TRANSACTIONS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 280, Number 2, December 1983

CORRECTION TO "THE STABLE GEOMETRIC DIMENSION OF VECTOR BUNDLES OVER REAL PROJECTIVE SPACES"

BY

DONALD M. DAVIS, SAM GITLER AND MARK MAHOWALD

ABSTRACT. The theory of *bo*-resolutions as utilized in *The stable geometric dimension of vector bundles over real projective spaces* did not give adequate care to the $K\mathbf{Z}_2$'s occurring at each stage of the resolution. This restricts somewhat the set of integers e for which we can prove that the geometric dimension of vector bundles of order 2^e on large real projective spaces is precisely $2e + \delta$.

1. Introduction. The theory of *bo*-resolutions played a central role in obtaining the liftings in the main theorem of [**2**]. W. Lellman pointed out that the analysis of *bo*-resolutions presented in [**3** and **2**] did not give adequate care to $K\mathbf{Z}_2$'s occurring at each stage of the resolution. In [**4**] Mahowald discussed a modification in the theorem of *bo*-resolutions which takes these $K\mathbf{Z}_2$'s into account, and showed that the application to v_1-periodic homotopy in [**5**] remains valid. In this note we discuss the changes in [**2**] required by these considerations.

The qualitative content of the main theorem (1.1(i)) of [**2**] remains the same, but the condition $e \geqslant 20$ must be changed to $e \geqslant 75$, i.e.

THEOREM 1.1(i)$'$. *If* $e \geqslant 75$ *and* $n \geqslant 4e + 16[\log_2(e+4)] + 42$, *the geometric dimension of any vector bundle of order* 2^e *over* RP^n *is* $2e + \delta(n, e)$, *where* δ *is as in* [**2**, 1.1]. ($|\delta(n,e)| \leqslant 2$.)

A table such as that on [**2**, p. 59] can be made showing in each congruence the smallest e for which we can prove that this stable geometric dimension of bundles of order 2^e equals the lower bound [**2**, 1.1(ii)] implied by K-theory. These values range from 64 ($e \equiv 0$ (4), $n \equiv 4$ (8)) to 75 ($e \equiv 3$ (4), $n \equiv 0$ (2)).

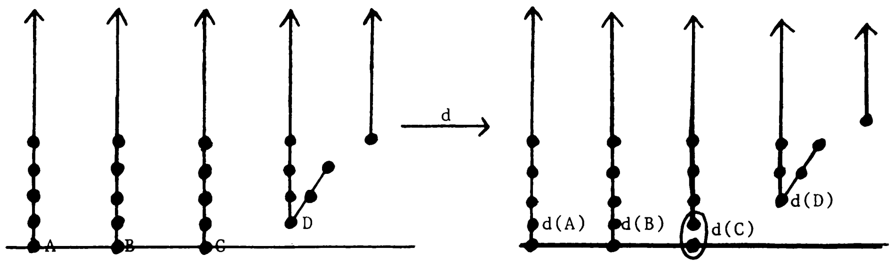
A second expansion of the argument of [**3**] is presented in [**4**]: the proof that the E_2-term of the *bo*-ASS for $\pi_* S^0$ and $\pi_*(S^0 \cup_2 e^1)$ vanishes above a line of slope $\frac{1}{5}$ is modified to meet an objection of K. H. Knapp. This strongest part of the theorem of *bo*-resolutions was not used in [**2**]. If it could be adapted to a broad class of spaces (see [**2**, 3.6]), the set of e for which the stable geometric dimension of bundles of order 2^e can be proved to be as expected could be substantially increased.

In [**1**] minor changes in the arguments of [**2**, 3.16, 3.17] were discussed. These are unrelated to the changes discussed herein.

Received by the editors December 8, 1982.
1980 *Mathematics Subject Classification*. Primary 55G35, 55G40; Secondary 55B15, 55B20.

2. Modifications to theorems of *bo*-resolutions. The changes required in §3 of [**2**] are: Let "$V_{s,i}$ is a $\mathbf{Z}_2$-vector space all elements of which have Adams filtration 0 or 1" and "if l: $Y \to X_s$ has Adams filtration $\geqslant 2$" replace the corresponding clauses of 3.6 and 3.6.1, respectively. Add to 3.7(ii) and 3.8(ii) the clause "after possibly being varied by a map into $K\mathbf{Z}_2$".

