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CORRECTIONS TO "FIRST STEPS IN DESCRIPTIVE THEORY OF LOCALES"

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The paper [I_2] contains one false result, 2.6, and two others whose proofs require substantial repair: 2.5 and 2.24. All errors were discovered by one critical reader, Till Plewe.

In 2.5 of [I_2], which characterizes those sober spaces X that have a largest pointless sublocale $\mathrm{pl}(X)$, the last six words of proof are not true, e.g. in the real line with a generic point adjoined. Argue instead:

The meet of $\{x\}^-$ and $\mathrm{pl}^+(X)$ is dense in the irreducible space $\{x\}^-$. Now *every dense sublocale of an irreducible space* Y *contains* (i.e. $D(Y)$ contains) *the generic point* y. For every sublocale of any locale is an intersection of complemented sublocales C whose complements are open $\cap$ closed [I_2], so it suffices to show that every such C dense in Y has y in it. Otherwise the complement C' would contain y, so no closed subspace except Y contains C', so C' is open; and as C is dense, $C' = 0$, contradicting $y \in C'$.

2.6 says that for a dense-in-itself regular space X the locale $\mathrm{pl}(X)$ has the same weight as X. This is false—refuted by many pairs (X, X') of regular spaces with the same pl $(\mathrm{pl}(X) \approx \mathrm{pl}(X'))$ but different weights, e.g. the space Q of rationals and a subspace of βQ consisting of Q and one more point.

The last result in the paper, 2.24, is that (with everything metrizable; in contrast to O_δ's) no nonzero pointless-absolute F_σ locale exists. The correct proof, now presented, amounts to showing that (1) any nonzero F_σ sublocale A of a pointless-absolute O_δ has a closed sublocale $B = \mathrm{pl}(\mathbf{C})$, $\mathbf{C}$ a Cantor set; that (2) B is not pointless-absolute F_σ, being not F_σ in a suitable metrizable extension $\mathrm{pl}(E)$; and (3) boosting the extension E of B to an extension of A. In [I_2], (1) is done correctly in three lines, and four more lines of the proof (the fourth and the last three) do (2). For (3), a pushout construction is proposed; but it is not hard to check that the pushout need not be first countable. Instead use F. Hausdorff's theorem [H]:

Theorem (Hausdorff). *A metric on a closed subspace of a metrizable space* Y *can be extended to a compatible metric on* Y.

This applies to the situation in 2.24: $\mathbf{C}\backslash Q$ closed in Y and densely embedded in $\mathbf{C}^2$. Induce a metric on $\mathbf{C}\backslash Q$ from $\mathbf{C}^2$, extend over Y by the theorem, and extend uniquely back over the limit points in $\mathbf{C}^2$.

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[H] F. Hausdorff, *Erweiterung einer Homöomorphie*, Fund. Math. **16** (1930), 353–360.

[I_1] J. Isbell, *Atomless parts of spaces*, Math. Scand. **31** (1972), 5–32.

[I_2] ——, *First steps in descriptive theory of locales*, Trans. Amer. Math. Soc. **327** (1991), 353–371.

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