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THE INTEGRABILITY TENSOR FOR BUNDLE-LIKE FOLIATIONS

BY

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ABSTRACT. A certain function is introduced which is useful in the study of a bundle-like foliation on a Riemannian manifold. Under the assumption that the leaves are totally geodesic, the Laplacian of this function is computed along a leaf. From this computation a sufficient condition is provided for the ambient manifold to be locally isometric to a product.

The principal result of this paper, Theorem 4.1, provides sufficient conditions for a compact foliated manifold which admits a bundle-like metric to be locally isometric to a product. The idea is to consider a function, f, defined on all of the ambient manifold and use the fact, that, under certain conditions, f is subharmonic when regarded as a function along a leaf. In §1 we recall the basic definitions needed in the remaining sections. §2 contains a technical result, and §3 gives the formula for the Laplacian of f along a leaf.

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1. The fundamental tensor. Let M be a C^∞ differentiable manifold, which throughout this paper is assumed to be connected and complete. Assume M has a codimension q foliation which is denoted by $\mathcal{V}$. Then this foliation may be defined by a maximal family of C^∞ submersions $f_\alpha\colon U_\alpha \to R^q$ where $\{U_\alpha\}_{\alpha\in\Lambda}$ is an open cover of M, and where for each $p \in U_\alpha \cap U_\beta$ there is a local C^∞ diffeomorphism, $\phi^p_{\beta\alpha}$, of R^q so that $f_\beta = \phi^p_{\beta\alpha} \circ f_\alpha$ in some neighborhood U_p of p. U_p may be chosen so that it is in $U_\alpha \cap U_\beta$. In fact, if $p' \in U_\alpha \cap U_\beta$, $\phi^p_{\beta\alpha} = \phi^{p'}_{\beta\alpha}$ on $f(U_p \cap U_{p'})$ and $\phi^p_{\beta\alpha} = \phi^p_{\beta\gamma} \circ \phi^p_{\gamma\alpha}$ whenever this equation makes sense (see [**6**, pp. 2–3]). Observe that a tangent vector V belongs to the tangent space of the distribution at p, $\mathcal{V}_p$, if and only if $f_{\alpha*p}V = 0$ or $V \in \ker f_{\alpha*p}$.

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Now fix a Riemannian metric $\langle\ ,\ \rangle$ on M and let ∇ denote its Levi-Civita connection. Then the metric $\langle\ ,\ \rangle$ determines an orthogonal distribution to $\mathcal{V}$ which we denote by $\mathcal{H}$. Following [**8**] we define two tensors T and A on M as follows:

$$T_E F = \mathcal{V}\,\nabla_{\mathcal{V}E}\,\mathcal{H}F + \mathcal{H}\,\nabla_{\mathcal{V}E}\mathcal{V}F, \tag{1}$$

$$A_E F = \mathcal{V}\,\nabla_{\mathcal{H}E}\,\mathcal{H}F + \mathcal{H}\,\nabla_{\mathcal{H}E}\mathcal{V}F. \tag{2}$$

Here E and F are arbitrary tangent vectors to M and $\mathcal{V}E$, $\mathcal{H}E$, etc., are the projections onto the distributions $\mathcal{V}$ and $\mathcal{H}$.

The tensor T is of type $(1, 2)$ and enjoys the following properties [**8**, p. 460].

(a) T_E is vertical; that is, $T_E = T_{\mathcal{V}E}$.

(b) At each point, T_E is a skew-symmetric linear operator on M reversing the horizontal and vertical subspaces.

(c) For vertical vector fields V and W (i.e. vector fields tangent to the foliation $\mathcal{V}$), $T_V W = T_W V$.

Likewise A enjoys the following properties:

(a′) A_E is horizontal; $A_E = A_{\mathcal{H}E}$.

(b′) At each point, A_E is a skew-symmetric linear operator reversing the horizontal and vertical subspaces.

