NONVANISHING LOCAL COHOMOLOGY CLASSES

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ABSTRACT. We discuss the nonvanishing of a top-dimensional canonical cohomology class of the space $\mathcal{B}\text{Diff}_ω M$. We treat parallelizable and odd-dimensional stably parallelizable manifolds.

Conventions. The following hold unless stated otherwise:

1. $M$ is a closed, oriented, $n$-dimensional smooth manifold with volume form $ω$.
2. All $H^*$ and $H_*$ are with $\mathbb{R}$-coefficients.
3. $\deg f$ is the degree of $f$.
4. $[η]$ is the cohomology class of the closed form $η$.

Introduction. Let $\mathcal{B}\text{Diff}_ω M$ denote the group of volume preserving (with respect to $ω$) diffeomorphisms of $M$ with the $C^\infty$ topology. We will investigate certain local cohomology classes of $\text{Diff}_ω M$, originally defined by McDuff [M-1]. These classes are related to the Gelfand-Fuks cohomology of divergence free (with respect to $ω$) vector fields on $M$. These classes also, in a certain sense, measure how $\text{Diff}_ω M$ twists $M$. In this paper we will show that the top-dimensional class in question is nonzero if $M$ is parallelizable or if $M$ is odd-dimensional and stably parallelizable.

Background. Let $\text{Diff}_ω^δ M$ denote $\text{Diff}_ω M$ with the discrete topology. If $G$ is a topological group, then $BG$ denotes its classifying space. The inclusion $\overline{i}: \text{Diff}_ω^δ M \to \text{Diff}_ω M$ passes to a continuous map

\[ i: \mathcal{B}\text{Diff}_ω^δ M \to \mathcal{B}\text{Diff}_ω M \]

Let $\overline{\text{Diff}_ω M}$ be the homotopy theoretic fibre of (1). This space is well-defined up to homotopy type. By the local cohomology of $\text{Diff}_ω M$ we mean the (real) singular cohomology of $\overline{\text{Diff}_ω M}$.

The space $\overline{\text{Diff}_ω M}$ is in its own right a classifying space. It classifies globally trivialized $M$-bundles with a flat structure. Sitting over $\overline{\text{Diff}_ω M}$ we have the universal bundle $\overline{\text{Diff}_ω M} \times M$. Let us now choose a specific model for $\overline{\text{Diff}_ω M}$ [Ma].

Let $\text{Sing}\text{Diff}_ω M$ be the smooth singular complex of $\text{Diff}_ω M$. Note that $\text{Diff}_ω M$ acts freely on $\text{Sing}\text{Diff}_ω M$ by multiplication on the right, so it also acts freely on the...
geometric realization $|\text{Sing} \Diff^\omega \! M|$. We can now form the quotient space $|\text{Sing} \Diff^\omega \! M|/\Diff^\omega \! M$. This will be our particular model for $\overline{\Diff}^\omega \! M [M-1]$.

Thus $\overline{\Diff}^\omega \! M$ has a PL-structure in which a $k$-simplex is a smooth map $\Delta^k \to \Diff^\omega \! M$, which is well defined up to composition on the right by an element of $\Diff^\omega \! M$. To get rid of this ambiguity we ask for the 0-vertex to go to the identity diffeomorphism;

$$\Delta^k \to (\Diff^\omega \! M, 1), \quad t \to h_t.$$

We are now in a position to define a foliation $\mathcal{F}$ on $\overline{\Diff}^\omega \! M \times M$.

To each $k$-simplex in $\overline{\Diff}^\omega \! M$ we can associate a foliation on $\Delta^k \times M$ with leaves

$$\mathcal{L}_m = \{(t, h_t(m)) : t \in \Delta^k\}.$$

This foliation is the pull-back of the point foliation on $M$ by the map $f: (t, m) \to h_t^{-1}(m)$. The foliation is volume preserving [L] and has $f^*\omega$ for its transverse volume form. The foliations and transverse volume forms on each $(k\text{-simplex}) \times M$ fit together to give a codimension-$n$ volume preserving (generalized) foliation $\mathcal{F}$ on $\overline{\Diff}^\omega \! M \times M$ with transverse volume form $\Omega$.

For more background information see [Mo].

