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CLASSIFICATION OF CONTINUOUS JBW^*-TRIPLES

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ABSTRACT. We show that every JBW^*-triple without a direct summand of type I is isometrically isomorphic to an l^∞-sum $\mathcal{R} \oplus^\infty H(A,\alpha)$ where $\mathcal{R}$ is a w^*-closed right ideal in a W^*-algebra B and $H(A,\alpha)$ are the elements of a W^*-algebra A which are symmetric under a $\mathbf{C}$-linear involution α of A. Both A and B do not have a direct (W^*-algebra) summand of type I. In order to refine the decomposition $\mathcal{R} \oplus^\infty H(A,\alpha)$ we define and characterize types of JBW^*-triples.

1. Introduction: Review and announcement of results. The objects of this paper are JBW^*-triples. These are generalizations of Jordan W^*-algebras and W^*-algebras, namely certain Jordan triple systems which are defined on complex Banach spaces and which satisfy axioms intertwining the Jordan triple and the Banach space structure. It is therefore only natural that our work uses techniques from two different areas: Jordan theory and functional analysis. It is the aim of this section to describe most of the necessary background material and state our results in a way which requires only the most basic knowledge from both areas. We hope that this approach will make the paper readable for researchers from both areas. In this paper a *Jordan-*-triple* consists of a complex vector space U and a *triple product*

$$\{\cdots\}\colon U \times U \times U \to U\colon (x,y,z) \to \{xyz\} =: L(x,y)z$$

which is $\mathbf{C}$-linear in x and z, $\mathbf{C}$-antilinear in y and satisfies

(1.1) $\{xyz\} = \{zyx\}$ (commutativity),

(1.2) $[L(x,y), L(u,v)] = L(\{xyu\}, v) - L(u, \{yxv\})$ (five-linear-identity).

Our basic references for Jordan triple systems are [**22**, **23**, **28** and **29**], in particular all unexplained concepts and notations can be found there. We mention that from the point of view of Jordan theory Jordan-*-triples are real Jordan triple systems. However, we will sometimes use results from Jordan theory as if U were a complex Jordan triple system. In all cases these deviations from the theory are easily checked.

In the algebraic part of Jordan theory, Jordan triple systems are defined in terms of the *quadratic representation*

$$P\colon U \to \operatorname{End}_{\mathbf{R}} U\colon x \to P(x), \qquad P(x)y = \tfrac{1}{2}\{xyx\}.$$

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However in our situation both definitions coincide [**22**, 2.2]. Note our normalization of the P-operator, which is the one used in our basic references.

All Banach spaces considered in this paper will be complex Banach spaces. Their norm will be denoted by $||\cdot||$. For two Banach spaces U, V let $\mathcal{L}(U,V)$ be the Banach space of all bounded linear operators from U to V endowed with the operator norm. We put $\mathcal{L}(U) = \mathcal{L}(U,U)$

A JB^**-triple* is a Jordan-*-triple $(U, \{\dots\})$ defined on a complex Banach space such that the following three properties hold:

(1.3) the triple product $L\colon U \times U \to \mathcal{L}(U)\colon (x,y) \to L(x,y)$ is continuous,

(1.4) all left multiplications $L(z,z)$, $z \in U$, are hermitian operators (i.e. for all $t \in \mathbf{R}$ $\exp itL(z,z)$ is an isometry) with nonnegative spectrum, and

(1.5) $||P(z)z|| = ||z||^3$ for all $z \in U$ (C^*-condition).

A basic example of a JB^*-triple is given by

(1.6) Any (complex) C^*-algebra with the triple product $\{xyz\} = xy^*z + zy^*x$ (i.e. $P(x)y = xy^*x$) is a JB^*-triple. This follows from standard properties of C^*-algebras (see e.g. [**30** or **34**]).

Being a generalization of C^*-algebras already indicates the significance of JB^*-triples for Jordan theory. Besides this, the interest in JB^*-triples arises from complex analysis: "Every bounded symmetric domain in a complex Banach space is biholomorphically equivalent to the open unit ball of a JB^*-triple, and in this way the category of all bounded symmetric domains with base points is equivalent to the category of JB^*-triples." This is the main result in [**18**]; for an exposition see [**34**, 20].

A *subtriple* of a Jordan-*-triple U is a complex (not necessarily closed) subspace W of U satisfying $\{WWW\} \subset W$. A *homomorphism* between Jordan-*-triples U, V is a $\mathbf{C}$-linear map $\Phi\colon U \to V$ preserving the triple product: $\Phi\{xyz\} = \{\Phi x, \Phi y, \Phi z\}$ (equivalently, $\Phi P(x)y = P(\Phi x)\Phi y$). *Isomorphisms* and *isometric isomorphisms* are then defined in an obvious way. The following result is fundamental.

(1.7) KAUP'S JB^*-CHARACTERIZATION [**18**, 5.3]. *Let U be a Jordan-*-triple defined on a complex Banach space satisfying* (1.3) *and part of* (1.4): *all operators $L(z,z)$ are hermitian. Then U is a JB^*-triple iff for every $z \in U$ the closed subtriple generated by z is isometrically isomorphic to a commutative C^*-algebra viewed as Jordan-*-triple as in* (1.6).

The big advantage of this characterization is that it "localizes" the definition of a JB^*-triple. In particular it implies

(1.8) *Every closed subtriple of a JB^*-triple is again a JB^*-triple. Hence every closed subspace W of a C^*-algebra satisfying $w_1 w_2^* w_1 \in W$ for $w_i \in W$ is a JB^*-triple.*

By (1.8) and (1.6) we have a large supply of examples of JB^*-triples which however do not exhaust all possibilities: not every JB^*-triple can be embedded in an associative algebra. This follows from the next example:

(1.9) Every positive hermitian Jordan triple system as defined and studied in [**23**] is a JB^*-triple with respect to the spectral norm [**23**, 3.17]. The classification of these triple systems [**23** or **29**, IV, §2] shows that there exist two finite-dimensional exceptional examples (denoted V and VI) which cannot be imbedded in an associative algebra [**24**].

(1.10) For any compact set S and any JB^*-triple U let $\mathcal{C}(S,U)$ be the Banach space of continuous functions from S to U, endowed with the sup-norm. With respect to the pointwise defined triple product $\{fgh\}(s) = \{f(s), g(s), h(s)\}$, $s \in S$, $\mathcal{C}(S,U)$ becomes a JB^*-triple as is easily seen.

(1.11) For any family $(U_i)_{i\in I}$ of JB^*-triples let $||(u_i)_{i\in I}||_\infty = \sup_{i\in I} ||u_i||$ and

$$\bigoplus_{i\in I}^{\infty} U_i = \left\{u \in \prod U_i;\ ||u||_\infty < \infty\right\}.$$

Then $\bigoplus_{i\in I}^{\infty} U_i$ with componentwise operations is again a JB^*-triple, called the l^∞-sum of $(U_i)_{i\in I}$.

We now know enough examples to give a first rough description of all JB^*-triples in the spirit of the classical Gelfand-Naimark theorem for C^*-algebras. Recall, that this theorem says that any C^*-algebra is isometrically isomorphic to a closed selfadjoint subalgebra of the C^*-algebra $\mathcal{L}(H)$ for some Hilbert space H.

(1.12) GELFAND-NAIMARK FOR JB^*-TRIPLES [**9**]. *Every JB^*-triple is isometrically isomorphic to a subtriple of $\mathcal{L}(H)\oplus^\infty \mathcal{C}(S,\mathrm{VI})$ where H is a Hilbert space and $\mathcal{C}(S,\mathrm{VI})$ as in* (1.10) *for $U = \mathrm{VI}$ the exceptional JB^*-triple of dimension* 27 (*see* (1.9)).

The proof of this theorem heavily depends on the theory of *JBW^*-triples.* These are JB^*-triples which are dual Banach spaces. Thus, the relation between JB^*- and JBW^*-triples is analogous to the relation between C^*- and W^*-algebras. Of course, the reader will have noted that every W^*-algebra is a JBW^*-triple with respect to the product of (1.6).

Being a dual Banach space, a JBW^*-triple has a second topology besides the norm topology. This is the w^*-topology (see e.g. [**12**, 1.1.16]). In the following topological notions with respect to the w^*-topology will be preceded by "w^*-". Since every w^*-closed subspace of U is a dual Banach space we have the following supplement to (1.8):

(1.13) *Every w^*-closed subtriple of a JBW^*-triple is again a JBW^*-triple. In particular, every w^*-closed subspace of a W^*-algebra satisfying $w_1w_2^*w_1 \in W$ for all $w_i \in W$ is a JBW^*-triple.*

A fundamental result for JBW^*-triples is

(1.14) [**4**]. *The triple product of a JBW^*-triple is separately w^*-continuous: $(x,y,z) \to \{xyz\}$ is w^*-continuous in each of the three variables if one fixes the remaining two.*

It then follows from [**14**, (3.21)] that the predual U_* is unique.

From the point of view of Jordan theory the main advantage of JBW^*-triples over JB^*-triples is their rich supply of tripotents (JB^*-triples may only have the trivial one) where a *tripotent* in U is an element e with $P(e)e = e$. For every such element $L(e,e)$ is an hermitian operator with eigenvalues 0, 1 and 2:

$$U = U_2(e) \oplus U_1(e) \oplus U_0(e) \qquad \text{(Peirce decomposition)}$$

where $U_j(e) = \{x \in U;\ L(e,e)x = jx\}$. The Peirce spaces $U_j = U_j(e)$ satisfy certain multiplication rules, e.g. $\{U_jU_kU_l\} \subset U_{j-k+l}$, in particular every U_j is a subsystem whence, by (1.13) and (1.14), a JBW^*-triple if U is a JBW^*-triple. It is important that a JBW^*-triple does not only contain nonzero tripotents, but that we even have

(1.15) *The maximal tripotents of a JB^*-triple U (i.e. the tripotents e with $U_0(e) = 0$) coincide with the extreme points of the unit ball. Hence, by the Krein-Milman theorem every JBW^*-triple contains a maximal tripotent.*

This follows from [**19**, (3.5) and **18**, §4] (see [**13**, (2.22)]). On the other hand, e is called a *minimal tripotent* if $U_2(e) = \mathbf{C}e$. More generally, e is called an *abelian tripotent*, if the subtriple $U_2(e)$ is abelian, i.e. $[L(x,y), L(u,v)]|U_2(e) = 0$ for all $x, y, u, v \in U_2(e)$. Minimal or abelian tripotents need not exist in a JBW^*-triple, for example they do not exist in W^*-algebras without direct summand of type I. However, one can always split off a part that is spanned by minimal resp. abelian tripotents. This is a trivial consequence of the next statement. Recall, a subspace I of a Jordan-*-triple U is called an *ideal* if $\{IUU\} + \{UIU\} \subset I$.

(1.16) [**29**, IV 3.6] *For any algebraic property* (P) *for elements of a JBW^*-triple U there exists a unique decomposition $U = U_{(\mathrm{P})} \oplus^\infty U_{(\mathrm{P})}^\perp$ of U into w^*-closed ideals $U_{(\mathrm{P})}$ and $U_{(\mathrm{P})}^\perp$, where $U_{(\mathrm{P})}$ is the w^*-closure of the span of all elements of U having property* (P).

Letting (P) = "$e \in U$ is a minimal tripotent" gives the splitting $U = U_a \oplus^\infty U_a^\perp$ where U_a is the *atomic* part of U, the w^*-closure of the span of all minimal tripotents [**8**]. Letting (P) = "$e \in U$ is an abelian tripotent" gives

$$U = U_I \oplus^\infty U_I^\perp \tag{1.17}$$

where U_I is the w^*-closure of the span of all abelian tripotents [**13**]. Motivated by the notation in W^*-algebra theory [**30**] we call U of *type* I (resp. *continuous*) if $U = U_I$ (resp. $U = U_I^\perp$). It is easily seen that our definition of type I is equivalent to the one given in [**14**, (4.11)].

The structure of type-I-JBW^*-triples has been determined up to isometric isomorphy in [**13**]. They are l^∞-sums of tensor products of Cartan factors with abelian W^*-algebras. Hence, the structure of JBW^*-triples will be completely known as soon as one has determined the structure of $U_I^\perp$. The object of this paper is to reduce the study of continuous JBW^*-triples to that of W^*-algebras and their involutions. The two building blocks of continuous JBW^*-triples are described in the next two examples.

(1.18) Any w^*-closed right ideal R of a W^*-algebra A is a JBW^*-triple by (1.13), and we will see that R is continuous if A is a continuous W^*-algebra (i.e. a W^*-algebra without direct summand of type I). Note that the w^*-closed right ideals of A are exactly the spaces pA for a unique projection p ($p = p^2 = p^*$).

(1.19) Let α be an involution of a W^*-algebra A, i.e. a $\mathbf{C}$-linear antiautomorphism of A of period 2 which commutes with *. Since α is w^*-continuous, (1.13) implies that $H(A, \alpha) = \{a \in A;\ a^\alpha = a\}$ is a JBW^*-triple. Again, we will find that $H(A, \alpha)$ is continuous if A is continuous.

