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# CORRECTION TO "CLOSED 3-MANIFOLDS WITH NO PERIODIC MAPS"

BY

FRANK RAYMOND AND JEFFREY L. TOLLEFSON

In [3] we considered a family of 3-manifolds $M_\Phi$ which were defined as mapping tori of specially constructed surface homeomorphisms $\Phi$. Contrary to our claim there, these 3-manifolds in fact admit involutions, as was pointed out to us by M. Sakuma. However, by introducing a slight change in the family of maps $\Phi$ we are able to recover all the results claimed in [3]. With the change in the maps $\Phi$ described below we correct the difficulty in [3], which occurs in Lemma 1, and are easily able to adapt the remaining arguments to the new maps $\Phi$, thus obtaining the desired examples of closed, orientable, aspherical 3-manifolds with no periodic maps.

Recall from [3] that $F$ denotes a closed orientable surface of genus $g$ ($g \geqslant 3$) situated in $R^3$ as shown in Figure 1 and that $\{a_i, b_i \mid 1 \leqslant i \leqslant g\}$ is a set of simple closed curves as shown.

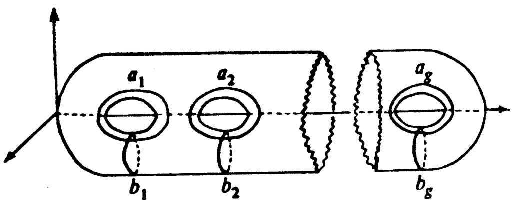


FIGURE 1

Fix a set of $g - 1$ arbitrary but distinct integers $\{n_2, \ldots, n_g\}$ such that each $n_i \neq 19$ and $n_i > 2$. We redefine the homeomorphisms $\Phi$: $F \to F$ by setting

$$\Phi = t(b_1)^{-1} t(a_1)^2 t(b_1)^{-3} t(a_1) \prod_{i=2}^{g} t(a_i) t(b_i)^{-n_i+1},$$

where the $t(a_i)$, $t(b_i)$ denote twist maps about the simple closed curves $a_i$, $b_i$, respectively. (The alteration of $\Phi$ occurs in the twists about the first pair of curves $a_1$, $b_1$.) The $2g \times 2g$ matrix $P$ corresponding to the induced automorphism $\Phi_*$ of $H_1(F)$ is

$$P = \begin{pmatrix} X(1) & 0 & \cdots & 0 \\ 0 & X(n_2) & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & X(n_g) \end{pmatrix}$$

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shown by blocks of $2 \times 2$ matrices, where

$$X(1) = \begin{pmatrix} 7 & 9 \\ 10 & 13 \end{pmatrix} \quad \text{and} \quad X(n) = \begin{pmatrix} n & 1 \\ n-1 & 1 \end{pmatrix} \quad \text{for } n > 2.$$

This matrix $P$ fulfills all the expectations of §3 in [3], principally Lemma 7 and Theorem B. Although Lemma 1 is satisfied only by $X(1)$, the remaining lemmas hold for all $X(n)$ as can easily be checked by following the arguments given there.

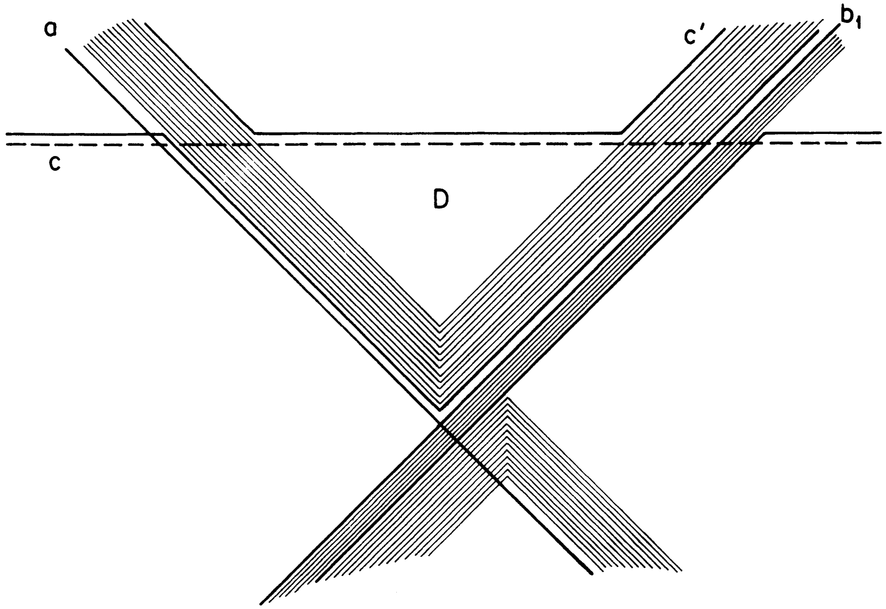


FIGURE 3(c)