DEFINITION 1.1. Let M be foliated as above and suppose $\langle\ ,\ \rangle$ is a Riemannian metric on M. $\langle\ ,\ \rangle$ is called *bundle-like* if and only if for each α, f_α: $U_\alpha \to R^q$ is a Riemannian submersion onto its image, or equivalently, the metric $\langle\ ,\ \rangle$ on U_α is projectible onto its image $f_\alpha(U_\alpha)$ in R^q for each α.

Notice, if $\langle\ ,\ \rangle$ is bundle-like, the local diffeomorphisms $\phi^{\beta}_{\beta\alpha}$ of R^q are isometries with respect to the projected metrics. In general the metric projected onto $f_\alpha(U_\alpha)$ does not coincide with the flat metric.

REMARK 1.2. The properties (a), (b), (c), (a′) and (b′) of the tensors T and A obtain whether or not the metric $\langle\ ,\ \rangle$ is bundle-like on M. (c) obtains because T is essentially the second fundamental form for the leaf. The reader familiar with [**8**] knows that the tensor A satisfies a third property called the *alternating property* on horizontal vectors:

(c′) $(A_X Y)_p = -(A_Y X)_p$ for all X, Y in $\mathcal{H}_p$ and for all $p \in M$. This property follows from the fact that $\langle\ ,\ \rangle$ in [**8**] is bundle-like on M. When $\langle\ ,\ \rangle$ is bundle-like on M, A is called the *integrability tensor* [**2**], since as O'Neill has shown [**8**, p. 461], $A_X Y = \frac{1}{2}\mathcal{V}[X, Y]$ for any horizontal X, Y. The distribution $\mathcal{H}$ is integrable if and only if $A \equiv 0$ on M (see [**2**, Corollary 1.5's proof]).

REMARK 1.3. A characterization of bundle-like metrics in terms of (c′) is given in [**4**, Lemma 1.2]. This result, communicated to me orally, was forgotten and rediscovered a year later. The Abstracts announcement [**3**, Theorem 1] did not take this conversation into account.

Now let us restrict ourselves to U_α and consider the submersions f_α: $U_\alpha \to R^q$ which define the foliation $\mathcal{V}$. We say a horizontal vector field X on U is f_α *basic* provided $f_{\alpha * p}X = f_{\alpha * p'}X$ for every p and p' in a connected component or *plaque* of $U_\alpha \cap L$ where L is any leaf of $\mathcal{V}$. If p and p' also lie on a plaque of $U_\beta \cap L$ with neighborhoods U_p and $U_{p'}$ of p and p', respectively, contained in $U_\alpha \cap U_\beta$ and with $\phi^p_{\beta\alpha}$ and $\phi^{p'}_{\beta\alpha}$ defined on $f_\alpha(U_p)$ and $f_\alpha(U_{p'})$, respectively, then

$$f_{\beta * p}X = \phi^p_{\beta\alpha *} f_{\alpha * p} X = \phi^{p'}_{\beta\alpha *} f_{\alpha * p'} X = f_{\beta * p'} X$$

whenever p and p' both lie in $U_p \cap U_{p'}$.

If p and p' are not contained in $U_p \cap U_{p'}$, we can choose a path in the plaque $U_\alpha \cap U_\beta \cap L$ connecting p to p' and can select for each x on that path an open $U_x \subset U_\alpha \cap U_\beta$ so $\phi^x_{\beta\alpha}$ is defined on $f_\alpha(U_x)$. By compactness of the path, every open cover has a finite subcover, so select one such finite subcover $\{U_{x_i}\}_{1 \leqslant i \leqslant n}$. If ε is the Lebesgue number of the subcover, then choose $\{p_k\}_{0 \leqslant k \leqslant m}$ with $p_0 = p$, $p_m = p'$ so $d(p_i, p_{i+1}) < \varepsilon$, where d is the distance function on the leaf induced from the metric on $\mathcal{V}$. Then

