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BOUNDARY UNIQUENESS THEOREMS IN $\mathbf{C}^n$

BY

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ABSTRACT. Let n-dimensional manifolds Γ_k, $k = 1, 2, \ldots$, be given in a smoothly bounded domain $\Omega \subset \mathbf{C}^n$. Assume that the Γ_k "converge" to an n-dimensional, totally real manifold $\Gamma \subseteq \partial\Omega$ and that a function f analytic in Ω has the property that its traces f_k on Γ_k have distributional limit zero as $k \to \infty$ (or assume that $f_k \to 0$ pointwise). Then under the assumption that f is polynomially bounded near $P \in \Gamma$ by $(\operatorname{dist}(z, \partial\Omega))^{-1}$ we conclude that f is identically zero.

1. Introduction. In this note we present boundary type uniqueness theorems for analytic functions of several complex variables. Consider n-dimensional manifolds Γ_k, $k \in N$, which "converge" to an n-dimensional, totally real manifold $\Gamma \subset \mathrm{b}\Omega$, the boundary of a C^∞-smooth, bounded domain $\Omega \subset \mathbf{C}^n$. (This convergence is only assumed to take place near a point $P \in I$.) Assume that the function f, analytic in Ω, has the property that its traces f_k on Γ_k have distributional limit 0 as $k \to \infty$, or that $f_k \to 0$ pointwise (see §2 for precise definitions). If f is polynomially bounded (near P) by $1/\operatorname{dist}(z, \mathrm{b}\Omega)$, then this implies that f is identically 0. This is the content of Theorem 2.2 and its Corollary 2.3. Theorem 2.2 is a consequence of results obtained in [**13** and **14**]. Note that the set of polynomially bounded analytic functions on a bounded domain Ω contains the classical Hardy and Bergman spaces and all partial derivatives of such functions. Also observe that there is no restriction concerning *tangential* (with respect to $\mathrm{b}\Omega$) approach of the Γ_k; osculation to arbitrary high order is possible (for $n > 1$). Examples 2.4 and 2.5 illustrate the role played by the hypotheses in Theorem 2.2 and Corollary 2.3, respectively.

If one knows that f omits a (finite) value, the growth condition can be relaxed. This yields a result (Corollary 2.7) which applies in particular to the Nevanlinna class (Example 2.8).

In §3 we briefly indicate how to use our technique in conjunction with a result of Sadullaev concerning bounded functions [**11**, Theorem 6] to extend this result so as to include *tangential* limits.

2. f polynomially bounded. Let Ω be a bounded, smooth (C^∞) domain in $\mathbf{C}^n$ with $P \in \mathrm{b}\Omega$. Assume $z \to \xi = \rho(z)$, $\rho := (\rho_1, \ldots, \rho_{2n})$ is a C^∞-diffeomorphism of a neighborhood V of P onto the open unit ball (for simplicity) in $\mathbf{R}^{2n}$, with $\rho(P) = 0$, such that near P the boundary of Ω is given locally by the equation $\rho_1(z_1, \ldots, z_n) = 0$, and such that $\rho^{-1}(C^+) \subset \Omega$. Here, C^+ is the "cone"

$$C^+ := \{\xi \in \mathbf{R}^{2n} \mid \xi_j > 0,\ 1 \le j \le n\}. \tag{1}$$

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Let $\Gamma \subset b\Omega$ be the manifold

$$\Gamma := \{z \in b\Omega \mid \rho_2(z) = \cdots = \rho_n(z) = 0\}. \tag{2}$$

We assume throughout that Γ *is totally real.*

We say that a function f defined on Ω is polynomially bounded if there exists $C > 0$ and $N \in \mathbf{N}$ such that

$$|f(z)| \leq C/d(z)^N. \tag{3}$$

where $d(z) := \operatorname{dist}(z, b\Omega)$.

The following constitutes a portion of Theorem 1.3 in [**13**] ($O(\Omega)$ denotes the holomorphic functions).

