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CORRECTION TO "CLOSED 3-MANIFOLDS WITH NO PERIODIC MAPS"

BY

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In [**3**] we considered a family of 3-manifolds M_Φ which were defined as mapping tori of specially constructed surface homeomorphisms Φ. Contrary to our claim there, these 3-manifolds in fact admit involutions, as was pointed out to us by M. Sakuma. However, by introducing a slight change in the family of maps Φ we are able to recover all the results claimed in [**3**]. With the change in the maps Φ described below we correct the difficulty in [**3**], which occurs in Lemma 1, and are easily able to adapt the remaining arguments to the new maps Φ, thus obtaining the desired examples of closed, orientable, aspherical 3-manifolds with no periodic maps.

Recall from [**3**] that F denotes a closed orientable surface of genus g ($g \geqslant 3$) situated in R^3 as shown in Figure 1 and that $\{a_i, b_i \mid 1 \leqslant i \leqslant g\}$ is a set of simple closed curves as shown.

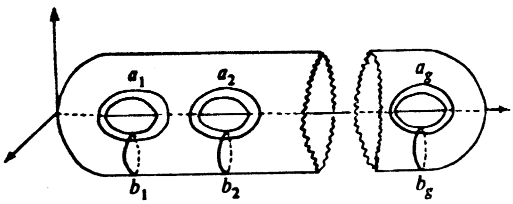

FIGURE 1

Fix a set of $g-1$ arbitrary but distinct integers $\{n_2,\ldots,n_g\}$ such that each $n_i \neq 19$ and $n_i > 2$. We redefine the homeomorphisms $\Phi\colon F \to F$ by setting

$$\Phi = t(b_1)^{-1}t(a_1)^2 t(b_1)^{-3} t(a_1) \prod_{i=2}^{g} t(a_i)t(b_i)^{-n_i+1},$$

where the $t(a_i)$, $t(b_i)$ denote twist maps about the simple closed curves a_i, b_i, respectively. (The alteration of Φ occurs in the twists about the first pair of curves a_1, b_1.) The $2g \times 2g$ matrix P corresponding to the induced automorphism Φ_* of $H_1(F)$ is

$$P = \begin{pmatrix} X(1) & 0 & \cdots & 0 \\ 0 & X(n_2) & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & X(n_g) \end{pmatrix}$$

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shown by blocks of 2×2 matrices, where

$$X(1) = \begin{pmatrix} 7 & 9 \\ 10 & 13 \end{pmatrix} \quad \text{and} \quad X(n) = \begin{pmatrix} n & 1 \\ n-1 & 1 \end{pmatrix} \quad \text{for } n > 2.$$

This matrix P fulfills all the expectations of §3 in [**3**], principally Lemma 7 and Theorem B. Although Lemma 1 is satisfied only by $X(1)$, the remaining lemmas hold for all $X(n)$ as can easily be checked by following the arguments given there.

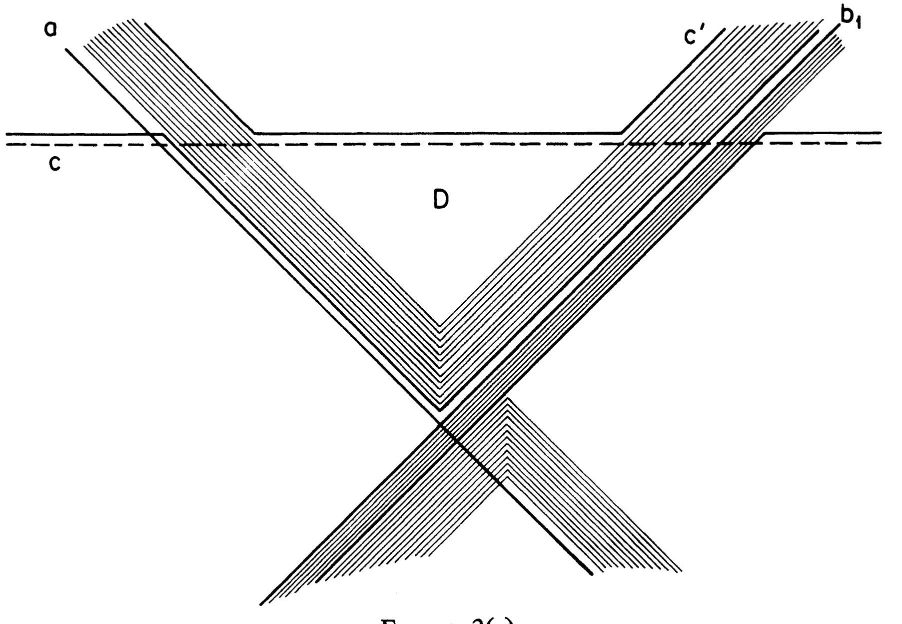

FIGURE 3(c)