Since a JB^*-sum of two JBW^*-triples is again a JBW^*-triple we can obtain new examples by adding examples (1.18) and (1.19). In this way we will get all continuous JBW^*-triples:

(1.20) CONTINUOUS JBW^*-TRIPLE CLASSIFICATION. *Every continuous JBW^*-triple is isometrically isomorphic to an l^∞-sum*

$$R \oplus^\infty H(A, \alpha) \tag{1.20.1}$$

where R is a w^*-closed right ideal in a continuous W^*-algebra and α is a $\mathbf{C}$-linear involution of the continuous W^*-algebra A commuting with *.

The proof of (1.20) will be given in §§2 and 3, in §4 we show uniqueness of the decomposition. It will be refined in §5, where we define and study types of JBW^*-triples. In an appendix we give a new proof of the Halving Lemma for JBW^*-triples.

The first step in establishing (1.20) is the classification of continuous JBW^*-triples containing a *unitary tripotent*, i.e. a tripotent e of U with $U = U_2(e)$. This is done in Theorem (2.1). This result is not (quite) new, we could have derived it from [**12**, 7.3.5] using the close connection between JW-algebras, JW^*-algebras and JBW^*-triples containing a unitary tripotent (see e.g. [**12**, 3.8]). However, a proof of (2.1) is included here, because our proof, which is easy and more Jordan theoretic than the proof of [**12**, 7.3.5], gives us precise information, which is needed later on and which is not immediate from [**12**, 7.3.5].

Since our main method is the study of the interaction of tripotents, it may be appropriate to conclude this introduction by reviewing the various types of tripotents in a prominent example.

(1.21) Let A be a C^*-algebra and denote by U the corresponding JB^*-triple structure on A (see (1.6)). Then

(i) $e \in U$ is a tripotent (i.e. $ee^*e = e$) iff e is partial isometry (i.e. e^*e is a projection, equivalently ee^* is a projection).

Indeed, if e is a partial isometry and $e = e_{11} + e_{10} + e_{01} + e_{00}$ is the associative Peirce decomposition relative to e^*e then $0 = (e^*e)_{00} = e_{10}^*e_{10} + e_{00}^*e_{00} = (e_{10}^* + e_{00}^*)(e_{10} + e_{00})$ forces $e = e_{11} + e_{01} = ee^*e$. In this case

$$\begin{aligned} &U_2(e) = ee^*Ae^*e, \quad U_1(e) = (1 - ee^*)Ae^*e + ee^*A(1 - e^*e), \\ &U_0(e) = (1 - ee^*)A(1 - e^*e). \end{aligned}$$

(ii) $e \in U$ is a maximal tripotent iff e is an extreme point of the unit sphere of A [**30**, 1.6.4] and e is a unitary tripotent iff e is a unitary element.

(iii) If A is even a W^*-algebra (and so U is a JBW^*-triple) then a tripotent $e \in U$ is a maximal tripotent iff there is a central projection $z \in A$ with $zee^* = z$ and $(1 - z)e^*e = 1 - z$. Indeed, we have $(1 - ee^*)A(1 - e^*e) = 0$ whence $c(1 - ee^*)c(1 - e^*e) = 0$ by [**30**, 1.10.7] where $c(\cdots)$ is the central support of the projection $1 - ee^*$ and $1 - e^*e$ resp. Let $z = c(1 - e^*e)$. Then $z(1 - ee^*) = 0 = (1 - z)(1 - e^*e)$ follows.

2. Continuous JBW^*-triples with unitary tripotents. In this section we will classify continuous JBW^*-triples with unitary tripotents. The result will then be applied to characterize the Peirce space $U_2(e)$ of a maximal tripotent e in a general continuous JBW^*-triple U. Our main method is the theory of grids which can be viewed as "matrix units" for Jordan triple systems [**15**, **26**, **27**, **29**].

(2.1) THEOREM. *Let $U = U_2(e)$ be a continuous JBW^*-triple with a unitary tripotent e. Then U is isometrically isomorphic (as a triple system) to an l^∞-sum*

$$U \cong \mathrm{Mat}(4,4;C) \oplus^\infty H_4(D,\pi,^*) \tag{2.1.1}$$

where

(i) *C is a continuous W^*-algebra and* $\mathrm{Mat}(4,4;C)$ *is the W^*-algebra of* 4×4 *matrices over C.*

(ii) *D is a continuous W^*-algebra with respect to the involution * and π is a* $\mathbf{C}$*-linear *-involution of D such that*

$$I = I^\pi \text{ for every } w^*\text{-closed ideal } I \text{ of } (D,^*). \tag{2.1.2}$$

The JBW^*-triple $H_4(D,\pi,^*)$ is defined on the π-hermitian matrices

$$\{x \in \mathrm{Mat}(4,4;D);\ x = x^{\pi t}\}$$

with triple product $P(x)y = xy^{*t}x$.

Before we give the proof we note

$$H_4(D,\pi,^*) = H(B,\alpha) \tag{2.1.3}$$

where $B = \mathrm{Mat}(4,4;D)$ is a continuous W^*-algebra with involution $x \to x^\alpha = x^{\pi t}$. Condition (2.1.2) will enforce uniqueness of the rectangular part in the decomposition (2.1.1)—see §4. It is easy to see that it is equivalent to

$$I \cap I^\pi \neq 0 \text{ for every nonzero } w^*\text{-closed ideal } I \text{ of } (D,^*). \tag{2.1.2$'$}$$

We again point out that this theorem could be derived from [**12**, 7.3.5]—see the end of §1. We also mention that the case of a JBW^*-factor is stated in [**1**].

PROOF. By [**12**, 5.2.15] applied to the JBW-algebra $\{u \in U;\ P(e)u = u\}$ there exist four orthogonal tripotents h_{ii}, $1 \le i \le 4$, such that $e = h_{11} + \cdots + h_{44}$ and $h_{ii}\,\tilde{1}\,h_{jj}$ for $i \neq j$, i.e. there also exist h_{ij} with $h_{ij} \dashv h_{ij} \vdash h_{jj}$ ($h \vdash g$ means $h \in U_1(g)$ and $g \in U_2(h)$). Hence, for $1, i, j \neq$ we have $h_{1j} \in U_1(h_{11} + h_{ii}) = U_1(h_{1i})$ and $h_{1i} \top h_{1j}$ ($h \top g$ means h and g are collinear, i.e. $h \in U_1(g)$ and $g \in U_1(h)$) follows. Therefore, by [**29**, II.1.4], $(h_{11}, h_{12}, h_{13}, h_{14})$ generates a hermitian grid $\mathcal{H}(4) = \{h_{ij};\ 1 \le i \le j \le 4\}$, i.e. a system of tripotents with the same multiplication table as the canonical hermitian matrix units (there is no harm to assume $h_{ii} = P(h_{1i})h_{11}$). $\mathcal{H}(4)$ covers $U = U_2(e)$, i.e. U is linearly spanned by $U_2(h_{ij})$, $1 \le i \le j \le 4$. Let U_{ij} be the Peirce spaces of U with respect to the orthogonal system $(h_{11}, h_{22}, h_{33}, h_{44})$. Then $L(h_{ij}, h_{ii})\colon U_{ii} \to U_{ij}$ $(i \neq j)$ is injective, since for every $z_{ii} \in \ker L(h_{ij}, h_{ii})$ we have

$$\begin{aligned} P(z_{ii})U &= P(z_{ii})U_{ii} = P(z_{ii})P(h_{ii})P(h_{ij})U_{jj} \\ &= P(\{z_{ii}h_{ii}h_{ij}\})U_{jj} \quad \text{(by the linearized fundamental formula [\textbf{27}, (0.9)])} \\ &= 0 \end{aligned}$$

and therefore in particular $P(z_{ii})z_{ii} = 0$ forcing $z_{ii} = 0$ by the C^*-condition (1.5).

We can now apply the Hermitian Coordinatization Theorem [**27**, 5.6], see also [**29**, III 1.9]: U is isomorphic to a hermitian matrix system $H_4(A,\pi,^*)$. More precisely, one can define a unital associative algebra A with involution * on U_{12} by

$$ab = \{\{ah_{12}h_{13}\}h_{13}b\}, \qquad a^* = P(h_{12})a, \quad 1_D = h_{12}.$$

Obviously, A is an algebra over $\mathbf{C}$ and * is $\mathbf{C}$-antilinear. The triple product of the JBW^*-triple U_{12} can be expressed by the algebra product $P(a)b = ab^*a$ for $a, b \in U_{12}$. Therefore, by [**5**, 2.14], $(A,^*)$ is a C^*-algebra, whence a W^*-algebra because U_{12} is w^*-closed. Obviously, A is continuous. The map $a \to a^\pi = P(h_{ij})P(h_{ii}, h_{jj})a$ is a $\mathbf{C}$-linear involution of A commuting with *. It induces an involution $x \to x^\alpha = x^{\pi t}$ of the W^*-algebra $B = \mathrm{Mat}(4,4;A)$, and $H_4(A,\pi,^*) = H(B,\alpha)$ as triple systems, in particular $H_4(A,\pi,^*)$ is a JBW^*-triple. Since every algebraic isomorphism between JB^*-triples is actually an isometry [**14**, (2.4)] we proved that U is isometrically isomorphic to $H_4(A,\pi,^*)$ as described in (ii) with the only missing point being the condition (2.1.2). It is exactly this condition which brings about the decomposition (2.1.1) via the following easy assertion which follows from [**12**, 7.3.4]:

(2.1.4) Let A be a W^*-algebra with a *-involution π. Then there are w^*-closed ideals C and D such that

(i) $A = C \oplus C^\pi \oplus D$, $D = D^\pi$ and

(ii) D satisfies (2.1.2)

Now let $A = C \oplus C^\pi \oplus D$ be a decomposition as in (2.1.4). Since D and $C \oplus C^\pi$ are $(\pi,^*)$-invariant we obtain a corresponding decomposition on the level of triple systems

$$H_4(A,\pi,^*) = H_4(C \oplus C^\pi, \pi,^*) \oplus H_4(D,\pi,^*)$$

is a decomposition into w^*-closed triple ideals whence a JB^*-sum [**14**, (4.4)]. The proof is now finished by observing that

$$\mathrm{Mat}(4,4;C) \to H_4(C \oplus C^\pi, \pi,^*) \colon x \to x \oplus x^{\pi t}$$

is an isomorphism of JB^*-triples. □

(2.2) REMARK. By Theorem (2.1) the study of continuous JBW^*-triples containing unitary tripotents is connected to the theory of W^*-algebras. In particular we like to mention the work of T. Giordano, V. Jones and E. Størmer on involutions of W^*-algebras [**10**, **11** and **31**].

We will now give an equivalent description of the condition (2.1.2) on $(D,\pi,^*)$ in terms of the Jordan triple system $H_n(D,\pi,^*)$. We will use the standard notation to describe elements in $H_n(D,\pi,^*)$:

$$a[ii] = aE_{ii}, \qquad b[ij] = bE_{ij} + b^\pi E_{ji} \qquad (i \neq j)$$

where E_{ij} are the usual matrix units, $a = a^\pi$ and $b \in D$. Also recall, that two tripotents $e, f \in U$ are called *rigid-collinear* if $U_2(e) \subset U_1(f)$ and $U_2(f) \subset U_1(e)$.

(2.3) LEMMA. *Let D be a W^*-algebra with a *-involution π and let $U = H_n(D,\pi,^*)$, $n \geq 2$, be a continuous JBW^*-triple. Then there are equivalent*

(i) *D contains a w^*-closed *-ideal $I \neq 0$ satisfying $I \cap I^\pi = 0$,*

(ii) *U contains nonzero rigid collinear tripotents $(c[11], d[12])$ where*

$$c = c^\pi = c^* = c^2 \quad and \quad d = d^* = d^2 \tag{2.3.1}$$

are projections in D.

PROOF. The most important multiplication rules in $H_n(D,\pi,^*)$ which we will use are

$$P(a[ii])b[ii] = ab^*a[ii], \qquad \{a[ii]b[ii] \times [ij]\} = ab^*x[ij],$$
$$P(x[ij])y[ij] = xy^*x[ij], \qquad P(x[ij])a[jj] = xa^*x^\pi[ii],$$
$$\{x[ij]y[ij]a[ii]\} = (xy^*a + (xy^*a)^\pi)[ii].$$

For (i)$\Rightarrow$(ii) let d be the unit element of I and put $c = d + d^\pi$. It easily follows that $U_2(d[12]) = I[12]$, $U_2(c[11]) = \{(a + a^\pi)[11];\ a \in I\}$ and $(c[11], d[12])$ is a rigid-collinear pair.