LEMMA 2.1. *$f \in O(\Omega)$ is polynomially bounded if and only if* $(\operatorname{Re} f)^+$, *the positive part of* $\operatorname{Re} f$, *is polynomially bounded.* □

Our first uniqueness theorem can be regarded as a generalization to the polynomially bounded functions of a result of Pinčuk [**9**] for continuous functions. For $U \subset \mathbf{R}^n$ open, denote by $\mathcal{D}(U)$ the usual space of smooth functions compactly supported inside U.

THEOREM 2.2. *Let Ω, P, and ρ be given as above. Let $f \in O(\Omega)$ satisfy $(\operatorname{Re} f)^+$ is polynomially bounded near P. Assume there exists a sequence of n-tuples $\{(c_1^k, \dots, c_n^k)\}$ with*

$$\lim_{k\to\infty} c_j^k = 0, \qquad c_j^k > 0,\ 1 \leq j \leq n, \tag{4}$$

such that

$$\lim_{k\to\infty} \int_{\Gamma_k} f(z)\psi(\rho_{n+1}(z), \dots, \rho_{2n}(z))\, dz_1 \wedge \cdots \wedge dz_n = 0 \tag{5}$$

$$\forall\, \psi \in \mathcal{D}\left(\left\{(\xi_{n+1}, \dots, \xi_{2n}) \mid \sum_{j=n+1}^{2n} \xi_j^2 < 1\right\}\right),$$

where the Γ_k are the manifolds

$$\Gamma_k := \{z \in V \mid \rho_j(z) = c_j^k,\ 1 \leq j \leq n\}. \tag{6}$$

Then f is identically 0.

Note that (5) says that the traces of f on Γ_k converge in $\mathcal{D}'(\Gamma)$ (compare Remark 1 in §2 and the beginning of §4 in [**14**]). In particular, therefore, (5) holds if these traces converge in an L^p-sense. We would like to emphasize that pointwise convergence *everywhere* also suffices. For simplicity, we set $z(\xi_1, \dots, \xi_{2n}) := \rho^{-1}(\xi_1, \dots, \xi_{2n})$. Then we have

COROLLARY 2.3. *The conclusion of Theorem 2.2 remains valid, if* (5) *is replaced by pointwise convergence*

$$\lim f(z(c_1^k, \dots, c_n^k, \xi_{n+1}, \dots, \xi_{2n})) = 0, \tag{7}$$

for all $(\xi_{n+1}, \dots, \xi_{2n})$ is some open set. Note that convergence is not required to be uniform.

PROOF. A Baire category argument shows that there exist an open set E and M such that

$$|f(z(c_1^k, \dots, c_n^k,\ \xi_{n+1}, \dots, \xi_{2n}))| \leq M, \quad \forall\, k \in N,\ \forall\, (\xi_{n+1}, \dots, \xi_{2n}) \in E. \tag{8}$$

(7) and (8) now imply that (5) holds for ψ supported in a ball contained in E; therefore Theorem 2.2 applies. □

As already mentioned in the introduction, Theorem 2.2 and, hence, Corollary 2.3 apply to the *Hardy spaces* as well as to the *Bergman spaces*.

Example 2.4 illustrates the necessity of requiring some bound on the growth of f. Example 2.5 shows that the requirement of convergence on an open set, in equation (7), cannot be replaced by almost everywhere convergence (in contrast to the bounded case, where convergence on sets of positive measure suffices; see §3). They work for $n = 1$ as well as for $n > 1$.

EXAMPLE 2.4. A result of Arakeljan [**2**, Satz 3, p. 139] implies that there exists a nonzero function f, analytic in the unit disc, which satisfies

$$|f((1-1/k)e^{i\theta})| \le 1/k, \qquad -\pi/4 \le \theta \le \pi/4 \ \forall\ k \in \mathbf{N}. \tag{9}$$

This example is easily lifted to higher dimensions (see the next example) to illustrate that without bounds on the growth of the functions near the boundary, Theorem 2.2 fails.

