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NONEXISTENCE OF NONTRIVIAL $\square''$-HARMONIC 1-FORMS ON A COMPLETE FOLIATED RIEMANNIAN MANIFOLD

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ABSTRACT. We study the nonexistence of nontrivial $\square''$-harmonic 1-forms on a complete foliated riemannian manifold with positive definite Ricci curvature. It is well known that the harmonic 1-form on a compact and orientable riemannian manifold with positive definite Ricci curvature is trivial. Our main theorem is an extension of this fact in the complete foliated riemannian case.

Introduction. B. L. Reinhart [4] showed that on a compact foliated manifold M with " bundle-like" metric, the cohomology of basic differential forms is isomorphic to the harmonic space of a certain semidefinite Laplacian. I. Vaisman [5] defined the second connection closely related to the foliated structure, and showed that there are no nontrivial foliated harmonic 1-forms on M with positive definite Ricci curvature of the second connection. In this note we shall discuss the square-integrable basic harmonic 1-forms in the complete case and obtain a similar result.

1. Definitions. Let M be an n-dimensional C^∞-manifold which, topologically, is a connected, orientable, paracompact, Hausdorff space. We shall assume that a foliation E of codimension q is given on M, and we may find about each point a coordinate neighbourhood with coordinates $(x^1, \ldots, x^p, y^1, \ldots, y^q)$ $(n = p + q)$ such that

(i) $|x^i| \leqslant 1, |y^\alpha| \leqslant 1$.

(ii) The integral manifolds of E are given locally by $y^1 = c^1, \ldots, y^q = c^q$ for constants c^α satisfying $|c^\alpha| \leqslant 1$. (Here and hereafter, Latin indices run from 1 to p and Greek indices from 1 to q.)

Such a coordinate neighbourhood will be called flat, while each of the slices given by a set of equations $y^\alpha = c^\alpha$ will be called a plaque.

We may assume that there exist in a flat neighbourhood U differential forms w^i and vectors v_α such that

(i) $\{\partial/\partial x^i\}$ forms the base for the space of cross-sections of E in U at each point.

(ii) $\{w^1, \ldots, w^p, dy^1, \ldots, dy^q\}$ and $\{\partial/\partial x^1, \ldots, \partial/\partial x^p, v_1, \ldots, v_q\}$ are dual bases for the cotangent and tangent spaces at each point of U respectively. Hence, $w^i = dx^i + \sum a^i_\alpha\, dy^\alpha$ and $v_\alpha = \partial/\partial y^\alpha + \sum b^i_\alpha \partial/\partial x^i$.

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Throughout this note, all local expressions for differential forms and vectors will be taken with respect to those bases.

2. Square-integrable basic cohomology spaces. On a foliated manifold we may have the decomposition of differential forms into components in the following way. Any C^∞-m-form ϕ may be expressed locally as

$$\sum_{\substack{i_1<\cdots<i_r\\ \alpha_1<\cdots<\alpha_s}} \sum_{r+s=m} \phi_{i_1\cdots i_r\alpha_1\cdots\alpha_s}(x,y)w^{i_1}\wedge\cdots\wedge w^{i_r}\wedge dy^{\alpha_1}\wedge\cdots\wedge dy^{\alpha_s}.$$

We may define $\Pi_{r,s}\phi$ to be the sum of all these terms with a fixed r and s. Since under a change of flat coordinate systems, $\{\{dy^\alpha\}\}$ goes into $\{\{dy^{*\alpha}\}\}$ and $\{\{w^i\}\}$ into $\{\{w^{*i}\}\}$, the operator $\Pi_{r,s}$ is independent of the choice of coordinate system. Here by $\{\{\cdot\}\}$ we mean the vector space generated by the set $\{\cdot\}$. $\Pi_{r,s}\phi$ is called the component of type (r, s) of ϕ. The type decomposition of forms induces a type decomposition of the exterior derivative d by the rule $(\Pi_{t,u}d)\phi = \sum_{r,s}\Pi_{r+t,s+u}d\Pi_{r,s}\phi$. Letting $\Pi_{1,0}\,d = d'$, $\Pi_{0,1}\,d = d''$ and $\Pi_{-1,2}\,d = d'''$, we have $d = d' + d'' + d'''$.