The converse direction is more complicated: Since the condition (2.3.1) obviously implies that $c[11]$ and $d[12]$ are tripotents, the main condition in (ii) is rigid collinearity. In particular, $d[12] = \{c[11]c[11]d[12]\}$ gives $d = cd$ whence $d = d^* = dc$ and d lies in the associative Peirce space $cDc =: C$. Because $U_2(c[11]) = \{a[11];\ a \in C, a^\pi = a\} \subset U_1(d[12])$ we have

$$da + ad^\pi = a, \qquad a = a^\pi \in C. \tag{2.3.2}$$

Since $d^\pi \in C^\pi = C$ this shows $d+d^\pi = c$, whence (d, d^π) are orthogonal idempotents and also

$$d^\pi a + ad = a, \qquad a = a^\pi \in C. \tag{2.3.3}$$

Now let

$$C = C_{11} \oplus C_{12} \oplus C_{21} \oplus C_{22}$$

be the associative Peirce decomposition of C with respect to (d, d^π). Then $C_{11}^\pi = C_{22}$, $C_{12}^\pi = C_{12}$, $C_{21}^\pi = C_{21}$ and (2.3.2), (2.3.3) imply

$$\pi | C_{12} \oplus C_{21} = -\,\mathrm{id}. \tag{2.3.4}$$

From (2.3.4) we obtain $a_{11}a_{12} = -(a_{11}a_{12})^\pi = a_{12}a_{11}^\pi$ for $a_{ij} \in C_{ij}$, whence

$$a_{11}b_{11}a_{12} = a_{12}(a_{11}b_{11})^\pi = (a_{12}b_{11}^\pi)a_{11}^\pi = b_{11}a_{12}a_{11}^\pi = b_{11}a_{11}a_{12}.$$

Similarly, $a_{21}a_{11}b_{11} = a_{21}b_{11}a_{11}$, thus

$$[C_{11}, C_{11}] \subset C_{11}^0 := \{a \in C_{11};\ aC_{12} = 0 = C_{21}a\}.$$

C_{11}^0 is a w^*-closed *-ideal of the W^*-algebra C_{11}. Let $C_{11}^\perp$ be its complementary ideal: $C_{11} = C_{11}^0 \oplus C_{11}^\perp$. Then $C_{11}^\perp$ is abelian. Denoting by f its unit element it follows that $(f + f^\pi)[11]$ is a tripotent of U which is abelian: $U_2((f + f^\pi)[11]) = \{(a + a^\pi)[11];\ a \in C_{11}^\perp\}$. Therefore $f = 0$, $C_{11}^\perp = 0$ and $C_{11} = C_{11}^0$. But then $C_{12} = 0 = C_{21}$, $C = C_{11} \oplus C_{22}$.

Finally, let $D = \bigoplus D_{ij}$ be the associative Peirce decomposition of D with respect to $(d, d^\pi, 1 - c)$. We proved $D_{12} = 0 = D_{21}$. It is then easy to check that $K = D_{11} \oplus D_{13} \oplus D_{31} \oplus D_{31}D_{13}$ is a *-ideal of D satisfying $KK^\pi = 0 = K^\pi K$. Hence the w^*-closure of K is an ideal as required in (i). $\square$

We will give two applications of Lemma 2.3. One concerns the uniqueness of the decomposition (2.1.1) or more generally (1.20.1)—see §4. The second application deals with the structure of $U_2(e)$ where e is a maximal tripotent in a general continuous JBW^*-triple U. In the next lemma we will find that it is enough to consider two types of maximal tripotents: unitary tripotents (then $U = U_2(e)$ is known by (2.1)) or *maximal faithful tripotents*, i.e. maximal tripotents e such that $I \cap U_1(e) \neq 0$ for every nonzero ideal I in U.

(2.4) LEMMA. *Every JBW^*-triple U is a JB^*-sum*

$$U = U_u \oplus^\infty U_f \tag{2.4.1}$$

where U_u contains a unitary tripotent and U_f a maximal faithful tripotent.

PROOF. Let e be a maximal tripotent in U. Then $Id = p_2^e + p_1^e$ where $p_2^e = Id - L(e,e)$ resp. $p_1^e = Id - 2L(e,e)$ are the Peirce projections onto the Peirce spaces $U_2(e)$ resp. $U_1(e)$. Hence

$$I = I \cap U_2(e) \oplus I \cap U_1(e), \qquad I \cap U_j(e) = p_j^e(I) \tag{2.4.2}$$

for every ideal I in U. Let $(I_j)_{j\in J}$ be a maximal family of ideals in U with $I \cap U_1(e) = 0$ and let U_u be the w^*-closure of the linear span of $(I_j)_{j\in J}$. Then U_u is an ideal of U contained in $U_2(e)$ as follows from (2.4.2) and the fact that $U_2(e)$ is w^*-closed. Let U_f be the ideal in U complementary to U_u (which exists by [**14**, Theorem 4.2]) and let $e = e_u + e_f$ be the corresponding decomposition of e. Then by construction, e_u is unitary in U_u and e_f is faithful in U_f. □

(2.5) EXAMPLE. We will show by an example that the decomposition (2.4.1) is in general not unique. Let H be an infinite-dimensional Hilbert space, $\mathcal{L}(H)$ the bounded operators on H and $U = \mathrm{Mat}(1,2;\mathcal{L}(H))$. This is a JBW^*-triple (as one sees for example by identifying U with the upper right corner in $\mathrm{Mat}(3,3;\mathcal{L}(H))$). It has $e = (\mathrm{id}, 0)$ as a faithful maximal tripotent. On the other hand, U contains a unitary tripotent: There exists a surjective isometry $\Phi\colon H \to H \oplus H$ and, interpreting U as bounded operators from $H \oplus H$ to H, the map $U \to \mathcal{L}(H)\colon T \to T\cdot\Phi$ becomes an isometric triple isomorphism. We remark in passing that this example also destroys the popular belief that the Jordan triple $\mathrm{Mat}(1,2;C)$, C an associative algebra with involution, does not contain invertible elements). □

By the results proven so far it is now enough to consider continuous JBW^*-triples containing a maximal faithful tripotent, if we want to classify continuous triples in general. In the next theorem we classify the Peirce-2-spaces of maximal faithful tripotents.

(2.6) THEOREM. *Let U be a continuous JBW^*-triple and $e \in U$ a maximal faithful tripotent. Then $U_2(e)$ is isometrically isomorphic to* $\mathrm{Mat}(4,4;C)$ *for a W^*-algebra C.*

PROOF. Let $U_2(e) \cong \mathrm{Mat}(4,4;C) \oplus H_4(D,\pi,^*)$ as in (2.1). Let $g \in U_2(e)$ be the tripotent corresponding to the unit matrix in $\mathrm{Mat}(4,4;C)$ and $(h_{ij};\ 1 \le i \le j \le 4) \subset U_2(e)$ the hermitian grid corresponding to the hermitian matrix units in $H_4(D,\pi,^*)$. Let $U = \bigoplus U_{ij}$ be the Peirce decomposition with respect to the orthogonal system $(h_{11}, h_{22}, h_{33}, h_{44}, g)$. Since $h_{11} + h_{22} + h_{33} + h_{44} + g = e$ we have $U_{00} = 0$. We claim $U_{i0} = 0$, $1 \le i \le 4$.

Assume otherwise, say $U_{10} \ne 0$. Choose a nonzero tripotent $f \in U_{10}$. Then $U_2(f) \subset U_{10}$ (since $U_{00} = 0$), $f \in U_1(h_{1i})$, $1 \le i \le 4$, thus f and h_{1i} are compatible, whence $h_{1i} = h_{1i}^1 + h_{1i}^0$, $h_{1i}^j \in U_2(h_{1i}) \cap U_j(f)$, is a decomposition into orthogonal tripotents with $h_i^1 \top f$. It is

$$\{fh_{11}^1\{h_{12}^1 f h_{11}^1\}\} - \{h_{12}^1 f\{fh_{11}^1 h_{11}^1\}\} = \{\{fh_{11}^1 h_{12}^1\} f h_{11}^1\} - \{h_{12}^1\{h_{11}^1 ff\} h_{11}^1\}$$

by (1.2), thus $\{h_{11}^1 h_{11}^1 h_{12}^1\} = h_{12}^1$ (since $\{h_{11}^1 h_{11}^1 f\} = f$), $\{h_{12}^1 h_{11}^1 f\} \in U_{20} \cap U_2(f) = 0$, $\{h_{11}^1 f h_{12}^1\} = 0$ and similarly $\{h_{12}^1 h_{12}^1 h_{11}^1\} = h_{11}^1$, so the tripotents h_{11}^1,

h_{12}^1 are collinear. They are even rigid collinear since $U_{11} = U_{11} \cap U_1(f) \oplus U_{11} \cap U_0(f)$, whence $P(h_{12}^1)U_{11} \subset U_{22} \cap (U_1(f) \oplus U_2(f)) = 0$, thus $U_2(h_{11}^1) \cap U_2(h_{12}^1) = 0$, which implies rigid collinearity. Also, the condition (2.3.1) is fulfilled. This follows from

$$\begin{aligned}(h_{11}^1)^* &= P(e)h_{11}^1 = P(h_{11})h_{11}^1 = h_{11}^1,\\ (h_{11}^1)^2 &= P(h_{11}^1)h_{11} = h_{11}^1,\\ (h_{12}^1)^* &= P(h_{12})h_{12}^1 = h_{12}^1,\end{aligned}$$

and (since the product in $D = U_{12}$ has the description $ab = \{\{ah_{12}h_{13}\}h_{13}b\}$),

$$(h_{12}^1)^2 = \{\{h_{12}^1 h_{12} h_{13}\} h_{13} h_{12}^1\} = h_{12}^1,$$

because (h_{12}^1, h_{13}^1) and (h_{12}^0, h_{13}^0) are two collinear families which are mutually orthogonal, as follows from the next lemma.

We can now apply Lemma 2.3: Since $f \neq 0$, the rigid collinear tripotents are nonzero, whence D does not satisfy the condition (2.1.2), a contradiction. It follows that $I = \bigoplus_{1 \le i \le j \le 4} U_{ij}$ is an ideal of U with $I \cap U_1(e) = 0$. So faithfulness forces $I = 0$, hence $U = U_{55} \oplus U_{50}$ which implies the result. □

In the proof of the previous theorem the following lemma was used.

(2.7) LEMMA. *Let U be a Jordan-*-triple. Let (h, g) be collinear tripotents, $h \in U_1(g)$ and $g \in U_1(h)$, and f a tripotent satisfying $U_2(f) \subset U_1(h) \cap U_1(g)$ and $\{fhg\} = 0$. Then*

$$\begin{aligned} h &= h_1 + h_0 \quad \text{with } h_i \in U_2(h) \cap U_i(f),\\ g &= g_1 + g_0 \quad \text{with } g_i \in U_2(g) \cap U_i(f)\end{aligned} \tag{2.7.1}$$

such that (h_1, g_1, f) and (h_0, g_0) are collinear families with $(h_1, g_1, f) \perp (h_0, g_0)$.

PROOF. (h, f) and (g, f) are compatible which implies the decompositions (2.7.1) using $U_2(f) \subset U_1(h) \cap U_1(g)$. It follows $h_1 \perp h_0, g_1 \perp g_0$ and $h_1 \top f \top g_1$. Since

$$0 = \{fhg\} = \{fh_1g\} = \{fh_1g_1\} \oplus \{fh_1g_0\} \in U_2(f) \oplus U_1(f)$$

we obtain

$$\{fh_1g_1\} = 0 = \{fh_1g_1\}$$

and so

$$\begin{aligned}\{g_0g_0h_1\} &= \{g_0g_0\{ffh\}\} = \{\{hff\}g_0g_0\}\\ &= (\text{by } (1.2)) \quad \{hf\{fg_0g_0\}\} + \{f\{fhg_0\}g_0\} - \{fg_0\{hfg_0\}\} = 0\end{aligned}$$

(because $f \perp g_0$ and $\{fhg_0\} = \{fh_1g_0\} = 0$). It follows $g_0 \perp h_1$, so by symmetry $g_1 \perp h_0$ which implies $g_1 \top h_1$ and $g_0 \top h_0$. □

3. Continuous JBW^*-triples with maximal faithful tripotents. In this section we will prove

(3.1) THEOREM. *Let U be a continuous JBW^*-triple containing a maximal faithful tripotent. Then U is isometrically isomorphic to pA where A is a continuous W^*-algebra and $p = p^* = p^2$ a projection in A.*

We note that (3.1) together with (2.4) and (2.1) easily gives the classification of continuous JBW^*-triples as stated in (1.20). The proof of (3.1) will occupy the whole section. We start out by proving (or stating) some lemmata. For any subset X of a JBW^*-triple U we denote $\bar{X} = w^*$-closure of X in U. For subspaces V_i of U we denote by $\{V_1V_2V_3\}$ the linear subspace spanned by all products $\{v_1v_2v_3\}$ with $v_i \in V_i$.

