

**CORRECTION TO "ON THE FREE BOUNDARY OF
 A QUASI-VARIATIONAL INEQUALITY ARISING IN
 A PROBLEM OF QUALITY CONTROL"**

BY

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In the proof of Theorem 4.1 of [1] we stated that $\partial z / \partial y_l > 0$ on ∂C_δ ; this is true if ∂C_δ lies in Ω_δ , but may possibly be false at points of $\partial C_\delta \cap \partial \Omega_\delta$. Thus the proof of Theorem 4.1 yields only the weaker result:

THEOREM 4.1'. *If (4.1), (4.2) hold then \bar{C} cannot lie in the interior of R_{n-1}^+ , that is, \bar{C} must intersect ∂R_{n-1}^+ .*

Stronger assertions about the continuation set C can be proved under additional restrictions on the $q_{i,j}$. Recall that we are dealing with the variational inequality for $w = u - \psi$:

$$w < 0, \quad -Lw < \frac{1}{Y} \sum_{i=1}^n B_i y_i \quad \left(Y = \sum_{i=1}^n y_i, y_1 \equiv 1 \right),$$

$$w \left(-Lw - \frac{1}{Y} \sum_{i=1}^n B_i y_i \right) = 0 \tag{1}$$

in R_{n-1}^+ ($y = (y_2, \dots, y_n)$ belongs to R_{n-1}^+ if and only if $y_i > 0$ for $2 \leq i \leq n$) under the assumptions:

$$B_i > 0 \quad \text{if } 2 \leq i \leq n, \quad B_1 < 0 \tag{2}$$

(if $B_1 > 0$ then $C = \emptyset$), where

$$Lw = \frac{1}{2} \sum_{i,j=2}^n \mu_{ij} y_i y_j \frac{\partial^2 w}{\partial y_i \partial y_j} + \sum_{j=2}^n b_j \frac{\partial w}{\partial y_j} - \alpha w,$$

$$b_j = \frac{1}{Y} \sum_{i=2}^n \mu_{ij} y_i y_j + \sum_{i=1}^n (q_{i,j} - q_{i,1}) y_i,$$

$$\mu_{ij} = (\lambda_i - \lambda_1) \cdot (\lambda_j - \lambda_1).$$

We shall now assume that

$$\mu_{ij} = \mu_i \delta_{ij}, \quad \mu_i > 0, \tag{3}$$

$$q_{l,j} = 0 \quad \text{if } l > j. \tag{4}$$

Then (see (4.5) of [1]), each $w_l = \partial z / \partial y_l$ ($z = Yw$) satisfies in C_δ an inhomogeneous elliptic equation to which the minimum principle applies, provided we already know that $w_{l+1} > 0, \dots, w_n > 0$. Thus, we can establish by induction that $w_n > 0$,

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$w_{n-1} > 0, \dots, w_k > 0$ provided we can verify these inequalities on the boundary of Ω_δ .

To do this we specialize to

$$\Omega_\delta = \left\{ y; y_i > \delta \text{ for } 2 < i < n, \sum_{i=2}^n y_i = \frac{1}{\delta} \right\}.$$

Denote by $\partial_0\Omega_\delta$ the part of $\partial\Omega_\delta$ where $\sum y_i = \delta$, and by $\partial_1\Omega_\delta$ the remaining part of $\partial\Omega_\delta$. We chose the boundary values of z as follows:

$$\begin{aligned} z &= 0 \quad \text{on } \partial_0\Omega_\delta, \\ z &= \gamma \left(\sum_{j=2}^n y_j - \frac{1}{\delta} \right) \quad \text{on } \partial_1\Omega_\delta, \quad \gamma > 0. \end{aligned} \quad (5)$$

Then

$$z < 0, \quad \frac{\partial z}{\partial y_i} > 0, \quad \frac{\partial^2 z}{\partial y_i^2} = 0 \quad \text{on } \partial_1\Omega_\delta \cap \{y_j = \delta\}, \quad j \neq i. \quad (6)$$

We choose γ sufficiently small so that $\gamma|q_{jj}| < \frac{1}{2}B_j$ ($2 < j < n$). Then

$$\sum_{i=1}^n \sum_{\substack{j=2 \\ j \neq i}}^n q_{ij} y_i \frac{\partial z}{\partial y_j} + \frac{1}{2} \sum_{i=2}^n B_i y_i > 0 \quad \text{on } y_i = \delta. \quad (7)$$

Further, if δ is sufficiently small,

$$-\alpha z + \frac{1}{2} \sum_{i=2}^n B_i y_i > -B_1 \quad \text{on } \partial_1\Omega_\delta. \quad (8)$$

Since

$$\sum_{j=2}^n q_{i,j} = \sum_{j=1}^n q_{ij} = 0 \quad \text{if } i > 1, \quad \sum_{j=2}^n q_{1,j} > 0,$$

we have

$$-\frac{z}{Y} \sum_{i=1}^n \sum_{j=2}^n q_{ij} y_i > 0. \quad (9)$$

We finally recall [1] that z satisfies

$$\frac{1}{2} \sum_{i=2}^n \mu_i \frac{\partial^2 z}{\partial y_i^2} + \sum_{i=1}^n \sum_{j=2}^n q_{ij} y_i \frac{\partial z}{\partial y_j} - \frac{z}{Y} \sum_{i=1}^n \sum_{j=2}^n q_{ij} y_i = \alpha z - \sum_{i=1}^n B_i y_i. \quad (10)$$

Assume now that

$$q_{i,n} = 0 \quad (1 < i < n). \quad (11)$$

Since $z = 0$ on $\partial_0\Omega_\delta$, $z < 0$ in Ω_δ , we have $w_n = \partial z / \partial y_n > 0$ on $\partial_0\Omega_\delta$. On $y_i = \delta$, $i < n$, $w_n > 0$ by (6). Since w_n cannot take a negative minimum in C_δ , it follows that $w_n > 0$ in C_δ if we can show that w_n cannot take a negative minimum at any point $y^0 \in \bar{C}_\delta$ which lies in $y_n = \delta$. Suppose it does; then $w_n(y^0) < 0$ and

$$\partial^2 z / \partial y_n^2 > 0 \quad \text{at } y^0. \quad (12)$$

In view of (11) the coefficient of $\partial z / \partial y_n$ in (10) vanishes. Using also (12), (7) with

$l = n$, (6), (8), (9), we conclude that the left-hand side of (10) is larger than the right-hand side; a contradiction.

We have thus proved that $w_n > 0$ in C_δ . Similarly we can prove that if $q_{i,n-1} = 0$ ($1 < i < n$) then $w_{n-1} > 0$, etc. We sum up:

THEOREM 1. *Assume that (3), (4) hold and that $q_{i,m} = 0$ if $1 < i < n$, $k < m < n$. Then for any j , $k < j < n$, there exists a function $\Psi_j(y_2, \dots, y_{j-1}, y_{j+1}, \dots, y_n)$ such that $y = (y_2, \dots, y_n)$ belongs to C if and only if*

$$y_j < \Psi_j(y_2, \dots, y_{j-1}, y_{j+1}, \dots, y_n).$$

COROLLARY 2. *If $q_{i,j} = 0$ ($1 < i, j < n$) then the assertions of Corollary 4.2 and Theorem 5.1 of [1] are valid.*

REFERENCES

1. A. Friedman, *On the free boundary of a quasi-variational inequality arising in a problem of quality control*, Trans. Amer. Math. Soc. **246** (1978), 95–110.

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