**McDuff’s classes.** The cohomology class $[\Omega]$ may be decomposed via the Künneth formula. So

$$[\Omega] = \bigoplus_{i=0}^{n} [\Omega]_i, \quad \text{where} [\Omega]_i \in H^{n-i}(\overline{\Diff}^\omega \! M) \otimes H^i(M).$$

The class $[\Omega]_i$ may be written as $\sum_j \alpha_{n-i}^j \otimes \beta_i^j$, where $j$ is indexed over the rank of $H^i(M)$, $\alpha_{n-i}^j \in H^{n-i}(\overline{\Diff}^\omega \! M)$, and $\beta_i^j \in H^i(M)$. Of course this representation is not unique.

We may now define the McDuff classes $c_k(M) \in H^k(\overline{\Diff}^\omega \! M; H^{n-k}(M))$ by the formula

$$c_k(M) = \sum_j \langle \alpha_{n-i}^j, \kappa \rangle \cdot \beta_i^j,$$

where $\kappa \in H_k(\overline{\Diff}^\omega \! M)$. Note that the class $c_k(M)$ is independent of the choices made. In fact $c_k(M)\kappa$ is just, neglecting sign, the slant product of $[\Omega]_{n-k}$ with $\kappa$. We will use absolute value signs to show that we are neglecting sign. Thus, we may also denote $c_k(M)\kappa$ by

$$(4') \, |[\Omega]/\kappa|.$$

In this paper we are concerned with the top class $c_n(M)$. We may express $[\Omega]_0$ as

$$\alpha_n \otimes 1 \in H^n(\overline{\Diff}^\omega \! M) \otimes H^0(M) \equiv H^n(\overline{\Diff}^\omega \! M) \otimes \mathbb{R}.$$

So we may identify $H^n(\overline{\Diff}^\omega \! M)$ with $H^n(\overline{\Diff}^\omega \! M) \otimes H^0(M)$ and thus identify $c_n(M)$ with $[\Omega]_0$. The class $[\Omega]_0$ is the same as $[\Omega]$ restricted to $\overline{\Diff}^\omega \! M \times \text{pt}$. Therefore,

$$c_n(M) \neq 0 \iff [\Omega]|_{\overline{\Diff}^\omega \! M \times \text{pt}} \neq 0.$$
Fundamental diagram. The diagram we use follows from [T]. Recall that $B\Gamma^n_{st}$ is the classifying space for codimension-$n$ volume preserving Haefliger structures. There is a special cohomology class $\bar{\mu} \in H^n(B\Gamma^n_{st})$ called the universal transverse volume class. If $f: X \to B\Gamma^n_{st}$ classifies a codimension-$n$ volume preserving Haefliger structure $H$, then $f^*\bar{\mu} \in H^n(X)$ is called the transverse volume class of $H$. If $H$ is actually the Haefliger structure coming from a codimension-$n$ volume preserving foliation with transverse volume form $\lambda$, then $[\lambda] = f^*\bar{\mu}$. The class $\bar{\mu}$ can be constructed directly from the topological groupoid of germs $\Gamma^n_{st}$, or it can be defined by a functorial principle.

Associated to $B\Gamma^n_{st}$ we have the normal bundle of its universal Haefliger structure. This bundle can be classified by a map $d: B\Gamma^n_{st} \to B\text{SL}(n, \mathbb{R})$. The map $d$ has a homotopy theoretic fibre $B\Gamma^n_{st}$. We choose models and maps so that we get an actual Hurewicz fibration

$$
B\Gamma^n_{st} \to B\Gamma^n_{st} \to B\text{SL}(n, \mathbb{R}).
$$

The space $B\Gamma^n_{st}$ is the classifying space for codimension-$n$ volume preserving Haefliger structures whose normal bundle is framed. This space is $(n-1)$-connected with $\pi_n(B\Gamma^n_{st}) = \mathbb{R}$. The class $i^*\bar{\mu} \equiv \mu \in H^n(B\Gamma^n_{st})$ corresponds to the identity homomorphism if we identify $H^n(B\Gamma^n_{st})$ with $\text{Hom}_\mathbb{Z}(\mathbb{R}, \mathbb{R})$ in the canonical manner.