(3.2) IDEAL-LEMMA. *Let U be a JBW^*-triple and $e \in U$ a tripotent.*

(a) *For every w^*-closed ideal $I \lhd U_2(e)$ there exists a w^*-closed ideal $J \lhd U$ with $J \cap U_2(e) = I$.*

(b) *If e is maximal and $I \lhd U_1(e)$ is a w^*-closed ideal, then $\overline{\{IIe\}} \oplus I$ is a w^*-closed ideal in U.*

(c) *If e is a faithful maximal tripotent, then $U_2(e) = \overline{\{U_1(e)U_1(e)e\}}$.*

(d) *If in* (c) *the Peirce space $U_1(e)$ is a direct sum of two w^*-closed ideals, $U_1(e) = I_0 \oplus^\infty I_1$, then the same holds for U, $U = (\overline{\{I_0I_0e\}} \oplus I_0) \oplus^\infty (\overline{\{I_1I_1e\}} \oplus I_1)$.*

PROOF. (a) and (b) can be proven algebraically using [**25**, 2.12], a functional analytic proof can be found in [**14**]. For (c) we let $U_i = U_i(e)$ and observe that

$$U = (\overline{\{U_1U_1e\}} \oplus U_1) \oplus J, \qquad J = \bar{J} \lhd U,$$

by (b) and the fact that every w^*-closed ideal of U is complemented. Since $J = (J \cap U_2) \oplus (J \cap U_1)$ we get $J \subset U_2$, thus $J = 0$ by faithfulness of e.

(d) By [**17**, (1.11)] we know $L(I_0, I_1) = 0 = L(I_1, I_0)$, then

$$\begin{aligned} L(I_0, \{I_1I_1e\}) &= L(I_0, \{eI_1I_1\}) - L(\{I_1eI_0\}, I_1) \\ &= (\text{by } (1.2)) \quad [L(I_0, I_1), L(I_1, e)] = 0 \end{aligned}$$

follows, hence $L(\{I_0I_0e\}, I_1) = L(I_1, \{I_0I_0e\})^* = 0$ by symmetry in 1 and 0. By (1.2) $\{I_0e\{I_1I_1e\}\} = 0$ whence

$$L(\{eI_0I_0\}, \{I_1I_1e\}) = (\text{by } (1.2)) \quad [L(e_1, I_0), L(I_0, \{I_1I_1e\})] = 0.$$

Thus, the two w^*-closed ideals $\tilde{I}_0 = \overline{\{I_0I_0e\}} \oplus I_0$ and $\tilde{I}_1 = \overline{\{I_1I_1e\}} \oplus I_1$ (by (b)) are orthogonal: $L(\tilde{I}_0, \tilde{I}_1) = 0$. Using (c) the claim now easily follows. □

The following obvious reduction principle will be very useful in establishing (3.1).

(3.3) LEMMA. *Theorem* (3.1) *holds for U, if $U = \bigoplus_{i\in I}^\infty U_i$ is an l^∞-sum of JBW^*-triples U_i such that* (3.1) *holds for each U_i.*

In proving (3.1) we will decompose U into three ideals, for each of which the following lemma can be applied:

(3.4) LEMMA. *Let U be a continuous JBW^*-triple containing a maximal faithful tripotent p such that*

(i) *U is a w^*-closed subsystem of a W^*-algebra B, considered as a JBW^*-triple, p is a projection in B: $p = p^* = p^2$ and*

(iii) *$U_2(p)$ is a subalgebra of B: $x, y \in U_2(p) \Rightarrow xy \in U_2(p)$.*

Then (3.1) *holds for U.*

PROOF. The first step of the proof consists in showing that for any $x \in U_1(p)$ both associative Peirce components of x also lie in U, i.e.

$$x \in U_1(p) \Rightarrow px \text{ and } xp \in U_1(p)$$

By our assumptions $U_2(p)$ is a W^*-algebra without abelian projections. It is therefore covered by a family $(p_{11}, p_{12}, p_{22}, p_{21})$ of (associative) rectangular matrix units such that $p = p_{11} + p_{22}$. Then

$$U_1(p) \ni \{p_{21}p_{11}\{p_{12}px\}\} = \{p_{21}, p_{11}, p_{12}x + xp_{12}\} = p_{21}p_{11}(p_{12}x + xp_{12}) = p_{22}x$$

since

$$p_{21}xp_{12} = (p_{21}xp_{12} + p_{12}xp_{21})p_{22} = [P(p_{21}, p_{12})P(p)x]p_{22} = 0$$

because $P(p)x \in P(U_2(p))U_1(p) \subset U_0(p) = 0$. By symmetry, $p_{11}x \in U_1(p)$ whence $px = (p_{11} + p_{22})x \in U_1(p)$ and $xp \in U_1(p)$ because $x = \{ppx\} = px + xp$.

Let $B = B_{11} \oplus B_{10} \oplus B_{01} \oplus B_{00}$ be the associative Peirce decomposition with respect to p. Then

$$U = U_{11} \oplus U_{10} \oplus U_{01}, \qquad U_{ij} = U \cap B_{ij}$$

where $U_{11} = U_2(p)$ and $U_{10} \oplus U_{01} = U_1(p)$ (the spaces U_{01} and U_{10} should not be confused with Jordan Peirce spaces). The associative multiplication rules show $U_1(p) = U_{10} \oplus U_{01}$ is a direct sum of w^*-closed ideals, whence, by (3.2.d),

$$(*) \qquad U = (U_{10} \oplus X) \oplus (U_{01} \oplus Y)$$

for Jordan triple ideals $X = \overline{U_{10}U_{10}^*}$, $Y = \overline{U_{01}^*U_{01}}$ of $U_2(p)$. It easily follows that X and Y are *-subalgebras of B. Indeed, regarding X it is enough to show that a product with factors in the w^*-dense set $U_{10}U_{10}^*$ stays in X. But such a product is a sum of products of type

$$(x_{10}y_{10}^*)(v_{10}w_{10}^*) = x_{10}y_{10}^*\{v_{10}w_{10}p\} = \{x_{10}y_{10}\{v_{10}w_{10}p\}\} \in X \quad \text{by } (*).$$

The proof for Y is similar. By what we have shown so far, the projection p decomposes as a sum of two projections, $p = p_X + p_Y$ with $X = U_2(p_X)$, $Y = U_2(p_Y)$. All assumptions (i)–(iii) are now also valid for the two ideals $X \oplus U_{10}$ and $Y \oplus U_{01}$. By (3.3) it is therefore enough to prove (3.1) for each of these ideals. Since the case of Y becomes the same as that of X if one passes from B to B^{op}, we are left with the case of X.

We may now assume $U = U_{11} \oplus U_{10}$, $U_{11} = \overline{U_{10}U_{10}^*}$. Also, we know that U_{11} is a subalgebra of B. Moreover, we have $x_{11}x_{10} = \{x_{11}px_{10}\} \in U_{10}$ and $x_{10}y_{10}^*z_{10} \in U_{11}U_{10} \subset U_{10}$. Using these rules it is easy to see that

$$A = U_{11} \oplus U_{10} \oplus U_{10}^* \oplus \overline{U_{10}^*U_{10}}$$

is a w^*-closed subalgebra of B, with $pA = U$. Let $A = A_I \oplus A_c$ be the W^*-algebra decomposition of A into type I and continuous part and decompose correspondingly $p = p_I \oplus p_c$. Then $U = p_IA_I \oplus p_cA_c$. Since the W^*-algebra $p_IA_Ip_I$ contains abelian tripotents, we have $U = pA_c$, i.e. (3.1) holds. □

(3.5) LEMMA. *Let U be a JBW^*-triple with a maximal faithful tripotent e such that $U_2(e)$ is covered by a rectangular grid $\mathcal{R} = (c_{11}, c_{12}, c_{22}, c_{21})$ of type $\mathcal{R}(2,2)$,*

$$(3.5.1) \qquad U_2(e) = U_{(2101)} \oplus U_{(1210)} \oplus U_{(0121)} \oplus U_{(1012)},$$

where $U_{(ijkm)} = U_i(c_{11}) \cap U_j(c_{12}) \cap U_k(c_{22}) \cap U_m(C_{21})$ are the Peirce spaces of the quadrangle $\mathcal{R}$. Then there exists another rectangular grid $\mathcal{R}' = (c_{11}, c_{12}^1, c_{22}, c_{21}^1)$ of type $\mathcal{R}(2,2)$ which also covers $U_2(e)$, i.e. (3.5.1) holds, and in addition satisfies

$$U_1(e) = U'_{(1100)} + U'_{(0011)}$$

where $U'_{(ijkl)}$ are the Peirce spaces of $\mathcal{R}'$.

PROOF. We specialize McCrimmon's Quadrangle Decomposition Theorem [**29**, I.2.2] to our situation noting that all Peirce spaces $U_{(ijkl)}$ vanish if $(ijkl)$ contains two 2's (by rigidity) or if $(ijkl) = (1111)$ (since $U_{(1111)} \subset U_2(e)$) or if $(ijkl) = (0000)$ (by maximality of e). Thus $U_1(e) = I_0 \oplus I_1$ where $I_0 = U_{(1001)} \oplus U_{(0110)}$, $I_1 = U_{(1100)} \oplus U_{(0011)}$. It is straightforward to check that $L(I_0, I_1) = 0$:

$$L(x_{(1001)}, y_{(1100)}) = (\text{by [\textbf{29}, (I.1.27)]})\ L(\{x_{(1001)}y_{(1100)}c_{12}\}, c_{12}) = 0$$

since $\{x_{(1001)}y_{(1100)}c_{12}\} \in U_{(1111)} = 0$. The other L's vanish by symmetry. Then also $L(I_1, I_0) = L(I_0, I_1)^* = 0$, and I_0, I_1 are two complementary ideals in $U_1(e)$. They are w^*-closed, since for example $I_0 = \{x \in U_1(e); \{I_1 I_1 x\} = 0\}$. By (3.2.d) the whole triple system splits, $U = \tilde{I}_0 \oplus \tilde{I}_1$, and correspondingly, $c_{ij} = c_{ij}^{(0)} + c_{ij}^{(1)}$. Clearly, $\{c_{ij}^{(\mu)};\ 1 \le i, j \le 2\}$, $\mu = 0$ or 1, is a quadrangle (possibly zero) in $\tilde{I}_\mu$, and, by construction, the claim holds for $\tilde{I}_1$ and also for $\tilde{I}_0$ if we exchange $c_{12}^{(0)}$ and $c_{21}^{(0)}$. Thus, in total, c_{11}, $c_{12}^1 = c_{12}^{(1)} + c_{21}^{(0)}$, c_{22}, $c_{21}^1 = c_{21}^{(1)} + c_{12}^{(0)}$ is a quadrangle of the form claimed. □

PROOF OF (3.1). It follows from [**3**, Theorem 6] that U is a w^*-closed subsystem of $B = \mathcal{L}(H)$ for some complex Hilbert space H,

(1) $$U < B, \qquad U\ w^*\text{-closed}.$$

Let $e \in U$ be a maximal tripotent of U. By [**14**] there exists a maximal tripotent $q \in B$ such that e is a q-projection, i.e. $e = qe^*q = eq^*e \in U_2(q)$. By (1.21)(iii) we have central projections z_i satisfying

$$1 = z_1 + z_2, \quad z_1 z_2 = 0, \quad (qz_1)(qz_1)^* = z_1, \quad (qz_2)^*(qz_2) = z_2.$$

To derive (2) below we may decompose $B = B_1 \oplus B_2$, $B_i = Bz_i$, and look at the ideals B_i separately. We do the case $B = B_1$, the other case follows similarly. For $B = B_1$ we have $q^*q = 1$. Then $\Phi\colon U \to B\colon x \to xq^*$ is an injective triple homomorphism: $xq^* = 0 \Rightarrow 0 = xq^*q = x$ and $P(\Phi x)\Phi y = P(xq^*)(yq^*) = xq^*qy^*xq^* = \Phi(P(x)y)$. Moreover $\Phi(U) = Uq^*$ is w^*-closed: If $(u_\lambda q^*)$ is a net converging to a limit point of Uq^*, then $u_\lambda = u_\lambda q^* q$ has a w^*-limit $u \in U$, since $x \to xq$ is w^*-continuous, thus $(u_\lambda q^*)$ has the w^*-limit uq^*. Finally, eq^* is a projection: $(eq^*)^* = qe^* = q(q^*eq^*) = eq^*$ since $qq^*e = e$ and $(eq^*)^2 = eq^*eq^* = eq^*$. Thus, in addition to (1), we may assume

(2) $$\begin{array}{l} U \text{ contains a maximal faithful tripotent } p \\ \text{such that } p = p^2 = p^*. \end{array}$$

Then $x \in U_2(p)$ satisfies $2x = \{ppx\} = px + xp$, hence $x = xp = px$. By (2.6), $U_2(p)$ is covered by a rectangular grid of type $\mathcal{R}(4,4)$, whence also by one of type $\mathcal{R}(2,2)$. Therefore, by (3.5), we have