PROPOSITION 2.1 (CF. [4]). *If ϕ is of type $(0, s)$, then $d\phi = d'\phi + d''\phi$. Moreover, $d'\phi = 0$ if and only if ϕ depends only upon y, in the sense that locally*

$$\phi = \sum \phi_{\alpha_1\cdots\alpha_s}(y)\,dy^{\alpha_1}\wedge\cdots\wedge dy^{\alpha_s}.$$

DEFINITION 2.1. A form of type $(0, s)$ which is annihilated by d' will be called a basic form.

DEFINITION 2.2. A riemannian metric $(\ ,\ )$ is bundle-like if it is representable in each flat neighbourhood U by an expression of the form

$$(\ ,\ )|_U = \sum g_{ij}(x,y)w^i\cdot w^j + \sum g_{\alpha\beta}(y)\,dy^\alpha\cdot dy^\beta.$$

Hereafter, we assume that the riemannian metric on M is bundle-like and all leaves are compact.

Let $\Lambda^{0,s}(M)$ be the space of all C^∞-basic forms of type $(0, s)$ and $\Lambda^{0,s}_\circ(M)$ the subspace of $\Lambda^{0,s}(M)$ composed of forms with compact support. Restricted to $\Lambda^{0,*}(M) = \sum_{s=0}^\infty \Lambda^{0,s}(M)$, $d''^2 = d^2 = 0$, so we may consider the cohomology of $\Lambda^{0,*}(M)$ and d''. (This is called the base-like cohomology by B. L. Reinhart [4].)

B. L. Reinhart [4] introduces the $*''$-operation on $\Lambda^{0,s}(M)$, and defined by

$$*''\phi = \sum_{\substack{\alpha_1<\cdots<\alpha_s\\ \beta_1<\cdots<\beta_{q-s}}} \operatorname{sgn}\begin{pmatrix} 1 & \cdots\cdots\cdots & q \\ \alpha_1,\ldots,\alpha_s, & \beta_1,\ldots, & \beta_{q-s}\end{pmatrix}(\det(g_{\alpha\beta}))^{1/2}$$
$$\cdot g^{\alpha_1\nu_1}\cdots g^{\alpha_s\nu_s}\phi_{\nu_1\cdots\nu_s}\,dy^{\beta_1}\wedge\cdots\wedge dy^{\beta_{q-s}}.$$

According to B. L. Reinhart [4], we may define a riemannian metric on $\Lambda^{0,s}(M)$ by

$$\langle\phi,\psi\rangle = \phi\wedge *''\psi\wedge dx^1\wedge\cdots\wedge dx^p,$$

and obtain a pre-Hilbertian metric on $\Lambda_\circ^{0,s}(M)$ by

$$\langle\langle \phi, \psi\rangle\rangle = \int_M \langle \phi, \psi\rangle = \int_M \phi \wedge *''\psi \wedge dx^1 \wedge \cdots \wedge dx^p.$$

The differential operator d'' maps $\Lambda^{0,s}(M)$ into $\Lambda^{0,s+1}(M)$. We define δ'': $\Lambda^{0,s}(M) \to \Lambda^{0,s-1}(M)$ by

$$\delta''\phi = (-1)^{qs+q+1} *'' d'' *'' \phi.$$

Then we have

$$\langle\langle d''\phi, \psi\rangle\rangle = \langle\langle \phi, \delta''\psi\rangle\rangle$$

for $\phi \in \Lambda_\circ^{0,s}(M)$, $\psi \in \Lambda_\circ^{0,s+1}(M)$.

Let $L_2^{0,s}(M)$ be the completion of $\Lambda_\circ^{0,s}(M)$ with respect to the inner product $\langle\langle\ ,\ \rangle\rangle$. We will denote by ∂ the restriction of d'' to $\Lambda_\circ^{0,s}(M)$ and by θ the restriction of δ'' to $\lambda_\circ^{0,s}(M)$. Define $\bar\partial = (\theta)^*$ and $\bar\theta = (\partial)^*$ where $(\ )^*$ denotes the adjoint operator of $(\ )$ with respect to the inner product $\langle\langle\ ,\ \rangle\rangle$. Then $\bar\partial$ (resp. $\bar\theta$) is a closed, densely defined operator of $L_2^{0,s}(M)$ into $L_2^{0,s+1}(M)$ (resp. $L_2^{0,s-1}(M)$). Let $D_{\bar\partial}^{0,s}$ (resp. $D_{\bar\theta}^{0,s}$) be the domain of the operator $\bar\partial$ (resp. $\bar\theta$) in $L_2^{0,s}(M)$. We put

