CORRECTION TO "CLOSED 3-MANIFOLDS WITH NO PERIODIC MAPS"

BY

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In [3] we considered a family of 3-manifolds $M_g$ which were defined as mapping tori of specially constructed surface homeomorphisms $\Phi$. Contrary to our claim there, these 3-manifolds in fact admit involutions, as was pointed out to us by M. Sakuma. However, by introducing a slight change in the family of maps $\Phi$ we are able to recover all the results claimed in [3]. With the change in the maps $\Phi$ described below we correct the difficulty in [3], which occurs in Lemma 1, and are easily able to adapt the remaining arguments to the new maps $\Phi$, thus obtaining the desired examples of closed, orientable, aspherical 3-manifolds with no periodic maps.

Recall from [3] that $F$ denotes a closed orientable surface of genus $g$ ($g \geq 3$) situated in $\mathbb{R}^3$ as shown in Figure 1 and that $\{a_i, b_i | 1 \leq i \leq g\}$ is a set of simple closed curves as shown.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Figure 1}
\end{figure}

Fix a set of $g - 1$ arbitrary but distinct integers $\{n_2, \ldots, n_g\}$ such that each $n_i \neq 19$ and $n_i > 2$. We redefine the homeomorphisms $\Phi: F \to F$ by setting

$$\Phi = t(b_1)^{-1}t(a_1)^2t(b_1)^{-3}t(a_1)^{g} \prod_{i=2}^{g} t(a_i)t(b_i)^{-n_i+1},$$

where the $t(a_i)$, $t(b_i)$ denote twist maps about the simple closed curves $a_i$, $b_i$, respectively. (The alteration of $\Phi$ occurs in the twists about the first pair of curves $a_1$, $b_1$.) The $2g \times 2g$ matrix $P$ corresponding to the induced automorphism $\Phi_*$ of $H_1(F)$ is

$$P = \begin{pmatrix}
X(1) & 0 & \cdots & 0 \\
0 & X(n_2) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & X(n_g)
\end{pmatrix}$$

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shown by blocks of $2 \times 2$ matrices, where

$$X(1) = \begin{pmatrix} 7 & 9 \\ 10 & 13 \end{pmatrix} \quad \text{and} \quad X(n) = \begin{pmatrix} n & 1 \\ n - 1 & 1 \end{pmatrix} \quad \text{for} \quad n > 2.$$  

This matrix $P$ fulfills all the expectations of §3 in [3], principally Lemma 7 and Theorem B. Although Lemma 1 is satisfied only by $X(1)$, the remaining lemmas hold for all $X(n)$ as can easily be checked by following the arguments given there.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3c}
\caption{FIGURE 3(c)}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3d}
\caption{FIGURE 3(d)}
\end{figure}
(Lemma 6 now needs the assumption $\eta_i \neq 19$.) We indicate the proof of Lemma 1 for $X(1)$ below.

§4 of [3] is unaffected by the above change in $\Phi$. Finally, the arguments in §5 are also unaffected once Lemma 10 has been verified for the new $\Phi$. But the essential character of the new $\Phi$ closely resembles that of the original and, as a consequence, the original proof of Lemma 10 can be repeated verbatim for the new $\Phi$. For completeness, we give two additional figures which should be included with Figures 3 and 5 of [3] and referred to during the course of the proof of Lemma 10.

![Figure 5(b)](image.png)

![Figure 5(c)](image.png)
All that remains then is Lemma 1 for the matrix \( X(1) \), which simply asserts that \( X(1) \) is not conjugate to its inverse in \( GL(2, \mathbb{Z}) \). For this, we follow Sakuma [4] and use an argument of Latimer and MacDuffee [2].

**Lemma 1.** There does not exist a matrix \( B \in GL(2, \mathbb{Z}) \) for which \( X(1)B = BX(1)^{-1} \).

**Proof.** Observe that the vectors \( (1 + \sqrt{11}) \) and \( (1 - \sqrt{11}) \) are eigenvectors of \( X(1) \) and \( X(1)^{-1} \), respectively, corresponding to the eigenvalue \( 10 + 3\sqrt{11} \). Suppose there does exist a matrix \( B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \) such that \( BX(1)B^{-1} = X(1) \). Then

\[
B \cdot \begin{pmatrix} 3 \\ 1 - \sqrt{11} \end{pmatrix} = \begin{pmatrix} 3a + (1 - \sqrt{11})b \\ 3c + (1 - \sqrt{11})d \end{pmatrix}
\]

is also an eigenvector of \( X(1) \) and thus is a complex multiple of \( (1 + \sqrt{11}) \). It follows that

\[
\frac{3c + (1 - \sqrt{11})d}{3a + (1 - \sqrt{11})b} = \frac{c + d\left(\frac{1 - \sqrt{11}}{3}\right)}{a + b\left(\frac{1 - \sqrt{11}}{3}\right)} = \frac{1 + \sqrt{11}}{3}.
\]

Hence \( (1 + \sqrt{11})/3 \) and \( (1 - \sqrt{11})/3 \) are equivalent (in the sense of [1]) and their expressions as simple continued fractions are the same after a finite number of terms. Since the irrational numbers in question are quadratic roots, their expressions as simple continued fractions are eventually periodic.

One easily computes that

\[
\frac{1 + \sqrt{11}}{3} = [1, 2, 3, 1, 1, 2, \ldots] = 1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{2 + \ldots}}}}
\]

and

\[
\frac{1 - \sqrt{11}}{3} = [-1, 4, 2, 1, 1, 3, 2, \ldots].
\]

Since the cycles are distinct, up to a cyclic permutation, \( (1 + \sqrt{11})/3 \) cannot be equivalent to \( (1 - \sqrt{11})/3 \), which is a contradiction. Therefore no such \( B \) exists.
REFERENCES


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