

ADDENDUM TO  
"BEHNKE-STEIN THEOREM FOR ANALYTIC SPACES"

BY

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**ABSTRACT.** A very simple argument shows that Theorem 3.1 in my paper *Behnke-Stein theorem for analytic spaces*, (these Transactions, 199 (1974), pp. 317-326) is enough, via a Narasimhan result, to obtain information about the torsion of the homology groups of a Runge pair of Stein spaces.

Let  $(X, Y)$ , with  $Y \subset X$ ,  $Y$  open, be a pair of reduced complex analytic spaces of (complex) dimension  $n$ . Andreotti and Narasimhan proved in [1], among many others, the following result:

(1.1) If  $(X, Y)$  is a Runge pair of Stein spaces (a 1-Runge pair in the terminology of [3]) and every singularity of  $X$  outside  $Y$  is isolated, then

$$H_r(X \text{ mod } Y, \mathbb{Z}) = 0$$

for  $r \geq n + 1$ .

We wish to show the following statements (1.2) and (1.3),

(1.2) (Narasimhan [2, Theorem 3]): if  $X$  is a Stein space, then:

$$H_r(X, \mathbb{Z}) = 0 \quad \text{for } r \geq n + 1$$

and

$H_n(X, \mathbb{Z})$  is without torsion;

(1.3) (Silva [3, Theorem 3.1]): if  $(X, Y)$  is a Runge pair of Stein spaces (or, equivalently, in the terminology of [3], a 1-Runge pair of cohomologically 1-complete spaces) then

$$H_{n+1}(X \text{ mod } Y, \mathbb{C}) = 0;$$

make us able to remove from (1.1) the assumption that the singularities of  $X$  outside  $Y$  are isolated.

Indeed, if we write the exact homology sequence for the pair  $(X, Y)$ :

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$$\cdots \rightarrow H_r(X, \mathbf{Z}) \rightarrow H_r(X \bmod Y, \mathbf{Z}) \xrightarrow{\varphi_r} H_{r-1}(Y, \mathbf{Z}) \rightarrow \cdots$$

from (1.2) we obtain that  $H_r(X \bmod Y, \mathbf{Z}) = 0$  for  $r > n + 1$ . Suppose now  $r = n + 1$ . (1.3) implies that  $H_{n+1}(X \bmod Y, \mathbf{Z})$  is a torsion group. If we look again at the exact homology sequence for  $(X, Y)$  we see that the morphism:

$$\varphi_{n+1}: H_{n+1}(X \bmod Y, \mathbf{Z}) \rightarrow H_n(Y, \mathbf{Z})$$

is injective, so that,  $H_n(Y, \mathbf{Z})$  being without torsion, we must have  $H_{n+1}(X \bmod Y, \mathbf{Z}) = 0$ .

In conclusion we have obtained the following

**THEOREM.** *If  $(X, Y)$  is a Runge pair of Stein spaces, then*

$$H_r(X \bmod Y, \mathbf{Z}) = 0,$$

for  $r \geq n + 1$ .

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