The possible problem in 3.7(ii) is illustrated by the charts below, where $\bar{m}$ is a 1-successor of $\bar{n}$.



$\pi_*(B_n^- \wedge bo)$ $\qquad\qquad$ $\pi_*(B_m^- \wedge bo)$

$d(C)$ is the sum of the filtration 1 generator of the infinite summand and a filtration 0 generator of a split $K\mathbf{Z}_2$, which was ignored in [**2**] (see third sentence in proof of 3.6). We do not know whether this phenomenon actually occurs, but it is conceivable that it might, and this would require the modifications listed above.

3. Modifications of application to geometric dimension. We will use the following addition to [**2**, 3.7].

COROLLARY 3.7.1 [**4**]. *$\overline{bo}^{\wedge s} \wedge bo$ is equivalent to a wedge of Eilenberg-Mac Lane spectra $K\mathbf{Z}_2$ and $K\mathbf{Z}_{(2)}$ in dimension less than $6s$.*

PROOF. By [**2**, 3.7, 3.9]

$$\overline{bo}^{\wedge s} \wedge bo \simeq K \vee \bigvee_{\bar{n} \in R_s} \Sigma^{4|\bar{n}|} \begin{cases} bo^{\langle 2|\bar{n}| - \alpha(\bar{n})\rangle}, & n \text{ even}, \\ bsp^{\langle 2|\bar{n}| - 1 - \alpha(\bar{n})\rangle}, & n \text{ odd}. \end{cases}$$

The result follows from the fact that $bo^{\langle i\rangle}$ and $bsp^{\langle i-1\rangle}$ are a wedge of $K\mathbf{Z}_2$'s and $K\mathbf{Z}_{(2)}$'s in dimension less than $2i$. □

COROLLARY 3.7.2. *If X is $(m-1)$-connected and X_s is the sth space in the bo-resolution of X [**2**, p. 46], then $X_s \wedge bo$ is a wedge of $K\mathbf{Z}_2$'s and $K\mathbf{Z}_{(2)}$'s in dimension less than $m + 5s$.*

[**2**, 4.1] was the case $e \equiv 0$ (4), $n \equiv 2$ (8) of Theorem 1.1(i), and served to illustrate the proof for all cases. We prove its modification.

THEOREM 4.1′. *If $e \equiv 0$ (4), $e \geqslant 72$, and f classifies $a \cdot 2^{e+6}\xi$ with a odd, then for $d \geqslant 2[\log_2(4e + 10)]$ the composite*

$$\Sigma^{-1}P_{2e+11+8d}^{4e+10+8d} \xrightarrow{\phi^d} \Sigma^{-1}P_{2e+11}^{4e+10} \xrightarrow{f} SO \to V_{2e+1}$$

is null-homotopic.

The only part of the proof in [2] requiring change is the first three complete sentences on p. 58. These should be replaced by:

Then 4.2 and 3.6.1′ imply that $f' \circ \phi^k$ lifts to X_{2k} (since ϕ is the composite of two filtration-2 maps) with $X_{2k} \wedge bo$ a wedge of $K\mathbf{Z}_2$'s and $K\mathbf{Z}_{(2)}$'s in dimension $\leq 2e - 2 + 2t + 10k$. Choose $k = [\log_2(4e + 10)] = e + 6 - t$ so that $4e + 10 + 8k - 1 \leq 2e - 2 + 2t + 10k$. Further lifting in the bo-resolution cannot encounter the difficulty discussed in §2; [2, 3.6.1] holds from this point on. $f' \circ \phi^{k+k'}$ lifts to $X_{2k+4k'}$ which is $(2e - 2 + 2t + 6k + 12k')$-connected. If $k' = [\log_2(4e + 10)]$ then $\Sigma^{-1}P^{4e+10+8(k+k')} \to X_{2k+4k'}$ is trivial for dimensional reasons, and hence so is

$$f'\phi^{k+k'}\colon \Sigma^{-1}P^{4e+10+8(k+k')} \to P_{2e+1}^{\langle t\rangle}.$$

The condition (*) on [2, p. 57] is satisfied if $9[\log_2(4e + 10)] \leq e$, i.e. $e \geq 72$.

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Department of Mathematics, Lehigh University, Bethlehem, Pennsylvania 18015

Department of Mathematics, Centro de Investigacion del I. P. N., Mexico 14DF, Mexico

Department of Mathematics, Northwestern University, Evanston, Illinois 60201