(3) $$\begin{array}{l} U_2(p) \text{ is covered by a rectangular grid} \\ (e_{11}, e_{12}, e_{22}, e_{21}) \text{ such that } p = e_{11} + e_{22} \text{ and} \\ U_{10} = U_1(e_{12}) \cap U_0(e_{21}),\ U_{20} = U_1(e_{21}) \cap U_0(e_{12}) \end{array}$$

where here and in the following U_{ij} are the (Jordan) Peirce spaces of U relative to (e_{11}, e_{22}). The rectangular covering grid $\mathcal{R}(2,2)$ induces on $U_2(p)$ the structure of a matrix algebra $\mathrm{Mat}(2,2;C)$ where the associative algebra C is defined on U_{11} by

$$a\pi b = \{\{ae_{11}e_{12}\}e_{12}b\} \qquad (a, b \in U_{11}).$$

It follows as in the proof of Theorem 2.1 that C and hence $\mathrm{Mat}(2,2;C)$ is a W^*-algebra. By (1) we know for $x, y \in U_2(p)$

(4) $$x\pi y + y\pi x = \{xpy\} = xy + yx$$

where the right side is computed in B. Thus the natural injection $U_2(p) \to B$ is a Jordan homomorphism. Since $(U_2(p), \pi)$ is a matrix algebra we can apply [**16**, Theorem 7]. There exist projections $e, f \in U_2(p)' = \{x \in B;\ xu = ux \text{ for all } u \in U_2(p)\}$ such that

(5) $$p = e + f, \qquad w\pi z = wze + zwf \quad \text{for all } w, z \in U_2(p).$$

The map $R_e\colon U_2(p)^{(p)} \to B^+\colon x \to xe$ is a w^*-continuous Jordan algebra homomorphism, whence $I = \ker R_e$ is a w^*-closed algebra ideal of $U_2(p)$ and $U_2(p)$ splits, $U_2(p) = I \oplus I^\perp$. We decompose p correspondingly, $p = p_I + p_I^\perp$. Then it follows from (2) that p_I and $p_I^\perp$ are projections in B. By (3.2) the splitting of $U_2(p)$ extends to a global splitting: $U = J \oplus J^\perp$, such that p_I (resp. $p_I^\perp$) is a maximal faithful tripotent in J (resp. $J^\perp$) and $J_2(p_I) = I$ is a subalgebra of $B\colon x, y \in I \Rightarrow xy \in I$. Thus, (3.1) holds for J by (3.4), and by (3.3) we can split off the ideal J. Since we made sure that all our assumptions remain valid for $J^\perp$, we can, equivalently, assume that R_e is injective. The same argument now applies to $R_f\colon U_2(p) \to B\colon x \to xf$, and after possibly splitting off another direct summand we may, in addition to (2)–(5), assume

(6) $$\begin{aligned} &R_e\colon U_2(p) \to B\colon x \to xe \quad \text{and} \\ &R_f\colon U_2(p) \to B\colon x \to xf \quad \text{are injective.} \end{aligned}$$

Let $(i,j) = (1,2)$ or $(2,1)$ and let $g \in U_{i0}$ be a nonzero tripotent. Then $(h_i = \{gge_{ii}\},\ d_i = \{gge_{ij}\},\ g)$ is a rigid-collinear family (by (2.7) and (3)) with $h_i \in U_{ii}$, $d_i \in U_2(e_{ij}) \subset U_2(p)$. For $a, b \in U_2(h)$ we claim

(7) $$a\pi b = \{\{ah_ig\}gb\}$$

expressing the W^*-algebra product π in terms of the Jordan triple product. Indeed, by (2.7), the tripotents e_{ii} and e_{ij} split as a sum of orthogonal tripotents

$$e_{ii} = h_i + h_i^\perp, \qquad e_{ij} = d_i + d_i^\perp,$$

and we have

$$a\pi b = \{\{ae_{ii}e_{ij}\}e_{ij}b\} = \{\{ahd\}db\} = \{\{ahg\}gb\} \quad \text{by [\mathbf{27}, 2.6(i)]}$$

(for $i = 1$ the first term on the right side is the definition of the π-product, for $i = 2$ this is an identity in $\mathrm{Mat}(2,2;C) \cong U_2(p)$).

Besides the Jordan Peirce spaces U_{ij} we will now also use the *associative* Peirce decomposition of B relative to the orthogonal system $(e, f, 1-p)$,

$$B = \bigoplus_{i,j=1,2,3} B_{ij}.$$

Since $e, f \in U_2(p)'$ we have

(8) $$U_2(p) \subset B_{11} + B_{22}, \qquad U_1(p) \subset B_{13} + B_{23} + B_{31} + B_{32}.$$

Our next claim is

(9) For any tripotent $g \in U_{i0}$, $i = 1, 2$, we have
$$gg^*p = e\{ggp\}e \in B_{11}, \qquad pg^*g = f\{ggp\}f \in B_{22}.$$

The proof of (9) is rather technical. We first introduce some notation.

$$r := gg^*p, \quad s := pg^*g, \quad h := r + s = \{ggp\}.$$

Since $g \in B_1(p)$ we have $gg^* \in B_2(p) \oplus B_0(p)$ whence $gg^*p = pgg^* = pgg^*p$. Similarly for g^*g. Also, $gpg \in U_0(p) = 0$. It follows

(9_1) $\qquad r$ and s are orthogonal projections in B.

For $a, b \in U_2(h) \subset U_{ii}$ we have, by (7),

$$a\pi b = \{\{ahg\}gb\} = ah^*gg^*b + gh^*ag^*b + bg^*ah^*g + bg^*gh^*a.$$

Since $a, b \in U_2(p)$ and $g \in U_1(p)$ the associative Peirce multiplication rules show $ag^*b = 0 = bg^*a$ also $h^*gg^* = hpgg^* = hr = r$ and $g^*gh^* = s$ follows similarly, hence

(9_2) $$a\pi b = arb + bsa, \qquad a, b \in U_2(h).$$

Comparing with (4) shows

(9_3) $$abe + baf = arb + bsa,$$

multiplying (9_3) with $L(e)$ and $R(e)$ and using $ef = 0$, $e, f \in U_2(p)'$ gives

(9_4) $$abe = a(ere)b + b(ese)a.$$

Note $(ere)h = erhe = ere$ and $hese = ese$. Therefore, (9_4) evaluated for $b = h$, says $ahe = a(ere) + (ese)a$. But

$$ahe = aehe = ae(r + s)e = aere + aese$$

whence $a(ese) = (ese)a$. Also,

$$\begin{aligned} ahe &= (a\pi h)e \quad \text{(by (4))} \\ &= (h\pi a)e \quad (\text{since } h = 1 \text{ in } U_2(h)) \\ &= hae = eha = e(r + s)a = (ere)a + (ese)a, \end{aligned}$$

whence $(ere)a = a(ere)$ and (9_4) becomes

$$abe = ab(ere) + ba(ese),$$

or, because $abe = abe^2 = (ae)(be)$,

(9_5) $$\tilde{a}\tilde{b} = \tilde{a}\tilde{b}(ere) + \tilde{b}\tilde{a}(ese), \qquad \tilde{a}, \tilde{b} \in U_2(h)e.$$

Since $U_2(h)$ is a π-subalgebra of $U_2(p)$ by (7) and since R_e is an injective w^*-continuous algebra homomorphism we conclude that $U_2(h)e$ is a w^*-closed subalgebra of B without abelian projections, which is therefore covered by associative 2×2 matrix units, say $(c_{11}, c_{12}, c_{21}, c_{22})$ satisfying

$$c_{11} + c_{22} = he \qquad (= \text{unit element of } U_2(h)e).$$

It follows

$$c_{12} = c_{11}c_{12} = c_{11}c_{12}ere \quad (\text{by } (9_5)) = c_{12}ere,$$

and analogously

$$\begin{aligned} c_{21} &= c_{21}ere, \\ c_{11} &= c_{12}c_{21} = c_{12}c_{21}ere = c_{11}ere, \\ c_{22} &= c_{22}ere, \end{aligned}$$

thus

$$he = c_{11} + c_{22} = (c_{11} + c_{22})ere = here = ehre = ere.$$

On the other hand, $he = ehe = ere + ese$, whence $ese = 0$. Since $s \in B_2(p)$ we have

$$s = psp = (e+f)s(e+f) = fse + esf + fsf.$$

By (9_1) $s^2 = s$, which implies

$$0 = (esf)(fse) = (esf)(esf)^*,$$

i.e. $esf = 0 = fse$. Therefore $pg^*g = s = fsf \in B_{22}$. The same argument using (f, r, s) instead of (e, s, r) yields $r = ere \in B_{11}$, and therefore

$$gg^*p = r = ere = ehe = e\{ggp\}e, \quad pg^*g = s = fsf = fhf = f\{ggp\}.$$

Thus, (9) is proven.

We will now prove (9) for arbitrary elements x in U_{i0}. By [**14**, (3.12)] there exists a tripotent $g \in U_{i0}$ such that $x \in U_2(g)$, i.e. $x = gg^*xg^*g$. It follows

$$xx^*p = pxx^*p = p(gg^*xg^*g)(g^*gx^*gg^*)p = gg^*pxg^*gx^*gg^*p \in eBe = B_{11} \quad \text{by (9)}$$

and similarly, $px^*x \in fBf = B_{22}$. Hence

$$xx^*p = e\{xxp\}e, \quad px^*x = f\{xxp\}f \qquad (x \in U_{i0}). \tag{10}$$

The next step is to establish

$$U_1(p) \subset B_{13} \oplus B_{32}, \tag{11}$$

i.e.

$$x = ex(1-p) + (1-p)xf \quad \text{for } x \in U_1(p). \tag{11$'$}$$

Since $(11')$ is linear in x it is enough to verify it for $x \in U_{i0}$, $i = 1$ or 2. By (8) we know the associative Peirce decomposition of x: $x = x_{13} + x_{23} + x_{31} + x_{32}$, whence

$$\begin{aligned} e\{xxp\}e &= xx^*p \quad \text{(by (10))} \\ &= xx^*(e+f) = x(x_{13}^* + x_{23}^*) = x_{13}x_{13}^* \oplus x_{13}x_{23}^* \oplus x_{23}x_{13}^* \oplus x_{23}x_{23}^* \in B_{11}, \end{aligned}$$

therefore $x_{23}x_{23}^* = 0$, $x_{23} = 0$. In the same way we derive $x_{31} = 0$ using $px^*x = f\{pxx\}f$.

The final step is to show that $\Phi_e \colon U \to B \colon u \to eu = u_{11} + u_{13}$ maps U isometrically onto a subsystem of U to which we can apply (3.4).

$$\Phi_e \text{ is a Jordan triple homomorphism,} \tag{12}$$

since it acts homomorphically on all four summands of the general product in U: for $a, b \in U_2(p)$ and $x, y \in U_1(p)$ we have

$$P(a+x)(b+y) = P(a)b + \{xya\} \oplus \{abx\} + P(x)y,$$

and

$$P(ea)(eb) = eab^*eea = e(ab^*a) \quad (\text{since } e \in U_2(p)') = eP(a)b,$$

$$\begin{aligned} \{ex, ey, ea\} &= exy^*ea + eay^*ex = exy^*ae \quad (\text{since } eay^*ex = a_{11}y_{13}^*x = 0) \\ &= e(xy^*a + ay^*x)e \quad (\text{since } eay^* = aey^* = 0 \text{ by (11)}) \\ &= e\{xya\}e = e\{xya\} \quad (\text{since } e \text{ commutes with } \{xya\} \in U_2(p)), \end{aligned}$$

$$\{ea, eb, ex\} = eab^*ex + exb^*ea = eab^*x + exeb^*a = eab^*x \quad (\text{since } exe = 0 \text{ by } (11))$$
$$= e(ab^*x + xb^*a) \quad (\text{since } ex \in U_2(p) \in x_{13}U_2(p) = 0) = e\{abx\}$$

and

$$P(ex)ey = x_{13}y_{13}^*x_{13} = x_{13}y^*x = exy^*x.$$

Therefore (12) is proven. Consequently, $\ker \Phi_e$ is a Jordan triple ideal in U and therefore $\ker \Phi_e = \bigoplus_{i,j}(U_{ij} \cap \ker \Phi_e)$. By (5), $U_2(p) \cap \ker \Phi_e = 0$ and for $x \in U_{i0} \cap \ker \Phi_e$ we get $x = x_{32}$ by (11), hence $0 = xx^*p = e\{xxp\}$ by (10) and $P(x)x = \{\{xxp\}px\} = 0$, thus $x = 0$ by (1.5) and we have

(13) Φ_e is injective.

Since $\Phi_e(u) = u_{11} + u_{13}$ and U is w^*-closed, also $\Phi_e U$ is w^*-closed, clearly ep is a projection, and, by (4), $eU_2(p)$ is a subalgebra of B. Therefore, by (3.4), (3.1) holds for $\Phi_e U$, proving (3.1) in full generality. □

4. Uniqueness. By our general classification theorem every continuous JBW^*-triple U is an l^∞-sum of ideals, $U = U_{as} \oplus U_{\text{herm}}$ where U_{as} is isometrically isomorphic to a w^*-closed right ideal in a continuous W^*-algebra and U_{herm} is isometrically isomorphic to a hermitian matrix triple $H_4(D, \pi, ^*)$ as in (2.1)(ii). This naturally leads to the question

(4.1.?) Are the ideals U_{as} and U_{herm} unique?