$$Z_{\bar\partial}^{0,s}(M) = \{\phi \in D_{\bar\partial}^{0,s} \mid \bar\partial\phi = 0\} \quad\text{and}\quad Z_{\bar\theta}^{0,s}(M) = \{\phi \in D_{\bar\theta}^{0,s} \mid \bar\theta\phi = 0\}$$

which are closed in $L_2^{0,s}(M)$. Let $B_{\bar\partial}^{0,s}(M)$ and $B_{\bar\theta}^{0,s}(M)$ be the closure of $\bar\partial(D_{\bar\partial}^{0,s-1})$ and $\bar\theta(D_{\bar\theta}^{0,s+1})$ respectively.

DEFINITION 2.3. $H_2^{0,s}(M) = Z_{\bar\partial}^{0,s}(M) \ominus B_{\bar\partial}^{0,s}(M)$ is the square-integrable basic cohomology space, where $\ominus$ denotes the orthogonal complement of $B_{\bar\partial}^{0,s}(M)$.

THEOREM 2.1 (CF. [1]) (THE ORTHOGONAL DECOMPOSITION THEOREM).

$$L_2^{0,s}(M) = H_2^{0,s}(M) \oplus B_{\bar\partial}^{0,s}(M) \oplus B_{\bar\theta}^{0,s}(M).$$

DEFINITION 2.4. The Laplacian acting on $\Lambda^{0,*}(M)$ is defined by $\square'' = d''\delta'' + \delta''d''$.

PROPOSITION 2.2 (CF. [1]). *Let the bundle-like metric on M be complete and all leaves be compact. If $\phi \in L_2^{0,s}(M) \cap \Lambda^{0,s}(M)$ such that $\square''\phi = 0$, then $d''\phi = 0$ and $\delta''\phi = 0$.*

THEOREM 2.2 (CF. [1]). *Let the bundle-like metric on M be complete and all leaves be compact. If $\phi \in L_2^{0,s}(M) \cap \Lambda^{0,s}(M)$ such that $\square''\phi = 0$, then $\phi \in H_2^{0,s}(M)$.*

3. The second connection. According to I. Vaisman (cf. [2], [5]) we define the second connection D on M induced from the bundle-like metric $(\ ,\ )$ as follows.

$$D_{\partial/\partial x^i}\partial/\partial x^j = \Gamma_{ij}^k \partial/\partial x^k, \qquad D_{v_\alpha}\partial/\partial x^j = \Gamma_{\alpha j}^k \partial/\partial x^k,$$

$$D_{\partial/\partial x^i} v_\beta = 0, \qquad D_{v_\alpha} v_\beta = \Gamma_{\alpha\beta}^\gamma v_\gamma, \tag{3.1.1}$$

$$(\partial/\partial x^i)(\partial/\partial x^j, \partial/\partial x^k) = (D_{\partial/\partial x^i}\partial/\partial x^j, \partial/\partial x^k) + (\partial/\partial x^j, D_{\partial/\partial x^i}\partial/\partial x^k), \tag{3.1.2}$$

$$v_\alpha(v_\beta, v_\gamma) = (D_{v_\alpha} v_\beta, v_\gamma) + (v_\beta, D_{v_\alpha} v_\gamma),$$
$$T(\partial/\partial x^i, \partial/\partial x^j) = 0, \qquad T(\partial/\partial x^i, v_\beta) = 0,$$
$$T(v_\alpha, \partial/\partial x^j) = 0, \qquad T(v_\alpha, v_\beta) = T^k_{\alpha\beta}\partial/\partial x^k, \tag{3.1.3}$$

where T denotes the torsion tensor of D; that is, for any vector fields X, Y on M, $T(X, Y) = D_X Y - D_Y X - [X, Y]$. Note that the torsion T of D does not always vanish. Then we get

$$\Gamma^k_{ij} = \Gamma^k_{ji} = \tfrac{1}{2} g^{hk}(\partial g_{hj}/\partial x^i + \partial g_{ih}/\partial x^j - \partial g_{ij}/\partial x^h), \tag{3.2.1}$$

$$\Gamma^\gamma_{\alpha\beta} = \Gamma^\gamma_{\beta\alpha} = \tfrac{1}{2} g^{\tau\gamma}(v_\alpha(g_{\tau\beta}) + v_\beta(g_{\alpha\tau}) - v_\tau(g_{\alpha\beta})), \tag{3.2.2}$$

$$\Gamma^k_{\alpha j} = -\partial b^k_\alpha/\partial x^j, \tag{3.2.3}$$

$$\partial\Gamma^\gamma_{\alpha\beta}/\partial x^i = 0. \tag{3.2.4}$$

REMARK 1. If the transversal (or normal) plane field $E^\perp$ to E with respect to (,) is integrable, then the second connection coincides with the Levi-Civita connection induced from (,) (cf. [5]).