The foliation $\mathcal{F}$ on $B\text{Diff}_c M \times M$ can be classified by the map $\Phi$ into $B\Gamma^n_{st}$. Consider the following diagram:

$$
\begin{array}{ccc}
B\text{Diff}_c M \times M & \xrightarrow{\Phi} & B\Gamma^n_{st} \\
\downarrow \pi & & \downarrow d \\
M & \xrightarrow{\tau} & B\text{SL}(n, \mathbb{R})
\end{array}
$$

Here $\pi$ is projection, and $\tau$ classifies $TM$ (the choice of $\omega$ gives an $\text{SL}$-structure). The normal bundle $\nu(\mathcal{F})$ is just $B\text{Diff}_c M \times TM$ since $\mathcal{F}$ is transverse to the $M$-factors. We classify $\nu(\mathcal{F})$ by $d \circ \Phi$. Note that $\nu(\mathcal{F})$ is also classified by $\tau \circ \pi$. Everything is chosen so that (7) commutes and $d$ is a Hurewicz fibration.

We are interested in $\mathcal{L}(M)$, the space of lifts of $\tau$. Let us define $\Pi: B\text{Diff}_c M \to \mathcal{L}(M)$ by setting $\Pi(b) = \Phi(b, \cdot)$ for each $b \in B\text{Diff}_c M$. Since $M$ is compact and $n$-dimensional while the fibre of $d$ is $B\Gamma^n_{st}$, which is $(n-1)$-connected, the space $\mathcal{L}(M)$ is not connected. Because $B\text{Diff}_c M$ is connected, its image under $\Pi$ is in one component, $\mathcal{L}_0(M)$, of $\mathcal{L}(M)$. McDuff [M-2], in the spirit of Thurston [T], has shown that $\Pi^*$ is a cohomology isomorphism. This enables us to view $c_n(M)$ as living in $H^n(\mathcal{L}_0(M))$. Let us exploit this philosophy.

Define $\varepsilon: \mathcal{L}_0(M) \to B\Gamma^n_{st}$ as evaluation at the fixed point $m_0$ in $M$. Consider $\mathcal{F}|\iota$, $\mathcal{F}$ restricted to $B\text{Diff}_c M \times m_0$. The bundle $\nu(\mathcal{F}|\iota)$ is isomorphic to $B\text{Diff}_c M \times R^n$. This tells us that $\mathcal{F}|\iota$ is a codimension-$n$ Haefliger structure with trivial normal bundle. Due to this $\Phi|\iota$, the map classifying $\mathcal{F}|\iota$, is homotopic to a map with image in $B\Gamma^n_{st}$. Without loss of generality we may assume $\Phi|\iota$ actually maps into $B\Gamma^n_{st}$. The following diagram homotopy commutes.
Recall that $c_n(M)$ is $[\Omega]|_{B\mathbb{R}^n_\infty \times \partial M}$. Therefore, $\Phi^*\mu = c_n(M)$. Since $\Pi^*$ is an isomorphism,

\begin{align*}
\Phi^*\mu &\neq 0 \iff \epsilon^*\mu \neq 0. 
\end{align*}

So the problem is now one of understanding the behavior of $\mathcal{L}_0(M) \to \overline{B\Gamma^n_{sl}}$. Inherent in $\mathcal{L}_0(M)$ is the twisting of $TM$. The simpler that $TM$ is, the easier it is to understand $\epsilon$. Such a case is provided when $M$ is stably parallelizable. By this we mean that $TM \oplus e^1 \cong \epsilon^{n+1}$. The most obvious example of such a manifold is $S^n$. In fact spheres "classify" stably parallelizable manifolds.

\textbf{(10) PROPOSITION.} The manifold $M$ is stably parallelizable if and only if $TM \cong \gamma^*(TS^n)$ for some map $\gamma$ from $M \to S^n$.

Before proceeding further we need a technical lemma which follows easily from obstruction theory.

\textbf{(11) LEMMA.} If $f_0$ and $f_1$ are two lifts in $\mathcal{L}(M)$, they are in the same component of $\mathcal{L}(M)$ if and only if $f_0^\ast\bar{\mu} = f_1^\ast\bar{\mu}$.

Since $\Pi(b)$ is $\Phi(b, \cdot)$ and $\Phi(b, \cdot)^*\bar{\mu}$ is $[\Omega]|_{b \times M} = [\omega]$, we have

\begin{align*}
f \in \mathcal{L}_0(M) \iff f^*\bar{\mu} = [\omega].
\end{align*}

\textbf{(13) THEOREM.} If $M$ is an odd-dimensional stably parallelizable manifold, then $c_n(M) \neq 0$.