To answer this question affirmatively is the object of this section. Of course, there are other uniqueness questions, like

(4.2.?) Are different coordinate systems $(D, \pi, ^*)$ for U_{herm} isomorphic? Answer: Yes, since all three data $(D, \pi, ^*)$ can be internally described in Jordan terms—see the Hermitian Coordinatization Theorem [**27**].

(4.3.?) To which extent is the associative structure on U_{as} unique?

To state this last question more clearly, we recall the following concept, due to O. Loos [**20**, **21**]: An *associative triple system* is a K-vector space V together with a K-trilinear map $V \times V \times V \to V: (x, y, z) \to \langle xyz\rangle$ satisfying

(4.4) $$\langle uv\langle xyz\rangle\rangle = \langle u\langle yxv\rangle z\rangle = \langle\langle uvx\rangle yz\rangle$$

for all $u, v, x, y, z \in V$. We call U an *associative B^*-* (resp. *BW^*-)triple*, if U is a JB^*- (resp. JBW^*-)triple which carries an associative triple structure $\langle\cdots\rangle$ satisfying

(4.5.1) $\langle\cdots\rangle$ is $\mathbf{C}$-linear in the two outer variables and $\mathbf{C}$-antilinear in the middle variable,

(4.5.2) $$P(x)y = \langle xyx\rangle \quad \text{for all } x, y \in U.$$

Obviously, any C^*- (W^*-) algebra or any norm (w^*-) closed subspace U satisfying $UU^*U \subset U$ is an associative B^*- (BW^*-)triple. In particular, this holds for w^*-closed right ideals, thus, by pulling back the associative triple structure, U_{as} becomes an associative BW^*-triple. Of course, different imbeddings give rise to different associative triple structures, at least initially. Thus, a more precise form of question (4.3.?) is

(4.3′.?) How do different associative triple structures on an associative BW^*-triple compare?

The concept of an associative BW^*-triple does not only provide a good framework to handle the uniqueness question of the U_{as}-part, it can also be used to show uniqueness in general. We will need the following two results.

(4.6) LEMMA. *Suppose* $U = H_n(D,\pi,^*)$, $n \geq 2$, *is a hermitian matrix triple and also an associative* B^**-triple. Then condition* (2.3)(ii) *is fulfilled*: U *contains rigid collinear tripotents* $(c[11], d[12])$ *with* $c = c^\pi = c^* = c^2$, $d = d^* = d^2$.

PROOF. Let c be the unit of D. Then $(c[11], \ldots, c[nn])$ is an orthogonal system of tripotents, hence an orthogonal system of idempotents with respect to the associative triple structure [**21**, 5.1]. By [**17**, 5.4] the Jordan Peirce space $U_{12} = C[12]$ splits: $U_{12} = A_{12} \oplus A_{21}$ where A_{12} and A_{21} are orthogonal associative triple ideals whence also orthogonal Jordan ideals. Let $c[12] = d_{12} \oplus d_{21}$. Then (d_{12}, d_{21}) are orthogonal tripotents such that $d_{12} + d_{21}$ and $c[11] + c[22]$ have the same Peirce spaces, therefore $U_{11} \oplus U_{22} \subset U_1(d_{12}) \cap U_1(d_{21})$ and $(c[11], d_{12})$ are rigid collinear. For $d_{12} = d[12]$ we have

$$d^2[12] = P(d[12])c[12] = P(d_{12})(d_{12} + d_{21}) = d_{12},$$

i.e. $d^2 = d$, and

$$d^*[12] = P(c[12])d[12] = P(d_{12} + d_{21})d_{12} = d_{12}$$

i.e. $d^* = d$. Obviously, $c = c^\pi = c^* = c^2$. □

(4.7) LEMMA. *In an associative* BW^**-triple the* w^**-closed Jordan ideals coincide with the* w^**-closed associative triple ideals.*

PROOF. Let I be a w^*-closed ideal. If I is an associative triple ideal, it is obviously also Jordan. Therefore we assume that I is a Jordan ideal of U.

We know $U = I \oplus I^\perp$ where the w^*-closed Jordan ideal $I^\perp$ has the description $I^\perp = \bigcap\{U_0(g);\ g \text{ is a tripotent in } I\}$ [**29**, IV, 3.5]. Since $U_0(g) = \{x \in U;\ \langle ggx\rangle = 0 = \langle xgg\rangle\}$ [**21**] is an associative subsystem, $I^\perp$ (and then also $I = (I^\perp)^\perp$) is an associative subsystem. Thus it remains to be shown that the mixed associative triple products vanish. Let $a, b \in I$ and $x \in I^\perp$. We decompose a and b relative to a fixed maximal tripotent e of I: $a = a_{11} + a_{10} + a_{01}$, $b = b_{11} + b_{10} + b_{01}$ with $a_{ij}, b_{ij} \in U_{ij} = \{u \in U;\ \langle eeu\rangle = iu,\ \langle uee\rangle = ju\} \subset I$. We have $x = x_{00}$. Using the associative Peirce multiplication rules [**21**] we obtain

$$\begin{aligned}\langle abx\rangle &= \langle ab_{01}x_{00}\rangle = \langle a_{11}b_{01}x_{00}\rangle + \langle a_{01}b_{01}x_{00}\rangle \\ &= \{a_{11}b_{01}x_{00}\} + \{a_{01}b_{01}x_{00}\} = 0, \\ \langle axb\rangle &= \langle a_{10}x_{00}b_{01}\rangle = \{a_{10}x_{00}b_{01}\} = 0.\end{aligned}$$

By changing from $\langle\cdots\rangle$ to $\langle\cdots\rangle^{\mathrm{op}}$ ($\langle xyz\rangle^{\mathrm{op}} = \langle zyx\rangle$), we also have $\langle xba\rangle = 0$. Therefore, by symmetry between I and $I^\perp$, all mixed products vanish. □

(4.8) UNIQUENESS THEOREM. *Let*

$$U = A \oplus^\infty H_4(D,\pi,^*) \tag{4.8.1}$$

be a continuous JBW^**-triple where* A *is an associative* BW^**-triple and* $H_4(D,\pi,^*)$ *is a hermitian matrix system as in* (2.1)(ii). *Then the decomposition* (4.8.1) *is unique.*

PROOF. Let $U = \tilde{A} \oplus H_4(D,\pi,^*)$ be a second decomposition. Then $\tilde{A} = \tilde{A} \cap A \oplus \tilde{A} \cap H_4(D,\pi,^*)$ and $V := \tilde{A} \cap H_4(D,\pi,^*) = H_4(C,\pi,^*)$ for a w^*-closed

*-ideal $C = C^\pi$ of D, since V is an ideal of $H_4(D, \pi, ^*)$. Note that C inherits the condition (2.1.2) from D. On the other hand, V is a w^*-closed Jordan ideal of $\tilde{A}$, whence an associative BW^*-triple by (4.7). Then (4.6) and (2.3) imply $C = 0$, i.e. $\tilde{A} \subset A$. By symmetry $\tilde{A} = A$, and uniqueness follows since complementary ideals in JBW^*-triples are unique. □

5. Types of continuous JBW^*-triples. By (1.20) every continuous JBW^*-triple U is isometrically isomorphic to an l^∞-sum $U = U_{\text{herm}} \oplus^\infty U_{as}$, where $U_{\text{herm}} = H(B, \alpha)$ is the fixed point space of a $\mathbf{C}$-linear involution α of a continuous W^*-algebra B and U_{as} is a w^*-closed right ideal in a continuous W^*-algebra. By (4.8) this decomposition is unique. In this section we will define types of JBW^*-triples and show that U_{herm} and U_{as} can be further decomposed as an l^∞-sum of the various types.

Since the ideals U_{herm} and U_{as} are unique, we can consider them separately. We begin with $U_{\text{herm}} = H(B, \alpha)$. In a natural way, the JBW^*-triple U_{herm} is a w^*-closed and α-closed subalgebra of B^+, therefore a JW^*-algebra, i.e. a w^*-closed Jordan-*-subalgebra of $\mathcal{L}(H)$, H a complex Hilbert space. Obviously, every unitary tripotent $u \in U_{\text{herm}}$ gives rise to a JW^*-algebra structure on U_{herm} (via the Jordan algebra product $a \cdot b = \{aub\}/2$ and the involution $a^* = P(u)a$), and these JW^*-algebra structures are nonisomorphic in general. Nevertheless, we will see in (5.3) that the type of the W^*-algebras generated by the various JW^*-algebra realizations of $U_{\text{herm}} \subset \mathcal{L}(H)$ is an invariant. We will use that the selfadjoint parts of JW^*-algebras are precisely the JW-algebras studied by Topping in [**33**]. He defined the types II_1, II_∞ and III of JW-algebras in terms of the lattice of projections (see also [**12**, 5.1.5, 5.1.6]). Keeping in mind that a continuous JW-algebra is always reversible [**12**, 5.3.10], we have the following result proven by Ayupov.

(5.1) THEOREM [**2**, THEOREM 8]. *A continuous JW-algebra is of type* II_1, II_∞ *or* III *as defined in* [**33**], *if and only if the W^*-algebra generated by it is of the corresponding type.*

(5.2) LEMMA. *Let $M \subset \mathcal{L}(H)$, $N \subset \mathcal{L}(K)$ be JW^*-algebras which are isomorphic as JBW^*-triples. Let M be of type* II_1 (II_∞, III *resp.*) *Then N is of the same type as M.*

PROOF. Since JW^*-algebras are unital, we may assume that the identity operator $\mathbf{1}_H$ lies in M and also $\mathbf{1}_K \in N$. Let $\phi\colon M \to N$ be a JBW^*-triple isomorphism. Then $u = \phi(\mathbf{1}_H)$ is a unitary tripotent of N, $2n = uu^*n + nu^*u$ for all $n \in N$. Since uu^* and u^*u are projections and $n = \mathbf{1}_K \in N$, it follows $uu^* = \mathbf{1}_K = u^*u$. Define $\Psi(m) = \phi(m)u^*$, $m \in M$. Then $\Psi(m)^2 = \phi(m\mathbf{1}_H m)u^* = \Psi(m^2)$ and $\Psi(m^*) = \Psi(m)^*$ hold, i.e. $\Psi\colon M \to Nu^*$ is an isomorphism of JW^*-algebras. Since N and Nu^* generate the same W^*-algebra, the lemma follows by (5.1). □

Expressed for a JBW^*-triple U_{herm} (5.2) becomes

(5.3) COROLLARY. *Let $U = U_{\text{herm}}$ be a continuous JBW^*-triple and let u, v be unitary tripotents of U. Then the JW^*-algebra induced on U by u is of type* II_1 (II_∞, III *resp.*) *iff the JW^*-algebra induced on U by v is of type* II_1 (II_∞, III *resp.*).

It now makes sense to call $U = U_{\text{herm}}$ a JBW^*-triple *of type* II_1 (II_∞, III resp.) if the JW^*-algebra induced on U by a unitary tripotent is of type II_1 (II_∞, III resp.).

The corresponding decomposition of JW-algebras implies now

(5.4) THEOREM. *Every continuous JBW^*-triple $U = U_{\text{herm}}$ is uniquely decomposed into a direct sum of three ideals of type* II_1, II_∞ *and* III.

We will now define types for the second summand in the decomposition (1.20), the associative BW^*-triple U_{as}. We know that, modulo isometric isomorphy, $U_{as} = pA$ where p is a projection is a continuous W^*-algebra A. Obviously, we may assume that p has central support 1. To define types for U_{as} we use the theory of types of W^*-algebras, cf. [**30**].

A JBW^*-triple is called *of type* II_1^a ($\mathrm{II}_{\infty,1}^a$, II_∞^a, III^a resp.) if it is isomorphic to pA, p a projection in a W^*-algebra A, where

(5.5.1) for type II_1^a: A is of type II_1 and p is (necessarily) finite,

(5.5.2) for type $\mathrm{II}_{\infty,1}^a$: A is of type II_∞ and p is finite,

(5.5.3) for type II_∞^a: A is of type II_∞ and p is properly infinite,

(5.5.4) for type III^a: A is of type III and p is (necessarily) purely infinite.

The exponent a (for "associative") in our type notation is added to distinguish the types defined in (5.5) from the previously defined types for U_{herm}.

It is clear from the theory of W^*-algebras that U_{as} decomposes into a direct sum of four ideals of the above types. Moreover, we will show that this decomposition is unique. The following remarks and lemmata are needed to prove uniqueness.

(5.6) REMARKS. (1) It will be repeatedly used without further comment that the Peirce-2-space $U_2(q)$ of a tripotent q in a Jordan-*-triple U carries a canonical Jordan-*-algebra structure with product $(x, y) \to \{xqy\}/2$ and involution $x \to P(q)x = \{qxq\}/2$.