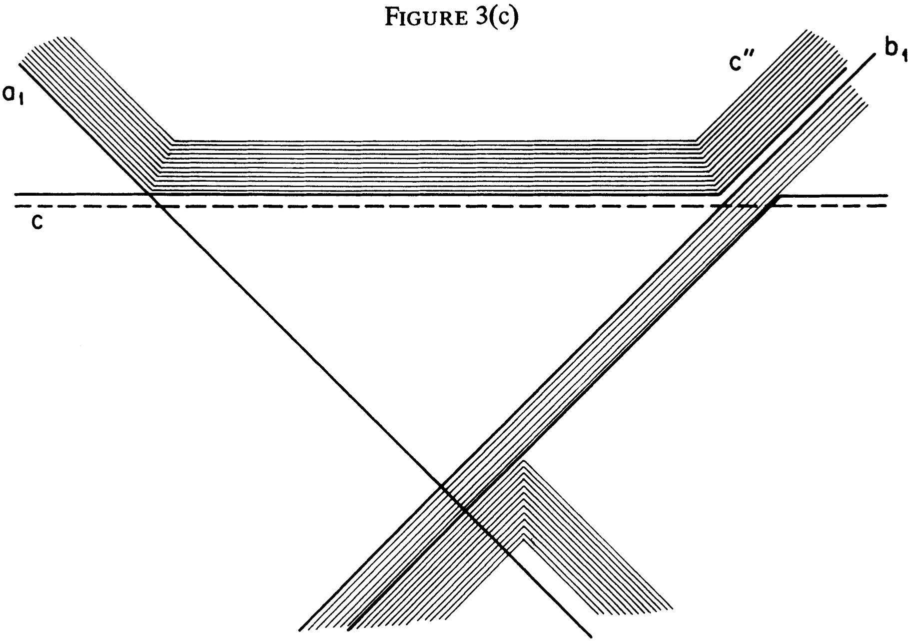


FIGURE 3(d)

(Lemma 6 now needs the assumption $\eta_i \neq 19$.) We indicate the proof of Lemma 1 for $X(1)$ below.

§4 of [3] is unaffected by the above change in $\Phi$. Finally, the arguments in §5 are also unaffected once Lemma 10 has been verified for the new $\Phi$. But the essential character of the new $\Phi$ closely resembles that of the original and, as a consequence, the original proof of Lemma 10 can be repeated verbatim for the new $\Phi$. For completeness, we give two additional figures which should be included with Figures 3 and 5 of [3] and referred to during the course of the proof of Lemma 10.

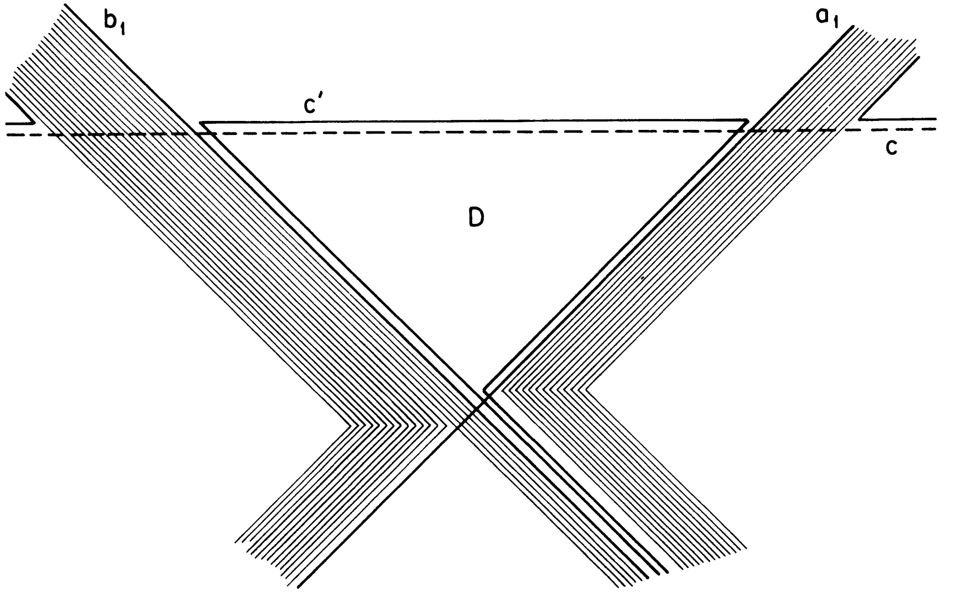


FIGURE 5(b)

