CORRECTION TO “VANISHING THEOREMS AND 
KÄHLERITY FOR STRONGLY PSEUDOCONVEX MANIFOLDS”

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It has been brought to my attention by N. Coltoiu that there was a gap in the proof of Theorem II in [3]. By using the same notations as in [3], the proof of our Theorem II can be corrected as follows.

THEOREM II. Let \((X, S)\) be a strongly pseudoconvex manifold. If \(\dim S = 1\), then \(X\) is Kählerian. In particular, any strongly pseudoconvex surface is Kählerian.

First of all the following result is needed.

LEMMA [1]. Let \(S\) be a compact \(C\) analytic space and let \(L\) be a holomorphic line bundle on \(S\). Let us assume that, for every positive dimensional subspace \(T\) of \(S\), there exist an integer \(n\) and a nonzero holomorphic section of \(L^n \otimes O_T\) which vanishes at some point of \(T\). Then \(L\) is positive.

Notations. From now on, a positive line bundle (resp. semipositive line bundle) will be denoted by \(L > 0\) (resp. \(L \geq 0\)).

PROOF OF THEOREM II. Without loss of generality, one can assume that \(S\) is irreducible. So let \(x\) be a point on \(S\) and let \(\pi: \hat{X} \to X\) be the blowing up of \(X\) at \(x\) inducing a biholomorphism \(\hat{X} \setminus D = X \setminus \{x\}\).

Let \(T\) be the strict transform of \(S\) under \(\pi\); it is clear the \(\hat{X}\) is a strongly pseudoconvex manifold with its exceptional set \(\hat{S} = D \cup T\).

CLAIM. There exists a positive line bundle \(L\) on \(\hat{X} \setminus D\).

In fact, let \(L_1 := [D]\) be the line bundle determined by \(D\). Since \(\dim T = 1\), the Lemma above tells us that \(L_1|T > 0\). In view of the compactness of \(T\), one can find a relative compact neighborhood \(V\) of \(T\) in \(\hat{X}\) such that

\((*)\) \(L_1 > 0\) on \(V\) and by construction \(L_1 \geq 0\) on \(\hat{X} \setminus D\).

Since \(\hat{X}\) is strongly pseudoconvex, one can find a line bundle \(L_2\) on \(\hat{X}\) such that

\((**)\) \(L_2 > 0\) on \(\hat{X} \setminus \hat{S}\) and \(L_2 \geq 0\) on \(\hat{X}\).

In view of (*) and (**), it follows that, for some \(N \gg 0\), the line bundle

\(L := L_1 \otimes L_2^N > 0\) on \((\hat{X} \setminus \hat{S}) \cup V \supset \hat{X} \setminus D\). Hence our claim is proved.

Consequently, \(\hat{X} \setminus D = X \setminus \{x\}\) is Kählerian. The result in [2] tells us that \(X\) itself is Kählerian. Q.E.D.
References


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