(2) We use the standard notation "$e \sim f$" for projections e, f in A if there is a tripotent u in A with $uu^* = e$ and $u^*u = f$. Also, "$e \leq f$" means $e = ef = fe$, and we write "$e \prec f$" if $e \sim e' \leq f$ for some $e' \in A$. By [**30**, 2.1.2], $e \prec f$ and $f \prec e$ implies $e \sim f$.

(5.7) LEMMA. *Let p be a projection in a W^*-algebra A, and let q be a tripotent which is maximal in $U = pA$. Then $U_2(p)$ and $U_2(q)$ are isomorphic as Jordan-*-algebras.*

PROOF. It is $(p - qq^*)A(1 - q^*q) = 0$ because q is maximal in U. So by [**30**, 1.10.7], $p - qq^*$ and $1 - q^*q$ are centrally orthogonal which implies

(1) There is a central projection z in A with $zqq^* = zp$ and $(1 - z)q^*q = 1 - z$. So we have

$$(1 - z) = (1 - z)q^*q \sim (1 - z)qq^* = (1 - z)pqq^*p \leq (1 - z)p \leq (1 - z),$$

hence by (5.6.2) we obtain $(1-z)qq^* \sim (1-z)p$. Together with (1) this gives $qq^* \sim p$. So there is a tripotent u such that $uu^* = qq^*$ and $u^*u = p$. Then $u^*qq^*u = p$ and $U_2(qq^*) \to U_2(p)\colon a \to u^*au$ is an isomorphism of Jordan-*-algebras. It is not hard to see that $U_2(q) \to U_2(qq^*)\colon a \to aq^*$ is also an isomorphism of Jordan-*-algebras. This concludes the proof. □

(5.8) COROLLARY. *Let A, B be W^*-algebras with projections $p \in A$, $\tilde{p} \in B$ such that pA and $\tilde{p}B$ are isomorphic Jordan-*-triples. Then $A_2(p) = pAp$ and $B_2(\tilde{p}) = \tilde{p}B\tilde{p}$ are isomorphic as Jordan-*-algebras. In particular, p is finite* *(properly infinite, purely infinite) if and only if $\tilde{p}$ is finite (properly infinite, purely infinite).*

PROOF. Let $\phi\colon \tilde{p}B \to pA$ be a Jordan-*-triple isomorphism, let $q := \phi(\tilde{p})$. Then q is maximal in $U := pA$ and the first statement follows from the fact that $U_2(p) = A_2(p) = (pAp)$ and $\phi^{-1}(U_2(q)) = B_2(\tilde{p})$.

Furthermore, the properties of the projection lattice of a W^*-algebra are already determined by its Jordan algebra structure from which the second statement follows. □

It is clear from (5.8) that two JBW^*-triples of different types (as in (5.5)) are nonisomorphic except possibly those of types II_1^a, and $\mathrm{II}_{\infty,1}^a$. To distinguish them we need some more results on projections in W^*-algebras.

(5.9) LEMMA. *Let A be a W^*-algebra.*

(1) *If p and q are orthogonal projections in A with $p \succ q$ and p is properly infinite, then $p + q \sim p$.*

(2) *Let $(p_i)_{i\in I}$ be an infinite orthogonal family of projections in A with $p_i \sim p_j$ $(i, j \in I)$ and let q be a projection with $p_iq = 0$ and $q \prec p_i$ for all $i \in I$. Then there is an orthogonal family of projections $(q_i)_{i\in I}$ with $q_i \sim p_i$ $(i \in I)$ and $\sum_{i\in I} q_i = \sum_{i\in I} p_i + q$.*

PROOF. (1) follows directly from [**7**, III, 8.6, Corollary 2], and (2) is shown in the proof of [**32**, 4.12]. □

The following lemma can be proved using standard methods in the theory of von Neumann algebras. Its proof is therefore omitted.

(5.10) LEMMA. *Let A be a properly infinite W^*-algebra and let p be a projection in A with central support* 1. *Then there is a family $(z_m)_{m\in M}$ of central projections with sum* 1 *such that one of the following conditions holds for each $m \in M$,*

(1) *$pz_m \sim z_m$,*

(2) *there is an orthogonal family of pairwise equivalent projections in A with sum z_m containing pz_m.*

(5.11) LEMMA. *Let p be a projection in a W^*-algebra A and let $U = pA$.*

(1) *Let $(p_i)_{i\in I}$ be an orthogonal family of pairwise equivalent projections which contains p and satisfies $\sum_{i\in I} p_i = 1$. Then there is a collinear family $(q_i)_{i\in I}$, i.e. $\{q_iq_iq_j\} = q_j$ for $i \neq j$, of maximal tripotents in U satisfying $\bigcap_{i\in I} U_1(q_i) = 0$.*

(2) *Let $(q_i)_{i\in I}$ be a collinear family of maximal tripotents in $U = pA$ with $q_iq_i^* = p$ for all $i \in I$. Then $(q_i^*q_i)_{i\in I}$ is an orthogonal family of pairwise equivalent projections in A.*

PROOF. (1) By assumption, there are tripotents q_i with $q_iq_i^* = p, q_i^*q_i = p_i$ $(i \in I)$. It is $q_i = q_iq_i^*q_i = pq_i \in pA$ and $pA = pAp_i + pA(1-p_i)$ with $pAp_i = U_2(q_i)$ and $pA(1-p_i) = U_1(q_i)$ since $\{q_iq_ipa\} = pa + pap_i$. Now all assertions easily follow.

(2) By assumption, q_i and q_j $(i \neq j)$ are collinear, in particular,

$$q_i = q_jq_j^*q_i + q_iq_j^*q_j = pq_i + q_iq_j^*q_j = q_i + q_iq_j^*q_j,$$

i.e. $q_iq_j^*q_j = 0$. So $q_i^*q_i$ and $q_j^*q_j$ are orthogonal. By construction, we have $q_i^*q_i \sim p \sim q_j^*q_j$. □

We can now distinguish between JBW^*-triples of type II_1^a and $\mathrm{II}_{\infty,1}^a$:

(5.12) THEOREM. *Every JBW^*-triple of type $\mathrm{II}^a_{\infty,1}$ contains an infinite collinear family of maximal tripotents, whereas a JBW^*-triple of type II^a_1 does not contain such a family.*

PROOF. Let U be of type $\mathrm{II}^a_{\infty,1}$, thus we may assume that $U = pA$ for a finite projection p with central support 1 in a W^*-algebra A of type II_∞. Let $(z_m)_{m\in M}$ be a family of central projections as in (5.10). Since $pz_m \sim z_m$ contradicts our assumptions we must have condition (2) of (5.10), i.e. for all $m \in M$ there are (necessarily infinite) orthogonal families of projections $(p_{im})_{i\in I[m]}$ containing pz_m with $\sum_{i\in I[m]} p_{im} = z_m$ and $p_{im} \sim pz_m$ for all $i \in I[m]$. As $I[m]$ is an infinite set of all $m \in M$ we may assume that $\mathbf{N} \subset I[m]$. By (5.11.1), for all $m \in M$ there are collinear families $(q_{nm})_{n\in\mathbf{N}}$ of maximal tripotents in pz_mA. Let $q_n := \sum_{m\in M} q_{nm}$ $(n \in \mathbf{N})$. Then $(q_n)_{n\in\mathbf{N}}$ is a collinear family of maximal tripotents in U. Let now U be of type II^a_1. We may assume that $U = pA$ for a (necessarily finite) projection p with central support 1 in a W^*-algebra A of type II_1.

Let $(q_i)_{i\in I}$ be a collinear family of maximal tripotents in U. By (5.7.1), there are central projections z_i such that $z_iq_iq_i^* = z_ip$ and $(1 - z_i)q_i^*q_i = 1 - z_i$ for all $i \in I$. So we have

$$(1 - z_i) \sim (1 - z_i)q_iq_i^* \le (1 - z_i)p \le (1 - z_i),$$

i.e. $(1 - z_i)q_iq_i^* \sim (1 - z_i)p$ and the finiteness of p implies $(1 - z_i)q_iq_i^* = (1 - z_i)p$. Together with $z_iq_iq_i^* = z_ip$ we have $q_iq_i^* = p$. Then (5.11.2) shows that $(q_i^*q_i)_{i\in I}$ is an orthogonal family of pairwise equivalent projections and hence I is a finite set by the finiteness of A. □

(5.13) THEOREM. *Every continuous JBW^*-triple $U = U_{as}$ is uniquely decomposed into a direct sum of four ideals of type II^a_1, $\mathrm{II}^a_{1,\infty}$, II^a_∞ and III^a respectively.*

PROOF. As noted earlier, the existence of such a decomposition follows from the decomposition theory of W^*-algebras: If $U = pA$, first decompose A into a sum of three ideals of type II_1, II_∞ and III and in case II_∞ further decompose p into a sum of a finite and a properly infinite projection.

Any w^*-closed ideal I of $U = pA$ is of the form $I = p_IA$ for a projection $p_I \le p$ and is therefore of the same type as U. Since w^*-closed ideals of U split: $I = (I \cap J) \oplus^\infty (I \cap J^\perp)$ by [**14**, (4.2)], the uniqueness of the decomposition in (5.13) follows from the fact that JBW^*-triples of different types are nonisomorphic (by (5.8) and (5.12)). □

The rest of this section is devoted to a description of JBW^*-triples of type $\mathrm{II}^a_{\infty,1}$, II^a_∞ and III^a as "matrix triples" in terms of tensor products.

Let $U \subset \mathcal{L}(H)$, $V \subset \mathcal{L}(K)$ be w^*-closed associative subtriples of $\mathcal{L}(H)$ ($\mathcal{L}(K)$ resp.) (cf. §4, i.e. $u_1u_2^*u_1 \in U$ for $u_i \in U$, similarly for V). Then the w^*-closure of the algebraic tensor product $U \otimes V$ in $\mathcal{L}(H)\bar{\otimes}\mathcal{L}(K)$ (tensor product of W^*-algebras) is an associative subtriple of $\mathcal{L}(H)\bar{\otimes}\mathcal{L}(K)$, so in particular, it is a BW^*-triple which will be denoted by $U\bar{\otimes}V$. For complex Hilbert spaces H and K, $\mathcal{L}(K, H)$ is canonically identified with a subspace of $\mathcal{L}(K \oplus H)$. The following result is a consequence of the coordinatization theorem obtained in [**15**].

(5.14) THEOREM. *Let U be a JBW^*-triple of type $\mathrm{II}^a_{\infty,1}$ (II^a_∞, III^a resp.). Then U is an l^∞-direct sum of w^*-closed ideals U_m where U_m is isometrically* *isomorphic to $B_m \bar{\otimes} \mathcal{L}(\mathbf{C}, H_m)$, B_m is a W^*-algebra of type* II_1 (II_∞, III *resp.*) *and H_m is a complex Hilbert space.*

PROOF. We may assume that $U = pA$ for a projection p with central support 1 in a W^*-algebra A. Then (5.10) and (5.11) show the existence of an orthogonal family $(z_m)_{m\in M}$ of central projections in A and of collinear families $(q_{im})_{i\in I[m]}$ of tripotents which are maximal in $pz_mA =: U_m$ with

$$(*) \qquad \bigcap_{i\in I[m]} (U_m)_1(q_{im}) = 0 \quad \text{for all } m \in M.$$

Let $B_m := z_m pAp$. A collinear family of tripotents is a rectangular grid (cf. [**27**, **29**]). The condition $(*)$ and the maximality of the q_{im} imply that this rectangular grid is complete in the sense of [**15**, (4.2)]. Then [**15**, (4.6)] yields the desired results. □

We will conclude this section with a criterion for $B\bar{\otimes}\mathcal{L}(\mathbf{C}, H)$ to be a W^*-algebra. We need the following result.