$$\phi^{p_i}_{\beta\alpha *} f_{\alpha * p_i} X = \phi^{p_{i+1}}_{\beta\alpha *} f_{\alpha * p_{i+1}} X,$$

so it follows that

$$\begin{aligned} f_{\beta * p} X = f_{\beta * p_0} X &= \phi^{p_0}_{\beta\alpha *} f_{\alpha * p_0} X = \phi^{p_1}_{\beta\alpha *} f_{\alpha * p_1} X \\ &= \cdots = \phi^{p_m}_{\beta\alpha *} f_{\alpha * p_m} X = \phi^{p'}_{\beta\alpha *} f_{\alpha * p'} X = f_{\beta * p'} X. \end{aligned}$$

We conclude X is f_β basic on $U_\alpha \cap U_\beta$ since we worked on an arbitrary L. Notice that we have not made any assumption that the metric $\langle\ ,\ \rangle$ is bundle-like. We have established the following result.

Proposition 1.4. *A horizontal vector field on $U_\alpha \cap U_\beta$ is f_α basic if and only if it is f_β basic.*

2. In this section we assume that $T \equiv 0$, that is, that the leaves of the foliation $\mathcal{V}$ are totally geodesic submanifolds of M. In addition, we will assume that the metric $\langle\ ,\ \rangle$ is bundle-like on M. We will show that if X and Y are f_α basic then $A_X Y = \frac{1}{2}\mathcal{V}[X, Y]$ satisfies the equation for a Killing vector field when restricted to a plaque of $U_\alpha \cap L$: that is, we show

$$\langle \nabla_V(A_X Y), W\rangle + \langle V, \nabla_W(A_X Y)\rangle = 0 \tag{1}$$

for all V, W tangent to the leaf on the given plaque. This result is more or less known [**1**], but our proof uses the equations of [8].

To begin, note

$$\begin{aligned} \langle \nabla_V(A_X, Y), W\rangle &= \langle (\nabla_V A)_X Y, W\rangle + \langle A_{\nabla_V X} Y, W\rangle + \langle A_X \nabla_V Y, W\rangle \\ &= \langle (\nabla_V A)_X Y, W\rangle + \langle A_{A_X V} Y, W\rangle + \langle A_X A_Y V, W\rangle. \end{aligned} \tag{2}$$

The second equality uses the definition of A, the fact that V and W are vertical, and that $\mathcal{H}\nabla_V X = A_X V$ for basic vector fields. Since A is alternating on horizontal vector fields and $A_{\mathcal{H}E}$ is skew-symmetric, we get

$$\langle \nabla_V(A_X Y), W\rangle = \langle (\nabla_V A)_X Y, W\rangle + \langle A_X V, A_Y W\rangle - \langle A_Y V, A_X W\rangle. \tag{3}$$

In a similar way one sees

$$\langle \nabla_W(A_X Y), V\rangle = \langle (\nabla_W A)_X Y, V\rangle + \langle A_X W, A_Y V\rangle - \langle A_Y W, A_X V\rangle. \tag{4}$$

Adding the last two expressions we get

$$\langle \nabla_V(A_XY), W\rangle + \langle \nabla_W(A_XY), V\rangle = \langle (\nabla_V A)_X Y, W\rangle + \langle (\nabla_W A)_X Y, V\rangle. \tag{5}$$

Thus, to show A_XY satisfies Killing's equation on a plaque, it suffices to show

$$\langle (\nabla_V A)_X Y, W\rangle + \langle (\nabla_W A)_X Y, V\rangle = 0. \tag{6}$$