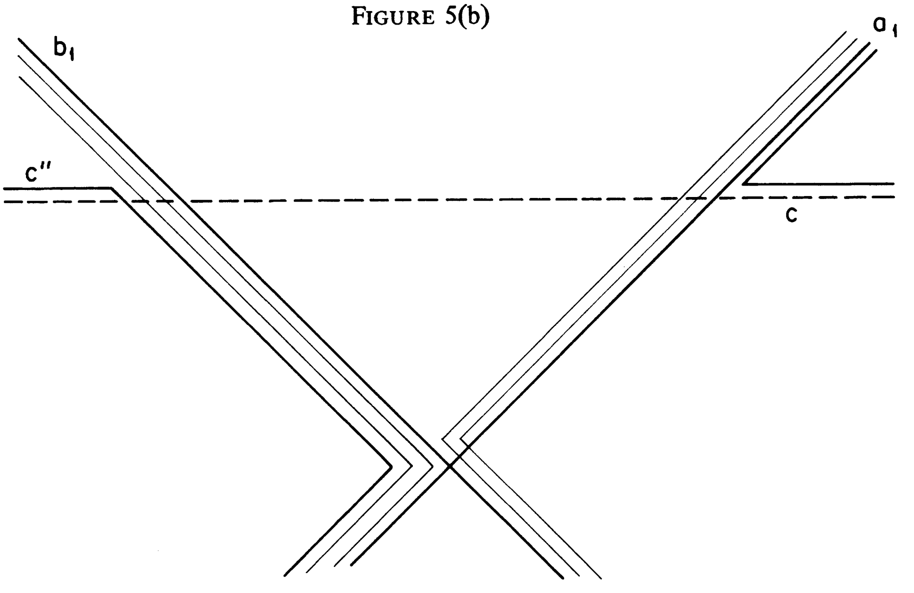


FIGURE 5(c)

All that remains then is Lemma 1 for the matrix $X(1)$, which simply asserts that $X(1)$ is not conjugate to its inverse in $GL(2, Z)$. For this, we follow Sakuma [4] and use an argument of Latimer and MacDuffee [2].

LEMMA 1. *There does not exist a matrix* $B \in GL(2, Z)$ *for which* $X(1)B = BX(1)^{-1}$.

PROOF. Observe that the vectors $\binom{3}{1+\sqrt{11}}$ and $\binom{3}{1-\sqrt{11}}$ are eigenvectors of $X(1)$ and $X(1)^{-1}$, respectively, corresponding to the eigenvalue $10 + 3\sqrt{11}$. Suppose there does exist a matrix $B = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ such that $BX(1)^{-1}B^{-1} = X(1)$. Then

$$B \cdot \begin{pmatrix} 3 \\ 1-\sqrt{11} \end{pmatrix} = \begin{pmatrix} 3a + \left(1-\sqrt{11}\right)b \\ 3c + \left(1-\sqrt{11}\right)d \end{pmatrix}$$

is also an eigenvector of $X(1)$ and thus is a complex multiple of $\binom{3}{1+\sqrt{11}}$. It follows that

$$\frac{3c + \left(1-\sqrt{11}\right)d}{3a + \left(1-\sqrt{11}\right)b} = \frac{c + d\left(\dfrac{1-\sqrt{11}}{3}\right)}{a + b\left(\dfrac{1-\sqrt{11}}{3}\right)} = \frac{1+\sqrt{11}}{3}.$$

Hence $(1 + \sqrt{11})/3$ and $(1 - \sqrt{11})/3$ are equivalent (in the sense of [1]) and their expressions as simple continued fractions are the same after a finite number of terms. Since the irrational numbers in question are quadratic roots, their expressions as simple continued fractions are eventually periodic.

One easily computes that

$$\frac{1+\sqrt{11}}{3} = [\underbrace{\dot{1},2,3,\dot{1}},1,2\ldots] = 1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \ldots}}}}}$$

and

$$\frac{1-\sqrt{11}}{3} = [-1,4,\underbrace{\dot{2},1,1,\dot{3}},2\ldots].$$

Since the cycles are distinct, up to a cyclic permutation, $(1 + \sqrt{11})/3$ cannot be equivalent to $(1 - \sqrt{11})/3$, which is a contradiction. Therefore no such $B$ exists.

## REFERENCES

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MICHIGAN, ANN ARBOR, MICHIGAN 48109

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CONNECTICUT, STORRS, CONNECTICUT 06268