\textbf{REMARK.} It is essential that $n$ be odd, for McDuff [M-I] has shown that $c_{2n}(S^{2n}) = 0$ and $c_{2n+1}(S^{2n+1}) \neq 0$.

\textbf{PROOF.} Case 1. Suppose that $M$ is stably parallelizable but not parallelizable. Then by (10) $TM \cong \gamma^*(TS^n)$, and $\gamma$ has nonzero degree. Certainly we can give $M$ a volume form $\omega_M$ such that $[\omega_M] = \gamma^*[\omega_S]$, where $\omega_S$ is the standard volume form on $S^n$. Consider the following diagram.

\begin{align*}
M &\xrightarrow{\tau} S^n \xrightarrow{\gamma} B\text{SL}(n, R) \\
\overline{B\Gamma^n_{sl}} &\downarrow \\
\end{align*}

The map $\tau$ classifies $TS^n$ and therefore $\tau \circ \gamma$ classifies $TM$. Say $f \in \mathcal{L}_0(S^n)$; then $f^*\bar{\mu} = [\omega_S]$. It follows that $f \circ \gamma \in \mathcal{L}_0(M)$. Hence, we have a map $\eta: \mathcal{L}_0(S^n) \to \mathcal{L}_0(M)$ given by $\eta(f) = f \circ \gamma$. Consider the next diagram, where $\epsilon_m$ is evaluation at $m_0 \in M$ and $\epsilon_s$ is evaluation at $\gamma(m_0) \in S^n$.

\begin{align*}
\mathcal{L}_0(S^n) &\xrightarrow{\eta} \mathcal{L}_0(M) \\
\downarrow \epsilon_s &\quad \epsilon_m \\
\overline{B\Gamma^n_{sl}} &
\end{align*}

\textbf{(15)}
Since $e_m \circ \eta(f) = e_m(f \circ \gamma) = (f \circ \gamma)(m_0) = e_s \circ f$, the above diagram commutes. Since $c_n(S^n) \neq 0$ for $n$ odd, (9) tells us that $e^*_\mu \neq 0$; therefore $e^*_\mu \neq 0$ and $c_n(M) \neq 0$.

Case 2. $M$ is parallelizable. Let $\tau$ now stand for a map classifying $TM$ into $BSl(n, R)$. Without loss of generality $\tau$ may be taken as a constant map. In this case (7) becomes

$$\overline{BDef}_{\mathbb{S}^n} M \times M \xrightarrow{\phi} \overline{B\Gamma}^n_{sl} \rightarrow B\Gamma^n_{sl}$$

$$\downarrow \pi \quad \downarrow d \quad \downarrow d$$

$$M \xrightarrow{\tau} * \rightarrow BSI(n, R)$$

Since $M$ is parallelizable, $\mathcal{L}(M) = Maps(M, \overline{B\Gamma}^n_{sl})$ and we will designate the component corresponding to $\mathcal{L}_0(M)$ as $Maps_1(M, \overline{B\Gamma}^n_{sl})$. Therefore, $s \in Maps_1(M, \overline{B\Gamma}^n_{sl})$ if and only if $s^*\mu = [\omega]$. The evaluation map $e: \mathcal{L}_0(M) \rightarrow \overline{B\Gamma}^n_{sl}$ becomes $e: Maps_1(M, \overline{B\Gamma}^n_{sl}) \rightarrow \overline{B\Gamma}^n_{sl}$. We wish to show that $e^*\mu \neq 0$. Choose $\omega$ so that $\langle [\omega], [M] \rangle = 1$.

