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# ERRATUM TO "GENERALIZED SUPER-SOLUTIONS OF PARABOLIC EQUATIONS"

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It has been brought to my attention that two terms were not defined correctly. The necessary changes will be made in Definitions 2, 3, and 6 and this will require a slight modification in the statement of Theorem 3. However, the content of Theorem 3 will not be changed.

DEFINITION 2. $u = u(x, t)$ is called a *super-solution* of $Lu = 0$ in $Q$ if $u \in L^2[0, T; H^{1,2}(\Omega)]$ and, for all $v \in C_0^1(Q^i)$ with $v \geqslant 0$,

$$\iint_Q \left[ a_{ij}u,_i v,_j + d_j v,_j u - b_j u,_j v - cuv - uv_t \right] dx\, dt \geqslant 0. \tag{7}$$

DEFINITION 3. Let $u \in L^2[0, T; H^{1,2}(\Omega)]$.

(a) $u$ is *nonnegative* on $\partial\Omega \times [0, T]$ if there is a sequence of functions $\{u^k(x, t)\} \subset C(\overline{Q})$ with $u^k$ Lipschitz continuous in $x$ on $\overline{\Omega}$ uniformly in $t$, $u^k > 0$ on $\partial\Omega \times [0, T]$, and $u^k \to u$ in $L^2[0, T; H^{1,2}(\Omega)]$.

(b) $u$ is *nonnegative* on $\Omega \times (0)$ if, for each function $v \in C_0(\Omega \times [0, T))$,

$$\liminf_{t \to 0} \int_\Omega u(x, t)v(x, t)\, dx \geqslant 0.$$

In particular, if $u$ has initial values $u_0(x) \in L^2(\Omega)$ on $\Omega \times (0)$, then $u$ is *nonnegative* on $\Omega \times (0)$ if $\int_\Omega u_0(x)v(x)\, dx \geqslant 0$ for all $v \in C_0(\Omega)$ with $v \geqslant 0$.

(c) $u$ is *nonnegative* on $\partial_p Q$ if $u$ is nonnegative on $\partial\Omega \times [0, T]$ and on $\Omega \times (0)$.

Then Theorem 3 should be restated as follows:

THEOREM 3 (MINIMUM PRINCIPLE). *Let $u$ be a super-solution of $Lu = 0$ in $Q$ and assume $u \geqslant 0$ on $\partial_p Q$ with initial values $u_0 \in L^2(\Omega)$. Then $u \geqslant 0$ a.e. in $Q$.*

The proof of Theorem 3 is as stated in the paper. We now define $\mathcal{S}_Q$.

DEFINITION 6. (i) $\mathcal{S}_Q = \{u$ lower semicontinuous super-solution on $Q\}$.

(ii) $\mathcal{S}_Q' = L_Q \cap \{u; u$ is SMV$\}$.

(iii) $\mathcal{S}_Q'' = L_Q \cap \{u;$ for each cylinder $W$, $\overline{W} \subset Q$, and $v \leqslant u$ on $\partial_p W$, $v \leqslant u$ on $W\}$.

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(iv) $\mathfrak{S}_Q''' = L_Q \cap \{u;\ u \text{ is LSMV}\}$.

It may be of interest to the reader to state the counterexample to Theorem 3 using Definition 3 in the original article. It was sent to me by Dr. Filippo Chiarenza of the University of Catania, Italy. Consider $Lu = u_t - u_{xx}$ in $Q = (-1, 1) \times (0, 1)$ and let $u(x, t) = x^2 + 2t - 1$. Then $Lu = 0$ in $Q$. Now let $\{u^k(x, t)\}$ be defined by

$$u^k(x, t) = \begin{cases} kt(x^2 + 2t - 1) + 1/k, & 0 \leqslant t < 1/k, \\ x^2 + 2t - 1 + 1/k, & 1/k \leqslant t \leqslant 1. \end{cases}$$

Then $u^k > 0$ on $\partial_p Q$, $u^k \in C(\bar{Q})$, and $u^k$ is lipschitz continuous in $x$ on $\Omega$ uniformly in $t$. Since $u^k \to u$ in $L^2[0, T; H^{1,2}(\Omega)]$ we should have $u \geqslant 0$ on $Q$. Since $u(x, t) = x^2 + 2t - 1 < 0$ in a neighborhood of $(0, 0)$, we have the desired counterexample.

## References

Neil A. Eklund, *Generalized super-solutions of parabolic equations*, Trans. Amer. Math. Soc. **220** (1976), 235–242.

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