EXAMPLE 2.5. Let Ω be a bounded smooth domain in $\mathbf{C}^2$ whose boundary $\mathrm{b}\Omega$ contains

$$\{e^{i\theta} \mid -\pi/4 < \theta < \pi/4\} \times \{w \mid |w| < 1\}. \tag{10}$$

Let $P = (1, 0)$ and let

$$\Gamma := \{(z,w) \mid (z,w) = (e^{i\theta}, u),\ -\pi/4 < \theta < \pi/4,\ -1/2 < u < 1/2\}. \tag{11}$$

Then $P \in \Gamma$, and Γ is totally real. Let g be a nonzero function analytic in the unit disc, polynomially bounded, and such that the radial limits exist and are equal to 0 a.e. on $\{e^{i\theta} \mid -\pi/4 < \theta < \pi/4\}$. That such a function exists follows from the fact that one can arbitrarily prescribe measurable boundary values and a bound for the growth (as long as it goes to infinity for $|z| \to 1$); see [**2**, p. 154] and [**3**]. Then the function $f(z,w) := g(z)$ has normal limits 0 a.e. on Γ (with respect to 2-dimensional euclidean measure on Γ) and is polynomially bounded near P, yet is not identically 0.

REMARK 2.6. For $n = 1$, Theorem 2.2 is known though somewhat hard to pinpoint in the literature. Compare, however, [**5**, Chapitre I; especially Théorème VIII and the remark following it]. For convenience, we outline a simple argument: by considering a smaller smooth domain which shares a piece of boundary with Ω near P, we may assume that f is polynomially bounded on all of Ω. Then f admits a distribution boundary value τ_f on $\mathrm{b}\Omega$ [**13**, Theorem 1.3]. Combining this with Cauchy's formula, we have for $z \in \Omega$:

$$f(z) = \frac{1}{2\pi i}\left\langle \frac{d\varsigma}{d\sigma} \cdot \tau_f(\varsigma), \frac{1}{z-\varsigma}\right\rangle_{\varsigma}, \tag{12}$$

where the pairing is between $\mathcal{D}'(\mathrm{b}\Omega)$ and $\mathcal{D}(\mathrm{b}\Omega)$ (the usual spaces of distributions and C^∞-functions, respectively, on $\mathrm{b}\Omega$). The term $d\varsigma/d\sigma$ is the unique C^∞-function on $\mathrm{b}\Omega$ such that $d\varsigma = (d\varsigma/d\sigma) \cdot d\sigma$, where $d\sigma$ is the length element of $\mathrm{b}\Omega$. By assumption (5), τ_f is supported away from P; thus the right side of (12) (and hence f) is analytic in a full neighborhood of P. Then (5) implies that this function vanishes on a full piece of $\mathrm{b}\Omega$ near P. It is therefore identically 0. □

PROOF OF THEOREM 2.2. By the previous remark we need only consider the case $n > 1$. Now [**13**, Theorem 1.3] implies that there exists

$$\lim_{\varepsilon \to 0+} \int_{\{\rho_1 = \varepsilon\}} f(z)\chi(\rho_2(z), \dots, \rho_{2n}(z))\, d\sigma_\varepsilon \tag{13}$$

for all χ smooth and supported in a neighborhood of the origin in $\mathbf{R}^{2n-1}$. Here $d\sigma_\varepsilon$ is the euclidean surface element on the hypersurface $\{\rho_1 = \varepsilon\}$. Actually, in [**13**] only the surfaces $\{z \mid d(z) = \varepsilon\} = b\Omega_\varepsilon$ are considered. However, the arguments easily carry over to cover the case of the level surfaces of any smooth defining function for Ω. We now proceed as in §5 of [**14**]: since Γ is totally real, there exists a vector in $T_p^{\mathbf{R}}(\Gamma)$ which is not contained in $T_p^{\mathbf{C}}(b\Omega)$, the complex tangent space of $b\Omega$ at P. We may assume that this vector is $\partial z/\partial \xi_{2n}(P)$, thus

$$\partial z/\partial \xi_{2n}(P) \notin T_p^{\mathbf{C}}(b\Omega) \tag{14}$$

(otherwise compose $(\rho_{n+1}, \dots, \rho_{2n})$ with a real linear transformation of $\mathbf{R}^n$ onto itself). (14) also holds in a neighborhood of P. Let us define $f_\varepsilon \in C^\infty(U_1 \times U_2)$ as