(5.15) LEMMA. *Let B be a W^*-algebra of type* II_∞ *or type* III, *let H be a separable complex Hilbert space. Then B and $B\bar{\otimes}\mathcal{L}(\mathbf{C}, H)$ are isomorphic as associative BW^*-triples.*

PROOF. Let $A := B\bar{\otimes}\mathcal{L}(H)$, fix an orthonormal basis for H, let $(e_{ij})_{i,j\in N}$ ($N = \{1, \dots, n\}$ or $N = \mathbf{N}$) be the canonical matrix units of $\mathcal{L}(H)$ with respect to this basis, let $p := 1_B \otimes e_{11}$. Then B is isomorphic to pAp and $B\bar{\otimes}\mathcal{L}(\mathbf{C}, H)$ can be naturally identified with $U := pA$. As B is properly infinite the same is true for A [**30**, 2.6.6]. We have $1_B \otimes e_{ii} \sim 1_B \otimes e_{jj}$ $(i, j \in N)$, and $\sum_{i\in N} 1_B \otimes e_{ii} = 1_A$. Because p is a properly infinite projection and N is countable this implies $p \sim 1_A$ (use [**7**, III, 8.6, Corollary 2]). This means that there is a tripotent u in A with $uu^* = p$ and $u^*u = 1_A$. Then $u = uu^*u \in pA$ and $U = uu^*Au^*u = U_2(u)$. Finally, $z \to zu^*$ is an associative triple isomorphism of $U = U_2(u)$ onto $U_2(p)$ which in turn is isomorphic to B. □

(5.16) THEOREM. *A JBW^*-triple U of type* II^a_∞ *or of type* III^a *with a separable predual is isomorphic to a W^*-algebra.*

PROOF. By (5.14) we may assume that $U = B\bar{\otimes}\mathcal{L}(\mathbf{C}, H)$ for a W^*-algebra B of type II_∞ or III. Then $\mathcal{L}(\mathbf{C}, H)$ can be embedded into U as a w^*-closed subspace, so the predual of $\mathcal{L}(\mathbf{C}, H)$ ($= \mathcal{L}(H, \mathbf{C})$) is a continuous image of the predual of U, hence separable. Then (5.15) yields the result. □

Appendix: The halving lemma, revisited. The only nontrivial result of the theory of JB-algebras, which we used in the previous sections, is [**12**, 5.2.15], which itself easily follows from the halving lemma [**12**, 5.2.14]. In order to be totally independent of [**12**] we will give here a new proof of the halving lemma. Besides being independent we also feel that a more Jordan theoretic proof is of some interest. In our approach the main point is to show

(A.1) THEOREM. *Every nonzero continuous JBW^*-triple U contains a nonzero triangle, i.e. tripotents $(e; e_1, e_2)$ such that $e_1 \in U_0(g)$ and $e_2 \in U_0(g)$ (e_1 and e_2 are orthogonal, $e_1 \perp e_2$) and $e_i \in U_2(e)$, $e \in U_1(e_i)$ ($e_i \dashv e$).*

PROOF. Let $U \neq 0$ be a continuous JBW^*-triple. We first want to show that

$$(\alpha) \qquad U \text{ contains nonmaximal tripotents.}$$

Assume otherwise and let $0 \neq e \in U$ be a tripotent. Then $U = U_2(e) \oplus U_1(e)$ by maximality. By [**29**, IV, Theorem 3.3], e is not minimal, whence there exists a tripotent $c \in U_2(e)$ such that $0 \neq U_2(c) \subsetneq U_2(e)$. Again by non-$(\alpha)$, we have $U_2(e) = U_2(c) \oplus (U_2(e) \cap U_1(c))$, in particular $e = e_2 + e_1$ with $e_i \in U_2(e) \cap U_i(c)$. In case $e_2 = 0$ we obtain $c \dashv e$, thus $0 \neq P(e)c \perp c$ contradicting non-(α). In case $e_2 \neq 0$ we get $e_1 = 0$ since $e_1 \perp e_2$ by the Compatibility Criterion [**29**, I.1.8]. But then $c \approx e$ by [**29**, I, Theorem 2.3] contradicting $U_2(c) \subsetneq U_2(e)$. This finishes the proof of (α).

By (α) we may now assume there exists an orthogonal system (c_1, c_2), $c_i \neq 0$. Let U_{ij} be the Peirce spaces relative to (c_1, c_2). We claim

$$(\beta) \qquad U_{12} \neq 0 \Rightarrow U \text{ contains a nonzero triangle.}$$

Indeed, let $0 \neq f \in U_{12}$ be a tripotent. By compatibility, U_{11} splits relative to f, $U_{11} = \bigoplus_{j=0,1,2} U_{11} \cap U_j(f)$. Suppose $U_{11} \cap U_2(f) \neq 0$ and choose a nonzero tripotent $d \in U_{11} \cap U_2(f)$. We decompose f relative to d: $f = f_1 + f_0$ because $P(d)f \in P(U_{11})U_{12} = 0$. Note $f_1 \perp f_0$, therefore $2d = \{ffd\} = \{ff_1d\} = \{f_1f_1d\}$, thus $d \dashv f_1$ and, by [**29**, I.2.5], $(f_1; d, P(f_1)d)$ is a triangle. We may now assume $U_{11} \cap U_2(f) = 0 = U_{22} \cap U_2(f)$. Then $c_i = c_{i1} + c_{i0}$ with $c_{ij} \in U_{ii} \cap U_j(f)$ and $f = \{c_ic_if\} = \{c_{i1}c_{i1}f\}$ because $c_{i1} \perp c_{i0}$. It follows that $c = c_{11} + c_{21}$ is a tripotent with $c \vdash f$, so we are done again.

In the following we will assume

$$(\gamma) \qquad U_1(e_1) \cap U_1(e_2) = 0 \text{ for any pair } (e_1, e_2) \text{ of orthogonal tripotents in } U,$$

and we will show that in this case any tripotent $e \in U$ is abelian, thus finishing the proof of the theorem by contradiction.

As an auxiliary result we will first derive

$$(\gamma_1) \qquad U = (U_2(c) \oplus U_1(c)) \oplus^\infty U_0(c) \quad (\text{direct } l^\infty\text{-sum}) \text{ for any tripotent } c \in U.$$

Indeed, for any tripotent $g \in U_0(c)$ we have $U_1(c) \subset U_0(g)$ by (c) whence $L(U_1(c), g) = 0 = L(g, U_1(c))$, and therefore $L(U_1(c), U_0(c)) = 0 = L(U_0(c), U_1(c))$, since the span of the tripotents in $U_0(c)$ is w^*-dense. Now (γ_1) easily follows.

In the following we may assume $U = U_2(e)$. Let $(c_1, c_2) \subset U$ be an orthogonal pair with Peirce spaces U_{ij}. By (γ_1)

$$U = (U_{11} \oplus U_{10}) \oplus^\infty (U_{22} \oplus U_{20}) \oplus^\infty U_{00},$$

and correspondingly $e = \sum e_{ij}$. Since e is invertible, $e_{11} + e_{10}$ is an invertible tripotent in $U_{11} \oplus U_{10}$, whence e_1 and $e_{11} + e_{10}$ are compatible with $e_{1i} \in U_{i+1}(c_1)$, consequently $e_{11} \perp e_{10}$ and $U_{11} + U_{10} = U_2(e_{11}) \oplus U_2(e_{10})$. It follows $U_{1i} = U_2(e_{1i})$, $U_{10} \subset U_0(e_{11})$, $e_{11} \approx c_1$. But then $U_{10} \subset U_1(c_1) = U_1(e_{11})$ implying $U_{10} = 0$. Thus, we showed

$$(\gamma_2) \qquad U = U_2(c) \oplus^\infty U_0(c)$$

for any nonmaximal tripotent $c \in U$. It is easily seen that (γ_2) implies

$$(\gamma_3) \qquad [L(c,c), L(y,y)] = 0$$

for any nonmaximal c and every $y \in U$. But then we get (γ_3) also for a maximal c, since any such c, being compatible with e, induces a splitting of e, $e = e_2 + e_1$, $e_2 \perp e_1$. If $e_1 = 0$ we have $e \approx c$, hence $L(c,c) = 2\,\mathrm{Id}$. Otherwise $U = U_2(e_2) \oplus^\infty U_2(e_1)$, thus for $f = f_2 + f_1$ with $f_i \in U_2(e_i)$ we get $\{f_1 f_1 e_1\} = \{f f e_1\} = e_1$ whence $f_1 \dashv e_1$ contradicting assumption (γ). Using again the w^*-denseness for the span of all tripotents in U we obtain $[L(x,x), L(y,y)] = 0$ for all $x, y \in U$, which implies, by polarization, that e is abelian. $\square$

(A.2) THE HALVING LEMMA [**12**, 5.2.14]. *Let U be a continuous JBW^*-triple and $p \in U$ be a tripotent. Then there exists a triangle $(e; e_1, e_2)$ covering $U_2(p)$, i.e. $U_2(p) = U_2(e)$.*

PROOF. Let, by Zorn, $\{(e_\alpha; e_{1\alpha}, e_{2\alpha}), \alpha \in A\}$ be a maximal family of orthogonal triangles: $e_\alpha \perp e_\beta$ for $\alpha \neq \beta$. Then $e = \sum_\alpha^* e_\alpha$ and $e_i = \sum_\alpha^* e_{i\alpha}$ are tripotents [**29**, IV, 3.11], and $(e; e_1, e_2)$ is a triangle as claimed. $\square$

REFERENCES

1. Sh. A. Ayupov, *Classification of injective JW-factors*, Functional Anal. Appl. **18** (1984), 234–235.
2. ——, *Extensions of traces and type criterions for Jordan algebras of self-adjoint operators*, Math. Z. **181** (1982), 253–268.
3. T. J. Barton, T. Dang and G. Horn, *Normal representations of Banach Jordan triple systems*, Proc. Amer. Math. Soc. **102** (1988), 551–555.
4. T. Barton and R. Timoney, *Weak*-continuity of Jordan triple products and applications*, Math. Scand. (to appear).
5. R. B. Braun, *A Gelfand-Neumark theorem for C^*-alternative algebras*, Math. Z. **185** (1984), 225–242.
6. S. Dineen, *The second dual of a JB^* triple system*, Complex Analysis, Functional Analysis and Approximation Theory (J. Mujica, ed.), North-Holland Math. Studies, vol. 125, North-Holland, Amsterdam, 1986, pp. 67–69.
7. J. Dixmier, *Von Neumann algebras*, North-Holland, 1981.
8. Y. Friedman and B. Russo, *Structure of the predual of a JBW^*-triple*, J. Reine Angew. Math. **356** (1985), 67–89.
9. ——, *The Gelfand-Naimark theorem for JB^*-triples*, Duke Math. J. **53** (1986), 139–148.
10. T. Giordano, *Antiautomorphismes involutifs des facteurs de von Neumann injectifs.* I, J. Operator Theory **10** (1983), 251–287.
11. T. Giordano and V. Jones, *Antiautomorphismes involutifs du facteur hyperfini de type* II_1, C. R. Acad. Sci. Paris Sér. A **290** (1980), 29–31.
12. H. Hanche-Olsen and E. Størmer, *Jordan operator algebras*, Pitman, 1984.
13. G. Horn, *Klassifikation der JBW^*-tripel vom Typ* I, Dissertation, Tübingen, 1984.
14. ——, *Characterization of the predual and ideal structure of a JBW^*-triple*, Math. Scand. (to appear).
15. ——, *Coordinatization theorems for JBW^*-triples*, Quart. J. Math. Oxford Ser. **38** (1987), 321–336.
16. N. Jacobson and C. E. Rickart, *Jordan homomorphisms of rings*, Trans. Amer. Math. Soc. **69** (1950), 479–502.
17. W. Kaup, *Über die Klassifikation der symmetrischen hermiteschen Mannigfaltigkeiten unendlicher Dimension.* I, Math. Ann. **257** (1981), 463–483.
18. ——, *A Riemann mapping theorem for bounded symmetric domains in complex Banach spaces*, Math. Z. **183** (1983), 503–529.
19. W. Kaup and H. Upmeier, *Jordan algebras and symmetric Siegel domains in Banach spaces*, Math. Z. **157** (1977), 179–200.
20. O. Loos, *Assoziative Tripelsysteme*, Manuscripta Math. **7** (1972), 103–112.
21. ——, *Alternative Tripelsysteme*, Math. Ann. **198** (1972), 205–238.

22. ——, *Jordan pairs*, Lecture Notes in Math., vol. 460, Springer-Verlag, Berlin and New York, 1975.

23. ——, *Bounded symmetric domains and Jordan pairs*, Lecture Notes, Univ. of California, Irvine, 1977.

24. O. Loos and K. McCrimmon, *Speciality of Jordan triple systems*, Comm. Algebra **5** (1977), 1057–1082.

25. K. McCrimmon, *Peirce ideals in Jordan triple systems*, Pacific J. Math. **83** (1979), 415–439.

26. ——, *Compatible Peirce decompositions of Jordan triple systems*, Pacific J. Math. **103** (1982), 57–102.

27. K. McCrimmon and K. Meyberg, *Coordinatization of Jordan triple systems*, Comm. Algebra **9** (1981), 1495–1542.

28. K. Meyberg, *Lectures on algebras and triple systems*, Lecture Notes, Univ. of Virginia, Charlottesville, 1972.

29. E. Neher, *Jordan triple systems by the grid approach*, Lecture Notes in Math., vol. 1280, Springer-Verlag, Berlin and New York, 1987.

30. S. Sakai, *C^*- and W^*-algebras*, Ergeb. Math. Grenzgeb., Band 60, Springer-Verlag, Berlin and New York, 1971.

31. E. Størmer, *Conjugacy of involutive antiautomorphisms of von Neumann algebras*, J. Funct. Anal. **66** (1986), 54–66.

32. S. Strătilă and L. Zsidó, *Lectures on von Neumann algebras*, Abacus Press, 1979.

33. D. M. Topping, *Jordan algebras of self-adjoint operators*, Mem. Amer. Math. Soc., vol. 53, 1965.

34. H. Upmeier, *Symmetric Banach manifolds and Jordan C^*-algebras*, North-Holland Math. Studies, vol. 104, North-Holland, Amsterdam, 1985.

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