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CORRIGENDUM TO "CHARACTERIZATIONS OF TURBULENT ONE-DIMENSIONAL MAPPINGS VIA ω-LIMIT SETS"

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The following statement is made without proof in [1].

Theorem 24 (Erroneous Version). *Let $f: I \to I$ be continuous. Then*

(i) *f is turbulent if and only if there is a point $x_0 \in I$ such that $\omega(x_0, f)$ is a unilaterally convergent sequence.*

(ii) *f^2 is turbulent if and only if there is a point $x_0 \in I$ such that $\omega(x_0, f)$ is a convergent sequence.*

The "only if" direction of statement (ii) is not true, as can be seen by considering the function obtained by applying doubling to the familiar "tent mapping" on the unit interval. This error was called to our attention by Professor Selvaratnam Sridharma, who observed that it is easy to construct a function having a period three point which has no convergent sequence as an ω-limit set. We wish to thank Professor Sridharma and wish to take this opportunity to present a correct replacement for the erroneous statement. All numbered results cited in the proof are found under those numbers in [1].

Theorem 24 (Corrected Version). *Let $f: I \to I$ be continuous. Then*

(i) *f is turbulent if and only if there is a point $x_0 \in I$ such that $\omega(x_0, f)$ is a unilaterally convergent sequence.*

(ii) *f^2 is turbulent if and only if there is a point $x_0 \in I$ such that either $\omega(x_0, f)$ is a convergent sequence or $\omega(x_0, f)\backslash\omega(x_0, f^2) = \omega(f(x_0), f^2) =$ a unilaterally convergent sequence.*

Proof of Theorem 24 (Corrected Version). Part (i) is simply the result of combining Corollaries 10 and 23, and the "if" direction of part (ii) is immediate from Corollaries 10 and 15. To prove the "only if" direction of part (ii), we begin by assuming that f^2 is turbulent. Hence, there is a point $y \in I$ for which $\omega(y, f^2)$ is a unilaterally convergent sequence. Let p denote the limit of this sequence. Note that $f(\omega(y, f^2))$ is a sequence converging to $f(p)$ and that $f(\omega(y, f^2)) = \omega(f(y), f^2)$. Furthermore, p is a fixed point of f^2. If p is a fixed point of f, then by **K11** $\omega(y, f)$ is a convergent sequence, convergent to p. So, letting $x_0 = y$, we would be done. On the other hand, if p is a period 2 point of f, then we let $x_0 = f(y)$ and employing **K11**, we may write

$$\omega(x_0, f) = \omega(y, f^2) \cup \omega(f(y), f^2).$$

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First observe that $p \notin \omega(f(y), f^2)$; for if $p \in \omega(f(y), f^2)$, then p is an isolated point of the infinite set $\omega(f(y), f^2)$, violating **K7**. Likewise, $f(p) \notin \omega(y, f^2)$. Again by **K7**, $\omega(x_0, f)$ can contain no periodic points other than p and $f(p)$. We have

$$\omega(x_0, f)\backslash\omega(x_0, f^2) = \omega(y, f^2)\backslash[\omega(y, f^2) \cap \omega(f(y), f^2)].$$

Now, $\omega(y, f^2) \cap \omega(f(y), f^2)$, being the intersection of two sequences with different limits, cannot be infinite. In fact, this intersection, which is obviously closed and invariant under f, will have to be empty. To see this, note that if it contained a point, then by invariance of this finite set under f it would contain a periodic point of f. However, we have already established that it contains neither p nor $f(p)$, the only periodic points of f in $\omega(x_0, f)$. Thus we have

$$\omega(x_0, f)\backslash\omega(x_0, f^2) = \omega(y, f^2),$$

which is a unilaterally convergent sequence. Clearly, $\omega(f(x_0), f^2) = \omega(y, f^2)$, and the proof is complete.

References

1. M. J. Evans, P. D. Humke, Ch.-M. Lee, and R. J. O'Malley, *Characterizations of turbulent one-dimensional mappings via ω-limit sets*, Trans. Amer. Math. Soc. **326** (1991), 261–280.

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