$$f_\varepsilon(\xi_2, \dots, \xi_{2n}) := f(z(\varepsilon, \xi_2, \dots, \xi_{2n})) \tag{15}$$

for $((\xi_2, \dots, \xi_{2n-1}), \xi_{2n}) \in U_1 \times U_2$, where U_1 and U_2 are (small) neighborhoods of 0 in $\mathbf{R}^{2n-2}$ and $\mathbf{R}$ respectively. We are now in a position to apply Theorem 4.1 in [**14**]. See also the discussion preceding that theorem. The theorem says that from the convergence of these $(2n-1)$-dimensional integrals we may conclude convergence of the integrals over lower-dimensional manifolds, in fact over 1-dimensional manifolds of the form $\{z/\rho_1(z) = \varepsilon,\ \rho_j(z) = c_j,\ 2 \le j \le 2_{n-1}\}$. Moreover, the convergence (as $\varepsilon \to 0_+$) is in the space of C^∞-functions with respect to $(\xi_2, \dots, \xi_{2n-1})$. More precisely, (13) implies that

$$\lim_{\varepsilon \to 0+} f_\varepsilon =: f_0 \tag{16}$$

exists in $C^\infty(U_1, \mathcal{D}'(U_2))$, the space of $\mathcal{D}'(U_2)$-valued C^∞-functions on U_1. This space carries the usual topology of locally uniform convergence of the functions and their derivatives (see [**15**, §40]). Now we split $(\xi_2, \dots, \xi_{2n-1})$ into two groups: $(\xi_2, \dots, \xi_n)$ and $(\xi_{n+1}, \dots, \xi_{2n-1})$ and assume that U_1 has the form $U_{1,1} \times U_{1,2}$ for small neighborhoods of 0 in $\mathbf{R}^{n-1}$. Then the convergence (16) implies a fortiori convergence in $C^\infty(U_{1,1}, \mathcal{D}'(U_{1,2} \times U_2))$. Note that, in particular, f_0 from (16) is in $C^\infty(U_{1,1}, \mathcal{D}'(U_{1,2} \times U_2))$. So for $(\xi_2, \dots, \xi_n) \in U_{1,1}$ fixed, f_0 defines a distribution in $\mathcal{D}'(U_{1,2} \times U_2)$ which we denote by $f_{0,(\xi_2, \dots, \xi_n)}$. Similarly, we define the traces

$$f_{\varepsilon,(\xi_2,\dots,\xi_n)}(\xi_{n+1}, \dots, \xi_{2n}) := f_\varepsilon(f_2, \dots, \xi_{2n}) \tag{17}$$

and conclude from the convergence in $C^\infty(U_{1,1}, \mathcal{D}'(U_{1,2} \times U_2))$ that

$$f_{\varepsilon,(\xi_2,\dots,\xi_n)} \underset{\varepsilon \to 0^+}{\to} f_{0,(\xi_2,\dots,\xi_n)} \quad \text{in } \mathcal{D}'(U_{1,2} \times U_2), \tag{18}$$

and the convergence is, moreover, locally uniform in $(\xi_2, \dots, \xi_n)$. Therefore, the following limit exists and may be calculated as shown:

$$\lim_{(c_1,\dots,c_n) \to 0} f_{c_1,(c_2,\dots,c_n)} = f_{0,(0,\dots,0)} = \lim_{k \to \infty} f_{c_1^k,(c_2^k,\dots,c_n^k)} = 0 \quad \text{in } \mathcal{D}'(U_{1,2} \times U_2). \tag{19}$$

Here, we have used (4) and (5). That (5) implies the last equality in (19) follows from standard distribution theory and the fact that $dz_1 \wedge \cdots \wedge dz_n$ is nonzero on

$\operatorname{supp}\psi \subset \Gamma_k$, for k sufficiently large (since then Γ_k will be totally real on $\operatorname{supp}\psi$); also compare the discussion at the beginning of §4 in [**14**]. Pinčuk's Edge of the Wedge Theorem [**8**, Theorem 1] now gives the desired result. In Pinčuk's notation, with $f^+ := f$, $f^- := 0$, (19) implies that the assumptions of Theorem 1 are fulfilled. Therefore, f and 0 have a common analytic continuation, whence $f = 0$. This concludes the proof of Theorem 2.2. □