Following McDuff's proof [M-1] that $\pi_n(B\Gamma^n_{sl}) \cong \mathbb{R}$, we choose a map $f: S^n \rightarrow \overline{B\Gamma}^n_{sl}$ such that $[f] = 1 \in \pi_n(\overline{B\Gamma}^n_{sl})$, i.e. $\langle f^*\mu, [S^n] \rangle = 1$. Let us give $S^n$ a volume form $\omega_S$ so that $f^*\mu = [\omega_S]$. We will say that $g: M \rightarrow S^n$ is of degree one if $g^*([\omega_S]) = [\omega]$. Let $Maps_1(M, S^n)$ be all the maps of degree one. Now define a map $\hat{f}: Maps_1(M, S^n) \rightarrow Maps_1(M, \overline{B\Gamma}^n_{sl})$ by setting $\hat{f}(g) = f \circ g$. Let $e$ and $e'$ be evaluation at $m_0$. The following diagram commutes:

$$Maps_1(M, S^n) \xrightarrow{j} Maps_1(M, \overline{B\Gamma}^n_{sl})$$

$$\downarrow e' \quad \downarrow e$$

$$S^n \xrightarrow{f} \overline{B\Gamma}^n_{sl}$$

Since $f^*\mu = [\omega_S]$, if we can show that $e'^*[\omega_S] \neq 0$, we will have shown that $e^*\mu \neq 0$.

$Maps_1(S^n, S^n)$ is the space of maps from $S^n$ to $S^n$ such that $f^*[\omega_S] = [\omega_S]$ (as before). Let $\xi$ be a fixed element of $Maps_1(M, S^n)$ such that $\xi(m_0) = s$, the south pole of $S^n$. Define $\hat{\xi}: Maps_1(S^n, S^n) \rightarrow Maps_1(M, S^n)$ by $\hat{\xi}(h) = h \circ \xi$. The following diagram, with $e''$ being evaluation at $s$, commutes.

$$Maps_1(S^n, S^n) \xrightarrow{\hat{\xi}} Maps_1(M, S^n)$$

$$\downarrow e'' \quad \downarrow e'$$

$$S^n \xrightarrow{\text{identity}} S^n$$

If we can show that $e''^*[\omega_S] \neq 0$, then we will have shown that $e^*[\omega_S] \neq 0$ and we will be done. The left-hand side of (17) is from the fibration

$$\Omega^nS^n_1 \rightarrow Maps_1(S^n, S^n) \rightarrow S^n,$$

where $\Omega^nS^n_1$ is the obvious component. Using the spectral sequence of (18) and the fact that $n$ is odd, we have $e''^*[\omega_S] \neq 0$. Q.E.D.
We will now prove

(19) **Theorem.** If $M$ is parallelizable then $c_n(M) \neq 0$.

**Remark.** Here we have removed the condition of $n$ being odd but yet we still use $S^n$ in our argument.

**Proof.** Since $M$ is parallelizable it may be immersed in $\mathbb{R}^{n+1}$ [H]. We will consider $M$ as being immersed in $\mathbb{R}^{n+1}$ with a fixed immersion. We will also freely identify $M$ with its image in $\mathbb{R}^{n+1}$. This causes no trouble, as will become apparent. Our goal is to define a map $\Psi: M \to \text{Maps}_1(M, S^n)$ such that if $\epsilon'$ is evaluation at $m_0$ then $\epsilon' \circ \Psi: M \to S^n$ has nonzero degree. If this is true then $\epsilon'^* [\omega_s]$ is nonzero, where $\omega_s$ is the volume form on $S^n$. Let $\overline{\Psi}$ be the composite $\epsilon' \circ \Psi$.

(20)

\[
\begin{array}{c}
M \\
\Psi \\
\downarrow \epsilon' \\
\overline{\Psi} \\
\text{Maps}_1(M, S^n)
\end{array}
\]

Choose positive $\epsilon$ less than the injectivity radius of $M$ and small enough so that every (open) ball $B(p, \epsilon)$ with center $p \in M$ and radius $\epsilon$ is embedded by the immersion. Consider $\text{Exp}^{-1}: B(p, \epsilon) \to TM_p$. In fact, the image is in $B(Q_p, \epsilon) \subset TM_p$. Let $\gamma$ be the Gauss map from $M$ to $S^n$. By parallel translation in $\mathbb{R}^{n+1}$ we get a congruence from $TM_p \to TS^n_{\gamma(p)}$. Remembering that $\pi$ is the injectivity radius of the sphere we see that $\text{Exp}$ maps $B(\tilde{\gamma}(p), \pi) \subset TS^n_{\gamma(p)}$ diffeomorphically onto $S^n - A(\gamma(p))$, where $A$ is the antipodal map on $S^n$. Let $\xi: TS^n \to TS^n$ by $\xi(\tilde{v}) = \pi/\epsilon \cdot \tilde{v}$. We are now in a position to define $\Psi: M \to \text{Maps}_1(M, S^n)$.