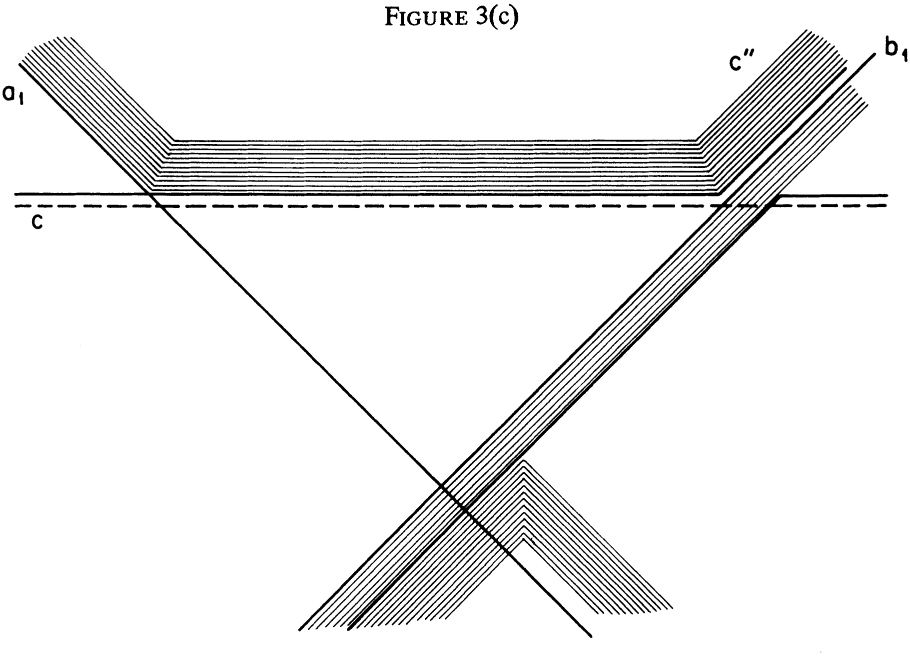

FIGURE 3(d)

(Lemma 6 now needs the assumption $\eta_i \neq 19$.) We indicate the proof of Lemma 1 for $X(1)$ below.

§4 of [3] is unaffected by the above change in Φ. Finally, the arguments in §5 are also unaffected once Lemma 10 has been verified for the new Φ. But the essential character of the new Φ closely resembles that of the original and, as a consequence, the original proof of Lemma 10 can be repeated verbatim for the new Φ. For completeness, we give two additional figures which should be included with Figures 3 and 5 of [3] and referred to during the course of the proof of Lemma 10.

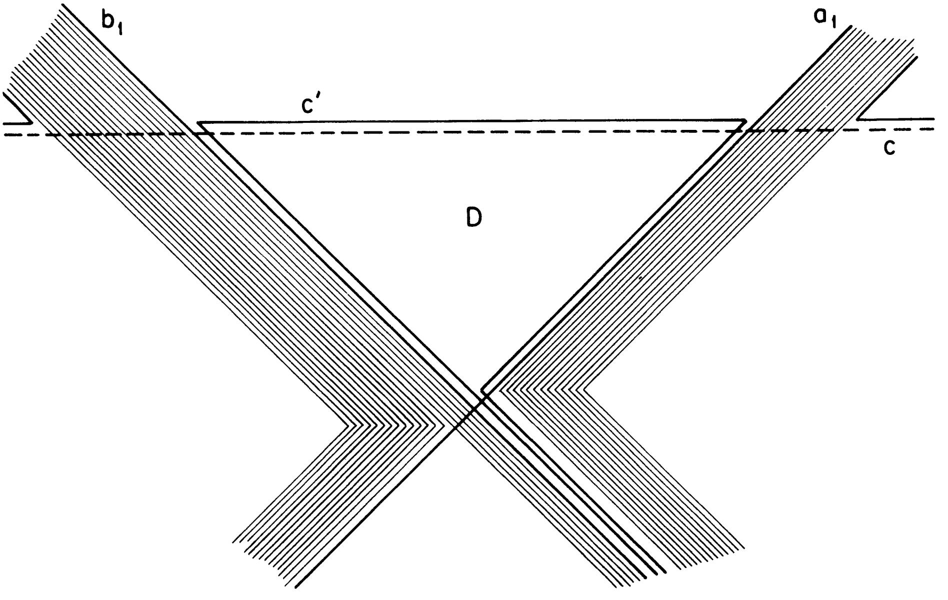

FIGURE 5(b)

