0 f} a_{rf} = q a_{w_0 r, f} + (q-1) a_{w_0 f} = a_{rf}\, a_{w_0 f},$$

since $l(rw_0) < l(w_0)$ and $l(w_0 r) < l(w_0)$. Therefore $a_{w_0 f}$ belongs to the center of A_f. Let ρ be an irreducible representation of $A_f^{\bar{Q}}$. Write $w_0 = r_1 r_2 \cdots r_N$, $r_i \in R$, where $N = l(w_0)$. Then $\rho(a_{w_0 f}) = c.1$, for some $c \in \bar{Q}$, since $a_{w_0 f}$ is a central element in A_f, and $\bar{Q}$ is a splitting field. Taking determinants, we obtain

$$\det(\rho(a_{w_0 f})) = c^{\deg \rho} = \prod_{i=1}^{N} \det(\rho(a_{r_i f})).$$

Since $a_{r_i f}^2 = q1 + (q-1)a_{r_i f}$, $\rho(a_{r_i f})$ has eigenvalues q and -1. By the argument on pp. 102–103 of the article *Hecke algebras and characters of parabolic type of finite groups with (B, N)-pairs* (C. W. Curtis, N. Iwahori and R. Kilmoyer, Inst. Hautes Etudes Sci., Publ. Math. No. 40 (1972) the multiplicity with which q appears in $\rho(a_{r_i f})$ is the same as the multiplicity of 1 in $\rho_0(r_i)$, where ρ_0 is the representation of the Coxeter group W corresponding to ρ. This multiplicity is $(½)(\chi_0(1) + \chi_0(r))$, where χ_0 is the character of ρ_0, and $r_i = r$ is an arbitrary element of R (since all elements in R are conjugate in W in these cases). Therefore

$$c^{\deg \rho} = \pm q^{N(\chi_0(1) + \chi_0(r))/2}$$

In case (W, R) is of type E_7, and ρ has degree 512, $N = 63$, $\chi_0(r) = 0$, and $N(\chi_0(1) + \chi_0(r))/(2 \deg \rho) = 63/2$. In type E_8, if ρ has degree $4096 = 2^{12}$, $N = 120$, and $N(\chi_0(1) + \chi_0(r))/(2 \deg \rho) = (15/2)(8 \pm 1)$. It follows that $c \notin Q$ in these cases, and hence that the corresponding characters of the generic algebra are not rational.

The fact that the characters of G corresponding to the exceptional characters of degree 512, in type E_7, and 4096, in type E_8, are definite exceptions also to Theorem (2.9), is shown as follows. The characters of $A_f^{\bar{Q}} \cong H_{\bar{Q}}(G(q), B(q))$ are the restrictions to the Hecke algebra of the corresponding characters of $G(q)$. Therefore, the nonrationality of the characters of $A_f^{\bar{Q}}$, in the above cases, implies the nonrationality of the corresponding characters of $G(q)$.

REFERENCE

1. C. T. Benson and C. W. Curtis, *On the degrees and rationality of certain characters of finite Chevalley groups*, Trans. Amer. Math. Soc. **165** (1972), 251–273.

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