Decompose $M$ as $B(p, \epsilon) \cup \{ M - B(p, \epsilon) \}$:

(21)

\[
\begin{array}{c}
B(p, \epsilon) \\
\text{Exp}^{-1} \\
\overline{\text{Exp}}^{-1} \\
\text{diffeomorphism} \\
B(\bar{\gamma}(p), \epsilon) \\
\downarrow \xi \\
B(\bar{\gamma}(p), \pi) \\
\downarrow \text{Exp} \\
S^n - A(\gamma(p))
\end{array}
\]

If $m \in B(p, \epsilon)$ define $\Psi(p)m$ to be the image of $m$ under the above composition. At this stage we want the orientation on $B(p, \epsilon)$, induced from the orientation of $M$, to go to the usual orientation on $S^n - A(\gamma(p))$. If it does not we give $M$ a different orientation and volume form. If $m \notin B(p, \epsilon)$ set $\Psi(p)m = A(\gamma(p))$.

As we vary $p$ we get a continuous map $\Psi: M \to \text{Maps}_1(M, S^n)$. Each $\Psi(p)$ is of degree one, for $\Psi(p)$ is just a standard collapsing map of degree one.

(22) **Lemma.** $\deg \overline{\Psi} = (-1)^{n+1} \deg \gamma + (-1)^n$, where $\deg$ stands for degree of the map.

**Proof of Lemma.** We just sketch the proof since the techniques are standard. If necessary, the first thing we do is adjust $\epsilon$ in our definition of $\Psi$ to make sure that in a neighborhood $D$ containing $B(m_0, \epsilon)$ we can slightly deform $M$ so that $D$ is flat.
Thus, the Gauss map $\gamma$ is constant on $D$, hence $\gamma(D) = \gamma(m_0)$. On $M - D$, $\Psi$ sends $x$ to $\Psi(x)m_0 = A(\gamma(x))$. If $x \in D$ then we must be careful. If $x \in D - B(M_0, \varepsilon)$, then $\Psi(x) = \Psi(x)m_0 = A(\gamma(x)) = A(\gamma(m_0))$ since $D$ is flat. The map $\Psi$ restricted to $B(m_0, \varepsilon)$ has degree $(-1)^n$. $\Psi$ is not smooth on $\partial B(m_0, \varepsilon)$. Therefore, we replace it by a map $\Psi'$ in the same homotopy class and very close to $\Psi$ and calculate the Brouwer degree. The $(-1)^{n+1}\deg \gamma$ part comes from $A \circ \gamma$, while $(-1)^n$ is due to the fact that $\overline{\Psi}$ near $m_0$ is a local diffeomorphism. By a judicious choice of a regular value it is easy to show that $\overline{\Psi}'$, and hence $\overline{\Psi}$, has the proper degree. The proof of the lemma is complete.

Remember that we are trying to show that if $M$ is parallelizable then $c_n(M) \neq 0$. By (16) if we can show that $\epsilon^*\omega_j = 0$ we will be done. By the previous lemma $\overline{\Psi}$ has degree $(-1)^n + (-1)^{n+1}\deg \gamma$. If we can show that this is nonzero, then $\epsilon^*\omega_j \neq 0$. This is our plan. We will vary our immersion of $M$ so that we get $\deg \gamma$ to our liking. To accomplish this we appeal to some results of Hopf [Ho, Mi].

(23) **Theorem (Hopf).** Let $L: M^n \to \mathbb{R}^{n+1}$ be an immersion with corresponding Gauss map $\gamma(L): M^n \to S^n$.

(a) If $n$ is even, $\deg \gamma(L) = \frac{1}{2} \chi(M)$, where $\chi(M)$ is the Euler characteristic of $M$.

(b) If $n$ is odd and $\deg \gamma(L) = k$, then given any $m$ one can find an immersion $j = j(m)$ such that $\deg \gamma(j) = k + 2m$.

Thus $\overline{\Psi}$ can be taken to have nonzero degree. Q.E.D.

**Remark.** It is worth pointing out that all 3-manifolds and products of spheres with one factor being odd are parallelizable and therefore have $c_n(M) \neq 0$.

**References**


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