By one of the equations of [**8**],

$$\langle R_{XV}Y, W\rangle = \langle (\nabla_V A)_X Y, W\rangle + \langle A_X V, A_Y W\rangle, \tag{7}$$

since $T \equiv 0$. Using a Bianchi identity and (7) we get

$$\langle R_{XV}Y, W\rangle = \langle R_{YW}X, V\rangle = \langle (\nabla_W A)_Y X, V\rangle + \langle A_Y W, A_X V\rangle, \tag{8}$$

or

$$\langle R_{WY}X, V\rangle = \langle (\nabla_W A)_X Y, V\rangle - \langle A_Y W, A_X V\rangle, \tag{9}$$

since $\mathcal{V}(\nabla_W A)_X Y = -\mathcal{V}(\nabla_W A)_Y X$ by [**8**, p. 462]. Adding (7) and (9) and using a Bianchi identity, we obtain

$$0 = \langle (\nabla_V A)_X Y, W\rangle + \langle (\nabla_W A)_X Y, V\rangle. \tag{10}$$

By (6) of this section A_XY is Killing on each plaque of $U_\alpha \cap L$. By Proposition 1.4 of the first section X and Y are f_β basic on $U_\alpha \cap U_\beta$, so we have the following result.

PROPOSITION 2.1. *If X and Y are basic on U_α then A_XY is a Killing vector field on each plaque of $U_\alpha \cap L$ and each plaque of $U_\alpha \cap U_\beta \cap L$.*

3. The function f. Throughout this and the next section the following conventions are observed:

(a) $1 \leqslant i, j, k, l \leqslant q$, where q is the codimension of the foliation X_i, X_j, etc., are assumed horizontal.

(b) $1 \leqslant r \leqslant n - q$ where n is the dimension of M. V_r always denotes a vertical vector field.

(c) Whenever $\Sigma_{i,r}$ is used, we allow the indices to run through all of the permitted values.

In this section we study the function

$$f(p) = \sum_{i,j} \langle A_{X_i}X_j, A_{X_i}X_j\rangle, \tag{1}$$

where $\{X_i\}_{1\leqslant i\leqslant q}$ is an orthonormal frame at p for the horizontal space $\mathcal{H}_p$, and where, as before, A denotes the integrability tensor. The same assumptions made in §2 obtain here: $\langle\ ,\ \rangle$ is bundle-like and $T \equiv 0$. We note that our definition of $f(p)$ does not depend on the choice of frame at p. If $\{Y_k\}$ is related to the frame $\{X_i\}$ by an orthogonal transformation of $\mathcal{H}_p$, then

$$\sum_{k,l} \langle A_{Y_k}Y_l, A_{Y_k}Y_l\rangle = \sum_{i,j} \langle A_{X_i}X_j, A_{X_i}X_j\rangle.$$

We omit this routine computation.

Our goal is to study the Laplacian of f on a leaf, $\Delta_{\text{leaf}} f$, and to try to compute it in some reasonable way. Let $\{V_r\}_{1\leqslant r\leqslant n-q}$ be a family of vertical vector fields orthonormal at p satisfying $(\nabla_{V_r} V_r)(p) = 0$. Then by Spivak [**10**, p. 194], for instance,

$$(\Delta_{\text{leaf}} f)(p) = \Big(\sum_r V_r V_r f\Big)(p).$$

Likewise, we choose a family of horizontal f_α basic vector fields $\{X_k\}$, orthonormal at and near $p \in M$, which are $f_{\alpha *}$ related to vector fields $\{X_k^*\}$ on $f_\alpha(U_\alpha)$ and where $1 \leqslant k \leqslant q$ in both cases. We have

$$V_r f = \sum_{i,j} V_r \langle A_{X_i} X_j, A_{X_i} X_j \rangle = \sum_{i,j} 2 \langle \nabla_{V_r}(A_{X_i} X_j), A_{X_i} X_j \rangle \tag{2}$$

and

$$\begin{aligned} V_r^2 f &= V_r V_r f \\ &= \sum_{i,j} \{ 2\langle \nabla_{V_r} \nabla_{V_r} A_{X_i} X_j, A_{X_i} X_j \rangle + 2 \langle \nabla_{V_r}(A_{X_i} X_j), \nabla_{V_r}(A_{X_i} X_j) \rangle \} \\ &= \sum_{i,j} \Big\{ 2 \langle \nabla_{V_r}(A_{X_i} X_j), \nabla_{V_r}(A_{X_i} X_j) \rangle - 2 \Big\langle R_{V_r, A_{X_i} X_j} V_r, A_{X_i} X_j \Big\rangle \Big\}. \end{aligned} \tag{3}$$