REMARK 2. Γ^k_{ij} and $\Gamma^\gamma_{\alpha\beta}$ coincide with the coefficients of the Levi-Civita connection (cf. [5]).

We express the operators d'', δ'' and $\Box''$ in terms of the second connection D. For any $\phi = \sum \phi_{\alpha_1 \cdots \alpha_s}(y)\, dy^{\alpha_1} \wedge \cdots \wedge dy^{\alpha_s} \in \Lambda^{0,s}(M)$,

$$(d''\phi)_{\beta_1 \cdots \beta_{s+1}} = \sum_{\nu=1}^{s+1} (-1)^{\nu-1} D_{\beta_\nu}\phi_{\beta_1 \cdots \hat{\beta}_\nu \cdots \beta_{s+1}}, \tag{3.3.1}$$

$$(\delta''\phi)_{\beta_1 \cdots \beta_{s-1}} = -D^\gamma \phi_{\gamma\beta_1 \cdots \beta_{s-1}}, \tag{3.3.2}$$

$$\begin{aligned}(\Box''\phi)_{\alpha_1 \cdots \alpha_s} = {} & -D^\alpha D_\alpha \phi_{\alpha_1 \cdots \alpha_s} + \sum_{k=1}^{s} (-1)^k R^{\gamma\cdot\alpha\cdot}_{\cdot\alpha\cdot\alpha_k}\phi_{\gamma\alpha_1 \cdots \hat{\alpha}_k \cdots \alpha_s} \\ & + 2\sum_{h<k} (-1)^{h+k} R^{\gamma\cdot\alpha\cdot}_{\cdot\alpha_h\cdot\alpha_k}\phi_{\alpha\gamma\alpha_1 \cdots \hat{\alpha}_h \cdots \hat{\alpha}_k \cdots \alpha_s},\end{aligned} \tag{3.3.3}$$

where R denotes the curvature tensor of D; that is,

$$R(X_A, X_B)X_C = D_{X_A} D_{X_B} X_C - D_{X_B} D_{X_A} X_C - D_{[X_A, X_B]} X_C = R^F_{CAB} X_F$$

for $X_A = \partial/\partial x^i$ or v_α.

4. $\Box''$-harmonic 1-forms. A differentiable curve C: $[0, 1] \to M$ is said to be tranversal if $\dot{C}(t)$ is in the transversal plane field $E^\perp$ for all t, where $\dot{C}(t)$ denotes the differential with respect to the parameter t. Let C be a transversal curve in M. Then, taking its local expression $C(t) = (C^i(t), C^\alpha(t))$, we have

$$\begin{aligned}\dot{C}(t) &= \dot{C}^i(t)\partial/\partial x^i + \dot{C}^\alpha(t)\partial/\partial y^\alpha \\ &= (\dot{C}^i(t) - b^i_\alpha \dot{C}^\alpha(t))\partial/\partial x^i + \dot{C}^\alpha(t) v_\alpha \\ &= \dot{C}^\alpha(t) v_\alpha \qquad \text{(by the transversality of } C(t)).\end{aligned}$$

A transversal curve C is called a geodesic if

$$D_{\dot{C}(t)}\dot{C}(t) = \left(\frac{d^2C^\alpha(t)}{dt^2} + \Gamma^\alpha_{\beta\gamma}\frac{dC^\beta(t)}{dt}\frac{dC^\gamma(t)}{dt}\right)v_\alpha = 0.$$

By a routine calculation, we may show that a transversal geodesic with respect to the second connection coincides with one with respect to the Levi-Civita connection. Hence a geodesic transversal to a leaf is transversal to all leaves (cf. [3]).

We fix a point o in M, and for each point p in M, we denote by $\rho(p)$ the distance between leaves through o and p.

We consider a differentiable function μ on $\mathbf{R}$ (the reals) satisfying

(i) $0 \leqslant \mu \leqslant 1$ on $\mathbf{R}$,

(ii) $\mu(t) = 1$ for $t \leqslant 1$,

(iii) $\mu(t) = 0$ for $t \geqslant 2$.