If the function f is a priori known to omit (at least) one value $a \in \mathbf{C}$, the growth condition can be relaxed (compare also Remark 3.2):

COROLLARY 2.7. *Same assumptions as in Corollary* 2.3, *but with the growth condition on* $(\operatorname{Re} f)^+$ *replaced by*

$$\text{range}(f) \subseteq \mathbf{C}\backslash\{a\}, \qquad a \in \mathbf{C}, \tag{20}$$

and

$$\log^+ |f| \text{ polynomially bounded near } P. \tag{21}$$

Then f *is identically* 0.

PROOF. Consider the covering map of $\mathbf{C}$ onto $\mathbf{C}\backslash\{a\}$:

$$\varsigma \to e^{\varsigma} + a. \tag{22}$$

Near P, f lifts to give an analytic branch of $\log(f - a)$, whose real part is polynomially bounded from above, by (21). Let $a \neq 0$. Let V be a small neighborhood of $0 \in \mathbf{C}$, such that its inverse image under the covering map consists of countably many disjoint open sets $V_j = V_0 + 2j\pi i$, $j \in \mathbf{Z}$. A Baire category argument yields an open set E in $(\xi_{n+1}, \ldots, \xi_{2n})$-space and k_0 such that $f(c_1^k, \ldots, c_n^k, \xi_{n+1}, \ldots, \xi_{2n}) \in V$ for $k \geq k_0$ and $(\xi_{n+1}, \ldots, \xi_{2n}) \in E$. Because the V_j are disjoint, we conclude that there is a subsequence Γ_{k_m} such that $\log(f-a)$ converges to a constant ς_0 along the Γ_{k_m} (in the sense of (7)) for all $(\xi_{n+1}, \ldots, \xi_{2n}) \in E$, or that there is a subsequence such that $\operatorname{Im}\log(f - a)$ converges to $+\infty$ or $-\infty$ uniformly on E along the subsequence. In the first case, Corollary 2.3 applies and yields that $\log'(f - a) \equiv \varsigma_0$. Note now that $e^{\varsigma_0} + a = 0$, so that we get $f \equiv 0$. The second case contradicts the fact that $\log(f - a)$ and hence $\operatorname{Im}\log(f - a)$ must have distributional limits along the Γ_k (existence of these limits follows from the polynomial boundedness; compare the arguments that led to the first part of formula (19) in the proof of Theorem 2.2). Consider now the case $a = 0$. Then we have an analytic branch of $\log f$, and $\operatorname{Re}\log f$ will tend to $-\infty$ pointwise along the Γ_k. This contradicts the fact that $\operatorname{Re}\log f$ must have distributional limit along the Γ_k (as above): by a Baire category argument, we may assume that

$$\operatorname{Re}\log f(c_1^k, \ldots, c_n^k, \xi_{n+1}, \ldots, \xi_{2n}) \leq 0 \tag{23}$$

for all $(\xi_{n+1}, \ldots, \xi_{2n})$ in some open set E and all $k \geq k_0$. By considering positive test functions supported in E and applying Fatou's lemma, one concludes from the existence of the distributional limit that the function which is identically $-\infty$ on some open set $E' \Subset E$ is integrable. This contradiction rules out the case $a = 0$. □

An important class of functions to which Corollary 2.7 applies is provided by the following

EXAMPLE 2.8. $f \in O(\Omega)$ is said to belong to the *Nevanlinna class* of Ω [**12**, pp. 47, 48; **10**]), if

$$\sup_{\varepsilon>0} \int_{b\Omega_\varepsilon} \log^+ |f| \, d\sigma_\varepsilon < \infty. \tag{24}$$

A standard argument involving the subharmonicity of $\log^+ |f|$ and polynomial estimates (uniform in ε) on the Poisson kernels of the domains Ω_ε imply that the functions in the Nevanlinna class satisfy (21). *Thus, if a Nevanlinna function omits a value and converges pointwise to a constant along the Γ_k, the function reduces to a constant.*

We conclude this section by pointing out that the geometric setup introduced at the beginning allows for *tangential approach* of the Γ_k to $b\Omega$, if $n > 1$. Indeed, if the sequence $\{(c_1^k, \ldots, c_n^k)\}$ is chosen appropriately, the sequences

$$\{z(c_1^k, \ldots, c_n^k, \xi_{n+1}, \ldots, \xi_{2n})\}$$

approach their limits $z(0, \ldots, 0, \xi_{n+1}, \ldots, \xi_{2n}) \in b\Omega$ tangentially to arbitrary high (prescribed) order.

