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CORRECTION TO "ON THE FREE BOUNDARY OF A QUASI-VARIATIONAL INEQUALITY ARISING IN A PROBLEM OF QUALITY CONTROL"

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In the proof of Theorem 4.1 of [1] we stated that $\partial z/\partial y_l \geq 0$ on ∂C_δ; this is true if ∂C_δ lies in Ω_δ, but may possibly be false at points of $\partial C_\delta \cap \partial\Omega_\delta$. Thus the proof of Theorem 4.1 yields only the weaker result:

THEOREM 4.1$'$. *If* (4.1), (4.2) *hold then* $\bar{C}$ *cannot lie in the interior of* R_{n-1}^+, *that is,* $\bar{C}$ *must intersect* ∂R_{n-1}^+.

Stronger assertions about the continuation set C can be proved under additional restrictions on the $q_{i,j}$. Recall that we are dealing with the variational inequality for $w = u - \psi$:

$$w \leq 0, \qquad -Lw \leq \frac{1}{Y} \sum_{i=1}^{n} B_i y_i \qquad \left(Y = \sum_{i=1}^{n} y_i, y_1 \equiv 1\right),$$

$$w\left(-Lw - \frac{1}{Y} \sum_{i=1}^{n} B_i y_i\right) = 0 \tag{1}$$

in R_{n-1}^+ ($y = (y_2, \dots, y_n)$ belongs to R_{n-1}^+ if and only if $y_i \geq 0$ for $2 \leq i \leq n$) under the assumptions:

$$B_i > 0 \quad \text{if } 2 \leq i \leq n, \quad B_1 < 0 \tag{2}$$

(if $B_1 \geq 0$ then $C = \varnothing$), where

$$Lw = \frac{1}{2} \sum_{i,j=2}^{n} \mu_{ij} y_i y_j \frac{\partial^2 w}{\partial y_i \partial y_j} + \sum_{j=2}^{n} b_j \frac{\partial w}{\partial y_j} - \alpha w,$$

$$b_j = \frac{1}{Y} \sum_{i=2}^{n} \mu_{ij} y_i y_j + \sum_{i=1}^{n} (q_{i,j} - q_{i,1}) y_i,$$

$$\mu_{ij} = (\lambda_i - \lambda_1) \cdot (\lambda_j - \lambda_1).$$

We shall now assume that

$$\mu_{ij} = \mu_i \delta_{ij}, \qquad \mu_i > 0, \tag{3}$$

$$q_{l,j} = 0 \quad \text{if } l > j. \tag{4}$$

Then (see (4.5) of [1]), each $w_l = \partial z/\partial y_l$ ($z = Yw$) satisfies in C_δ an inhomogeneous elliptic equation to which the minimum principle applies, provided we already know that $w_{l+1} \geq 0, \dots, w_n \geq 0$. Thus, we can establish by induction that $w_n \geq 0$,

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$w_{n-1} > 0, \ldots, w_k > 0$ provided we can verify these inequalities on the boundary of Ω_δ.

To do this we specialize to

$$\Omega_\delta = \left\{ y; y_i > \delta \text{ for } 2 < i < n, \sum_{i=2}^{n} y_i = \frac{1}{\delta} \right\}.$$

Denote by $\partial_0\Omega_\delta$ the part of $\partial\Omega_\delta$ where $\sum y_i = \delta$, and by $\partial_1\Omega_\delta$ the remaining part of $\partial\Omega_\delta$. We chose the boundary values of z as follows:

$$z = 0 \quad \text{on } \partial_0\Omega_\delta,$$
$$z = \gamma\left(\sum_{j=2}^{n} y_j - \frac{1}{\delta}\right) \quad \text{on } \partial_1\Omega_\delta, \quad \gamma > 0. \tag{5}$$

Then

$$z < 0, \qquad \frac{\partial z}{\partial y_i} > 0, \qquad \frac{\partial^2 z}{\partial y_i^2} = 0 \quad \text{on } \partial_1\Omega_\delta \cap \{y_j = \delta\}, \quad j \neq i. \tag{6}$$

We choose γ sufficiently small so that $\gamma|q_{j,j}| < \frac{1}{2}B_j$ $(2 < j < n)$. Then

$$\sum_{i=1}^{n} \sum_{\substack{j=2 \\ j\neq l}}^{n} q_{i,j}y_i \frac{\partial z}{\partial y_j} + \frac{1}{2}\sum_{i=2}^{n} B_i y_i > 0 \quad \text{on } y_l = \delta. \tag{7}$$

Further, if δ is sufficiently small,

$$-\alpha z + \frac{1}{2}\sum_{i=2}^{n} B_i y_i > -B_1 \quad \text{on } \partial_1\Omega_\delta. \tag{8}$$

Since

$$\sum_{j=2}^{n} q_{i,j} = \sum_{j=1}^{n} q_{ij} = 0 \quad \text{if } i > 1, \quad \sum_{j=2}^{n} q_{1,j} > 0,$$

we have

$$-\frac{z}{Y}\sum_{i=1}^{n}\sum_{j=2}^{n} q_{i,j}y_i > 0. \tag{9}$$

We finally recall [1] that z satisfies

$$\frac{1}{2}\sum_{i=2}^{n} \mu_i \frac{\partial^2 z}{\partial y_i^2} + \sum_{i=1}^{n}\sum_{j=2}^{n} q_{i,j}y_i \frac{\partial z}{\partial y_j} - \frac{z}{Y}\sum_{i=1}^{n}\sum_{j=2}^{n} q_{i,j}y_i = \alpha z - \sum_{i=1}^{n} B_i y_i. \tag{10}$$

Assume now that

$$q_{i,n} = 0 \qquad (1 < i < n). \tag{11}$$

Since $z = 0$ on $\partial_0\Omega_\delta$, $z < 0$ in Ω_δ, we have $w_n = \partial z/\partial y_n > 0$ on $\partial_0\Omega_\delta$. On $y_i = \delta$, $i < n$, $w_n > 0$ by (6). Since w_n cannot take a negative minimum in C_δ, it follows that $w_n > 0$ in C_δ if we can show that w_n cannot take a negative minimum at any point $y^0 \in \overline{C}_\delta$ which lies in $y_n = \delta$. Suppose it does; then $w_n(y^0) < 0$ and

$$\partial^2 z/\partial y_n^2 > 0 \quad \text{at } y^0. \tag{12}$$

In view of (11) the coefficient of $\partial z/\partial y_n$ in (10) vanishes. Using also (12), (7) with

$l = n$, (6), (8), (9), we conclude that the left-hand side of (10) is larger than the right-hand side; a contradiction.

We have thus proved that $w_n > 0$ in C_δ. Similarly we can prove that if $q_{i,n-1} = 0$ $(1 < i < n)$ then $w_{n-1} > 0$, etc. We sum up:

THEOREM 1. *Assume that* (3), (4) *hold and that* $q_{i,m} = 0$ *if* $1 < i < n$, $k < m < n$. *Then for any* j, $k \leq j < n$, *there exists a function* $\Psi_j(y_2, \ldots, y_{j-1}, y_{j+1}, \ldots, y_n)$ *such that* $y = (y_2, \ldots, y_n)$ *belongs to* C *if and only if*

$$y_j < \Psi_j(y_2, \ldots, y_{j-1}, y_{j+1}, \ldots, y_n).$$

COROLLARY 2. *If* $q_{i,j} = 0$ $(1 < i, j < n)$ *then the assertions of Corollary* 4.2 *and Theorem* 5.1 *of* [**1**] *are valid.*

REFERENCES

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