Then we set

$$w_k(p) = \mu(\rho(p)/k) \quad \text{for } k = 1, 2, 3, \dots.$$

LEMMA 4.1 (CF. [1]). *Under the above notations, there exists a number A depending only on μ, such that*

(i) $\|d''w_k \wedge \phi\|^2 \leqslant qA^2\|\phi\|^2/k^2$,

(ii) $\|d''w_k \wedge *''\phi\|^2 \leqslant qA^2\|\phi\|^2/k^2$, *for all* $\phi \in \Lambda^{0,s}_o(M)$, *where* $\|\phi\|^2 = \langle\langle\phi, \phi\rangle\rangle$.

For any $\phi \in L_2^{0,1}(M) \cap \Lambda^{0,1}(M)$, we have

$$(d''\phi, d''\psi)_{B(k)} + (\delta''\phi, \delta''\psi)_{B(k)} = (\square''\phi, \psi)_{B(k)} \tag{4.1}$$

for all $\psi \in \Lambda^{0,1}_{B(k)}(M)$, where $\Lambda^{0,1}_{B(k)}(M)$ is the space of all forms of type (0, 1) with compact support in $B(k)$ and $B(k)$ is an open tube of radius k of the leaf through the fixed point o in M. For $\psi = w_k^2\phi$, we have

$$d''\psi = w_k^2 d''\phi + 2w_k d''w_k \wedge \phi, \tag{4.2.1}$$

$$\delta''\psi = w_k^2\delta''\phi - *''(2w_k d''w_k \wedge *''\phi). \tag{4.2.2}$$

We consider the 1-form Φ of type (0, 1) defined by

$$\Phi = (D_\alpha\phi_\beta)\phi^\beta\, dy^\alpha = g^{\beta\gamma}(D_\alpha\phi_\beta)\phi_\gamma\, dy^\alpha$$

for $\phi \in L_2^{0,1}(M) \cap \Lambda^{0,1}(M)$. Since $w_k^2\Phi$ is compactly supported in $B(2k)$, the Stokes formula gives the equality

$$\int_M *''\delta''(w_k^2\Phi) \wedge dx^1 \wedge \cdots \wedge dx^p = 0. \tag{4.3}$$

In fact, $*''(w_k^2\Phi) \wedge dx^1 \wedge \cdots \wedge dx^p$ being a form of type $(p, q-1)$ with compact support in $B(2k)$,

$$\int_M d\left(*''(w_k^2\Phi) \wedge dx^1 \wedge \cdots \wedge dx^p\right) = 0.$$

And

$$\begin{aligned} d\left(*''\left(w_k^2\Phi\right) \wedge dx^1 \wedge \cdots \wedge dx^p\right) &= (d' + d'' + d''')\left(*''\left(w_k^2\Phi\right) \wedge dx^1 \wedge \cdots \wedge dx^p\right) \\ &= d''\left(*''\left(w_k^2\Phi\right) \wedge dx^1 \wedge \cdots \wedge dx^p\right) \\ &= *''\left(*''d'' *''\left(w_k^2\Phi\right) \wedge dx^1 \wedge \cdots \wedge dx^p\right) \\ &= -*''\delta''\left(w_k^2\Phi\right) \wedge dx^1 \wedge \cdots \wedge dx^p. \end{aligned}$$

By (3.3.2) and (4.2.2), (4.3) becomes the equality

$$\langle\langle 2w_k d''w_k \wedge \phi, D\phi\rangle\rangle_{B(2k)} + \langle\langle w_k D^2\phi, w_k\phi\rangle\rangle_{B(2k)} + \langle\langle w_k D\phi, w_k D\phi\rangle\rangle_{B(2k)} = 0,$$

where $(D^2\phi)_\beta = D^\alpha D_\alpha \phi_\beta$. And (3.3.3) gives the equality

$$\langle\langle \Box''\phi, w_k^2\phi\rangle\rangle_{B(2k)} = -\langle\langle w_k D^2\phi, w_k\phi\rangle\rangle_{B(2k)} + \langle\langle w_k \mathfrak{R}\phi, w_k\phi\rangle\rangle_{B(2k)}, \tag{4.4}$$

where $\mathfrak{R}$ is the symmetric linear transformation on 1-forms defined by $(\mathfrak{R}\phi)_\beta = -R_{\cdot\alpha\cdot\beta}^{\gamma\cdot\alpha\cdot}\phi_\gamma$.