3. The bounded case. In [**11**] it is shown that a bounded function f whose normal limits for a set of positive (n-dimensional Lebesgue) measure of Γ lie in a polar set [**4**, 2.1 and 2.2] must necessarily be constant. Clearly, the normal limits can be replaced by suitable limits along more general curves, but if one wants these limits to yield normal limits in a pointwise fashion (so as to then invoke Sadullaev's Theorem), severe restrictions concerning tangential approach must be imposed. These restrictions are discussed and illustrated by examples in §3 of [**1**] and in [**16**]. It is therefore interesting that the technique used in the proof of Theorem 2.2, which is nonpointwise in nature, allows us to extend Sadullaev's result to the setting introduced in §2, that is, to the tangential approach as discussed at the end of that section. We let $n > 1$.

THEOREM 3.1. *Let Ω, P, ρ, $\{\Gamma_k\}$, and Γ be as in §2. Let*

$$E \subset \left\{ (\xi_{n+1}, \ldots, \xi_{2n}) \mid \sum_{j=n+1}^{2n} \xi_j^2 < 1 \right\}$$

have positive measure. Assume that $f \in O(\Omega)$ is bounded and that there exists a sequence of n-tuples $\{(c_1^k, \ldots, c_n^k)\}$ as in Theorem 2.2 for which the limits

$$\lim_{k\to\infty} f(z(c_1^k, \ldots, c_n^k, \xi_{n+1}, \ldots, \xi_{2n})) \tag{1}$$

exist and are contained in a (fixed) polar set for all $(\xi_{n+1}, \ldots, \xi_{2n}) \in E$. Then f is constant.

PROOF. As mentioned, the proof is by reduction to [**11**, Theorem 6]. We keep the notations as in the proof of Theorem 2.2. The sequence $\{f_{c_1^k,(c_2^k,\ldots,c_n^k)}\}$ is bounded in $L^\infty(\xi_{n+1}, \ldots, \xi_{2n})$. It converges weak-* on $\mathcal{D}(\xi_{n+1}, \ldots, \xi_{2n})$, which is dense in $L^1(\xi_{n+1}, \ldots, \xi_{2n})$. Therefore, $f_{0,(0,\ldots,0)}$ is the weak-* limit of this sequence. On the other hand, the normal limits of f exist for almost all $(\xi_{n+1}, \ldots, \xi_{2n})$; see [**11**, Theorem 3], or combine [**7**, Theorem 1] with a Fubini argument. We call the resulting $\mathcal{L}^\infty$ function h. Then

$$h = f_{0,(0,\ldots,0)} \quad \text{in } \mathcal{L}^\infty_{(\xi_{n+1},\ldots,\xi_{2n})} \tag{2}$$

because both are the traces on Γ of the boundary value f_0 of f (note that this boundary value is independent of the defining function for Ω, with whose level surfaces it is defined; it is in any case given by the function defined on $b\Omega$ a.e. by the nontangential limits of the bounded function f). On the set E, the weak-* limit coincides a.e. with the pointwise limit, so that

$$f_{0,(0,\ldots,0)}(\xi_{n+1},\ldots,\xi_{2n}) = \lim_{k\to\infty} f(z(c_1^k,\ldots,c_n^k,\xi_{n+1},\ldots,\xi_{2n})) \tag{3}$$

a.e. on E. The reduction to [**11**, Theorem 6] is complete (by (1) and (2)). □

REMARK 3.2. By taking into account that the universal cover of $\mathbf{C}\setminus\{a,b\}$ is the unit disc (see [**6**, p. 15] for the covering map), one obtains uniqueness theorems for functions which omit two values; no growth condition is then required.

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