In the last equality R is the curvature tensor of ∇. The equality obtains since $A_{X_i} X_j$ satisfies Killing's equations on each plaque of $U_\alpha \cap L$ and so the formulas of [**5**, p. 56] apply. (Note in [**5**], Kobayashi considers $\frac{1}{2}\langle W, W \rangle$, where W is a Killing vector field, and his curvature tensor differs from the above one by a sign.) It follows from the above computations that if we regard f as a function along a single leaf, then the Laplacian of f on the leaf, $\Delta_{\text{leaf}} f$, has the following expression at the point p:

PROPOSITION 3.1. *Under the assumptions of this section the Laplacian of f at p, $(\Delta_{\text{leaf}} f)(p)$, has the following expression with respect to the adapted system at p:*

$$\begin{aligned} (\Delta_{\text{leaf}} f)(p) = &\sum_{i,j,r} 2 \langle \nabla_{V_r}(A_{X_i} X_j), \nabla_{V_r}(A_{X_i} X_j) \rangle \\ &- \sum_{i,j} 2\, S_{\text{leaf}}(A_{X_i} X_j, A_{X_i} X_j), \end{aligned}$$

where S_{leaf} is the Ricci tensor of the leaf.

PROOF. The result follows immediately from the definition of the Ricci tensor.

REMARK 3.2. A nice formula for $(\Delta f)(p)$ where Δf denotes the Laplacian of f on M can be computed. Results of this kind will be discussed elsewhere.

4. A geometric application. We are now in a position to state our main theorem as an application of the work of §§2 and 3. As before, $\mathcal{H} = \mathcal{V}^\perp \subset TM$.

THEOREM 4.1. *Let M be a compact connected Riemannian manifold with foliation $\mathcal{V}$. Assume the metric on M is bundle-like with respect to $\mathcal{V}$. If the leaves of $\mathcal{V}$ are totally geodesic and have quasi-negative Ricci curvature, then locally M is isometric to a product of plaques of the foliations $\mathcal{V}$ and $\mathcal{H}$.*

PROOF.[1] We follow the reasoning in [**11**]. Since M is compact, $f(x) = \Sigma_{i,j}\langle A_{X_i}X_j, A_{X_i}X_j\rangle$ attains a maximum at some $p \in M$ and so f is bounded on each leaf of $\mathcal{V}$. Suppose L is the leaf on which f attains its maximum. Since $S_{\text{leaf}}(W, W) \leqslant 0$ for all vertical W, we have by Proposition 3.1 that $\Delta_{\text{leaf}} f \geqslant 0$, so f is subharmonic on L. Recall the formula given in 3.1:

$$(\Delta_{\text{leaf}} f)(x) = 2\left\{ \sum_{i,j,r} \langle \nabla_{V_r} A_{X_i}X_j, \nabla_{V_r} A_{X_i}X_j\rangle - \sum_{i,j} S_{\text{leaf}}\left(A_{X_i}X_j, A_{X_i}X_j\right)\right\}.$$

The assumption that f attains its maximum on L means f equals some constant c on L. If $c > 0$, then f is nowhere zero on L. By assumption $S_{\text{leaf}}(W, W) < 0$ at some point q on L for all nonzero W. On the other hand $\Delta_{\text{leaf}} f \equiv 0$ on L since f is constant. We have a contradiction unless $A_{X_i}X_j = 0$ for all i, j at q so $f(q) = 0$. Since f is constant on L, $f(p) = 0$. But p was the point, where f attained its maximum on M. We conclude $f \equiv 0$ on M. If L is not orientable take the oriented double cover of L and apply the same argument extending f and $\Delta_{\text{leaf}} f$ in the obvious way.