On the other hand, the Schwartz inequality and Lemma 4.1 give the following.

$$\begin{aligned} \left|\langle\langle 2w_k d''w_k \wedge \phi, D\phi\rangle\rangle_{B(2k)}\right| &\leqslant \frac{2q^{1/2}A}{k}\|w_k D\phi\|_{B(2k)}\|\phi\|_{B(2k)} \\ &\leqslant \frac{q^{1/2}A}{k}\left(\|w_k D\phi\|^2_{B(2k)} + \|\phi\|^2_{B(2k)}\right), \end{aligned}$$

and

$$\left|\langle\langle \Box''\phi, w_k^2\phi\rangle\rangle_{B(2k)}\right| \leqslant \frac{1}{2}\left(\frac{1}{\sigma}\|w_k\phi\|^2_{B(2k)} + \sigma\|w_k\Box''\phi\|^2_{B(2k)}\right)$$

for every $\sigma > 0$.

Then we have

$$\begin{aligned} \sigma\|w_k\Box''\phi\|^2_{B(2k)} + \frac{1}{\sigma}\|w_k\phi\|^2_{B(2k)} \geqslant{}& 2\langle\langle w_k\mathfrak{R}\phi, w_k\phi\rangle\rangle_{B(2k)} - \frac{2q^{1/2}A}{k}\|\phi\|^2_{B(2k)} \\ &+ 2(1 - q^{1/2}A/k)\|w_k D\phi\|^2_{B(2k)}. \end{aligned} \tag{4.5}$$

In particular, setting $\Box''\phi = 0$ and letting $\sigma \to \infty$, we have

$$\begin{aligned} 0 \geqslant{}& 2\langle\langle w_k\mathfrak{R}\phi, w_k\phi\rangle\rangle_{B(2k)} - 2q^{1/2}A/k\|\phi\|^2_{B(2k)} \\ &+ 2(1 - q^{1/2}A/k)\|w_k D\phi\|^2_{B(2k)}. \end{aligned}$$

Letting $k \to \infty$, we have

$$0 \geqslant \limsup_{k\to\infty}\langle\langle w_k\mathfrak{R}\phi, w_k\phi\rangle\rangle_{B(2k)} + \|D\phi\|^2.$$

Suppose that the minimal eigenvalue λ of $\mathfrak{R}$ is nonnegative. Then there exists a constant $K \geqslant 0$ satisfying

$$\limsup_{k\to\infty}\langle\langle w_k\mathfrak{R}\phi, w_k\phi\rangle\rangle_{B(2k)} \geqslant K\|\phi\|^2.$$

In fact, there exists a constant $K \geqslant 0$ such that $\lambda \geqslant K$. Hence we have $0 \geqslant \|D\phi\|^2 + K\|\phi\|^2$.

DEFINITION 4.1. A basic form ϕ of type $(0, s)$ is $\square''$-harmonic if $\square''\phi = 0$.

DEFINITION 4.2. A basic 1-form ϕ of type $(0, 1)$ is D-parallel if $D\phi = 0$.

Therefore we have

MAIN THEOREM. *Let the bundle-like metric on M be complete and all leaves be compact. If the minimal eigenvalue λ of the Ricci tensor $-R^{\gamma\cdot\alpha\cdot}_{\cdot\alpha\cdot\beta}$ is positive and bounded away from zero, then there are no nontrivial basic $\square''$-harmonic 1-forms in $L_2^{0,1}(M)$. Moreover, if λ is zero, then the basic $\square''$-harmonic 1-form of type $(0, 1)$ is D-parallel.*

REMARK. "There are no nontrivial basic $\square''$-harmonic 1-forms in $L_2^{0,1}(M)$" means that we consider the operator $\square''$ in $L_2^{0,1}(M)$ only and we do not consider the extension $\overline{\square}''$ to $L_2^{0,1}(M)$.

If we consider the operator $\square''$ in $\Lambda^{0,1}(M)$, we may have the following statement. Let the bundle-like metric on M be complete. If the minimal eigenvalue λ of the Ricci tensor $-R^{\gamma\cdot\alpha\cdot}_{\cdot\alpha\cdot\beta}$ is positive and bounded away from zero, then there are no nontrivial global square-integrable basic $\square''$-harmonic forms ϕ of type $(0, 1)$ such that $\|d''\phi\| < \infty$ and $\|\delta''\phi\| < \infty$.

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