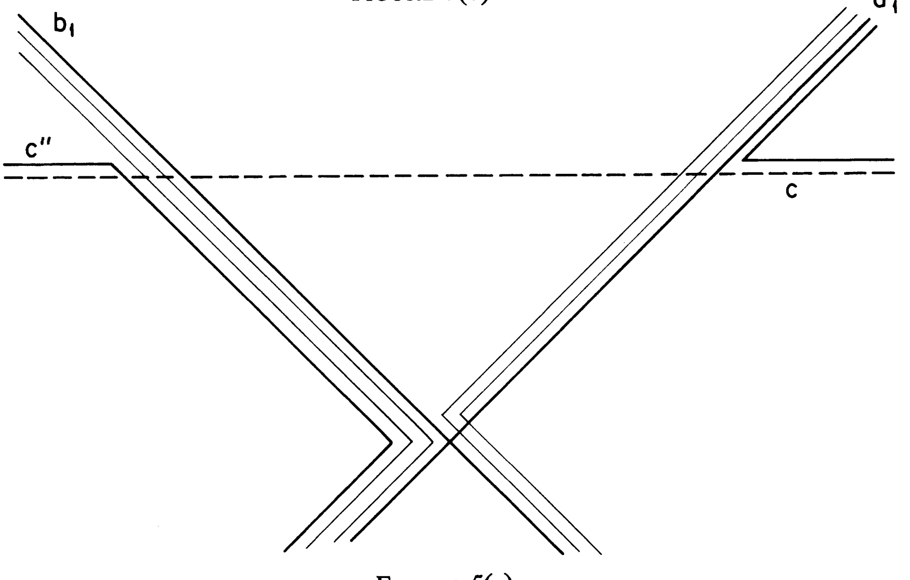

FIGURE 5(c)

All that remains then is Lemma 1 for the matrix $X(1)$, which simply asserts that $X(1)$ is not conjugate to its inverse in $GL(2, Z)$. For this, we follow Sakuma [**4**] and use an argument of Latimer and MacDuffee [**2**].

LEMMA 1. *There does not exist a matrix $B \in GL(2, Z)$ for which $X(1)B = BX(1)^{-1}$.*

PROOF. Observe that the vectors $\binom{3}{1+\sqrt{11}}$ and $\binom{3}{1-\sqrt{11}}$ are eigenvectors of $X(1)$ and $X(1)^{-1}$, respectively, corresponding to the eigenvalue $10 + 3\sqrt{11}$. Suppose there does exist a matrix $B = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ such that $BX(1)^{-1}B^{-1} = X(1)$. Then

$$B \cdot \begin{pmatrix} 3 \\ 1-\sqrt{11} \end{pmatrix} = \begin{pmatrix} 3a + \left(1-\sqrt{11}\right)b \\ 3c + \left(1-\sqrt{11}\right)d \end{pmatrix}$$

is also an eigenvector of $X(1)$ and thus is a complex multiple of $\binom{3}{1+\sqrt{11}}$. It follows that

$$\frac{3c + \left(1-\sqrt{11}\right)d}{3a + \left(1-\sqrt{11}\right)b} = \frac{c + d\left(\dfrac{\left(1-\sqrt{11}\right)}{3}\right)}{a + b\left(\dfrac{1-\sqrt{11}}{3}\right)} = \frac{1+\sqrt{11}}{3}.$$

Hence $(1+\sqrt{11})/3$ and $(1-\sqrt{11})/3$ are equivalent (in the sense of [**1**]) and their expressions as simple continued fractions are the same after a finite number of terms. Since the irrational numbers in question are quadratic roots, their expressions as simple continued fractions are eventually periodic.

One easily computes that

$$\frac{1+\sqrt{11}}{3} = \left[\underbrace{\dot{1},2,3,\dot{1}},1,2\ldots\right] = 1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \ldots}}}}}$$

and

$$\frac{1-\sqrt{11}}{3} = \left[-1,4,\underbrace{\dot{2},1,1,\dot{3}},2\ldots\right].$$

Since the cycles are distinct, up to a cyclic permutation, $(1+\sqrt{11})/3$ cannot be equivalent to $(1-\sqrt{11})/3$, which is a contradiction. Therefore no such B exists.

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