Since $f \equiv 0$ on M, this means $A_{X_i}X_j = 0$, or A_E annihilates $\mathcal{H}$. Since A_E is skew-symmetric reversing $\mathcal{H}$ and $\mathcal{V}$, it follows A_E annihilates $\mathcal{V}$ so $A_E \equiv 0$ for all E tangent to M. Now $\langle\ ,\ \rangle$ on M is bundle-like so by Remark 2, §1, $A_X Y = \mathcal{V}\frac{1}{2}[X, Y] = 0$ for any horizontal X, Y. This means $\mathcal{H}$ is integrable. By Proposition 2 of [**9**], a geodesic horizontal at one point is everywhere horizontal. It follows that the leaves of $\mathcal{H}$ are totally geodesic in M. By assumption, the leaves of $\mathcal{V}$ are also totally geodesic.

To conclude the proof we note that since $\mathcal{V}$ and $\mathcal{H}$ are both integrable, locally M is diffeomorphic to the product of a connected open set of a leaf of $\mathcal{V}$ and a connected open set of a leaf of $\mathcal{H}$. Call these open sets F and J, respectively. We may choose F and J so that $F \times J \subset U_\alpha$ where U_α is one of the open sets of §1. Recalling that f_α: $U_\alpha \to f_\alpha(U_\alpha)$, we note that for each $z \in F$, f_α, when restricted to $z \times J$, is bijective onto its image. Restricting f_α to $F \times J$ we have a Riemannian submersion onto its image with totally geodesic fibers and complementary distribution integrable ($T \equiv 0$ and $A \equiv 0$). These two conditions are precisely the conditions for the total space of a Riemannian submersion which has the structure of a fibered space to be a local Riemannian product by 6.4 and 6.5 of [7]. Since $F \times J$ is already diffeomorphic to a product we conclude it is isometric to a product. Hence, M is locally isometric to $F \times J$ or M is a local Riemannian product.

EXAMPLE 4.2. Consider R^3 with the standard axes. Choose a family of parallel planes each of which makes an angle α (α irrational) with the $(x, y, 0)$ plane so that the plane passing through $(0, 0, 0)$ contains the x axis. This construction gives a foliation of R^3 by the family of parallel planes. Now say $(x, y, z) \sim (x', y', z')$ if and only if $x - x'$, $y - y'$ and $z - z'$ are all integers. The relation $\sim$ induces an equivalence on R^3. We observe that (x, y, z) and (x', y', z') both in the same plane of the foliation are equivalent if and only if $y = y'$, $z = z'$ and $x - x'$ is an integer.

[1] We can relax the compactness condition on M and obtain the same conclusions, provided we keep the other hypotheses and, in addition, require that each leaf of $\mathcal{V}$ be compact.

Put the following metric on R^3. The leaves of the foliation (the planes which cut the xy plane *and* which are either parallel to the x axis or contain it) are equipped with the hyperbolic metric of constant curvature -1. Any line orthogonal to the plane of the foliation is given the usual Euclidean metric. Thus, we get a product metric on R^3. $\sim$ preserves the metric so $R^3 \to R^3/\sim = T^3$ is an isometric covering. The images of the planes in R^3 described above are dense leaves in T^3, so we have a codimension one foliation of T^3. Take $M = S^1 \times T^3$ where S^1 has the usual metric and T^3 has the metric given above. Each point $p \in M$ still passes through one and only one two-dimensional leaf. Obviously, the distribution complementary to the leaves is integrable and both distributions are totally geodesic since they arise from a local product structure [**4**, Proposition 1.3].

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