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AN INVERSE PROBLEM FOR SCATTERING BY A DOUBLY PERIODIC STRUCTURE

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Abstract. Consider scattering of electromagnetic waves by a doubly periodic structure $S = \{x_3 = f(x_1, x_2)\}$ with $f(x_1 + n_1\Lambda_1, x_2 + n_2\Lambda_2) = f(x_1, x_2)$ for integers n_1, n_2. Above the structure, the medium is assumed to be homogeneous with a constant dielectric coefficient. The medium is a perfect conductor below the structure. An inverse problem arises and may be described as follows. For a given incident plane wave, the tangential electric field is measured away from the structure, say at $x_3 = b$ for some large b. To what extent can one determine the location of the periodic structure that separates the dielectric medium from the conductor? In this paper, results on uniqueness and stability are established for the inverse problem. A crucial step in our proof is to obtain a lower bound for the first eigenvalue of the following problem in a convex domain Ω:

$$\begin{cases} -\triangle u = \lambda u \quad \text{in} \quad \Omega, \\ \nabla \cdot u = 0 \quad \text{in} \quad \Omega, \\ n \times u = 0 \quad \text{on} \quad \partial\Omega. \end{cases}$$

1. Introduction

Consider scattering of electromagnetic waves by a doubly periodic (or biperiodic) structure $S = \{x_3 = f(x_1, x_2)\}$ of period $\Lambda = (\Lambda_1, \Lambda_2)$, that is,

$$f(x_1 + n_1\Lambda_1, x_2 + n_2\Lambda_2) = f(x_1, x_2)$$

for integers n_1, n_2, and some positive constants Λ_1, Λ_2. Above the structure, the medium is assumed to be homogeneous with a constant dielectric coefficient $\epsilon_0 > 0$. The medium is a perfect conductor below the structure. The magnetic permeability is assumed to be a positive constant μ_0 throughout $\mathbf{R}^3$. Given the structure and a time-harmonic electromagnetic plane wave incident on the structure, the (direct) scattering problem is to predict the field distributions away from the structure. We are interested in studying the inverse problem: To determine the periodic structure or the shape of the interface from the measured scattered field. Our first result in this paper is a local uniqueness theorem for the inverse problem. Basically, our theorem indicates that any two doubly periodic surface profiles are identical if they generate the same scattered fields (or patterns) and the area in between the two

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profiles is sufficiently small. Our proof is based on a combination of unique continuation and an estimation of the first eigenvalue of the corresponding eigenvalue problem. We obtain a lower bound for the first eigenvalue and prove by using an earlier result of Payne and Weinberger [11] that the eigenvalue goes to infinity as the domain diameter shrinks to zero. We also prove a local stability result for the inverse problem: If S_1 is another doubly periodic structure "close" to S, then for any $\delta > 0$, the measurements of the two tangential electric fields being δ-close implies that the two surfaces are $O(\delta)$-close. This work is motivated by the study of optimal design problems of gratings, where one wishes to design a grating (or periodic) structure that generates some specified scattered field.

Scattering of electromagnetic waves in a doubly periodic structure has recently received considerable attention. We refer to Dobson [8], Dobson and Friedman [9], Abboud [1], and Bao [5] for results on existence, uniqueness, and numerical approximations of solutions. With a lossy medium, *i.e.*, its dielectric coefficient is complex, above the conductor, Ammari [3] obtained a global uniqueness result for the inverse problem in doubly periodic structures. A similar uniqueness result was proved in [4] for singly periodic structures. The local uniqueness theorem in this paper deals with a more complicated doubly periodic *dielectric* medium (with a real dielectric coefficient). We point out that it is well known that, in general, if the periodic structure is ruled on a dielectric medium as studied here, global uniqueness is impossible with one incident plane wave. Regarding stability of the inverse problem, little is known. The only available results are proved in Bao and Friedman [6] for singly periodic structures. Note that in [6] a more general class of inverse diffraction problems is studied. Our stability result in this paper extends one of the stability results of [6] to the doubly periodic case.

The scattering theory in periodic structures has many applications in micro-optics, where doubly periodic structures are often called *crossed diffraction gratings*. A good introduction to the problem of electromagnetic diffraction through periodic structures, along with some numerical methods, can be found in Petit [12]. A complete account of the general theory of inverse scattering problems in general (nonperiodic) structures may be found in the book of Colton and Kress [7] and references therein.

The outline of the paper is as follows. The direct scattering problem is formulated in the next section. We also present some auxiliary results in Section 2. Section 3 is devoted to the study of the eigenvalue problem. Our main result is a lower bound for the first eigenvalue. A uniqueness theorem for the inverse problem is proved in Section 4. In Section 5, we establish a local stability result for the inverse problem.

2. The direct problem and some auxiliary results

The electromagnetic wave propagation is governed by the time harmonic Maxwell's equations (time dependence $e^{-i\omega t}$):

$$\nabla \times E - i\omega\mu H = 0\,, \tag{2.1}$$

$$\nabla \times H + i\omega\epsilon E = 0\,, \tag{2.2}$$

where E and H denote the electric and magnetic fields, respectively. Here ω is the (scaled) angular frequency, and recall that $\mu = \mu_0$ is the magnetic permeability which is assumed to be a fixed positive constant everywhere.

Let the scattering profile be described by the periodic surface $S = \{(x_1, x_2, x_3) : x_3 = f(x_1, x_2)\}$ of period $\Lambda = (\Lambda_1, \Lambda_2)$. The function f is supposed to be sufficiently smooth, for example C^2. The space below S is filled with some perfectly reflecting material (a conductor). Let $\Omega = \{(x \in \mathbf{R}^3 : x_3 > f(x_1, x_2)\}$ be filled with a material whose dielectric coefficient is a fixed constant $\epsilon = \epsilon_0 > 0$. Suppose that a plane wave is incident on S from the top. The direct problem is concerned with predicting the behavior of the outgoing reflected waves, given the incident field and the periodic structure. Since the medium is a conductor, it does not support any transmitted waves.

Consider a plane wave in Ω,

$$E_I = se^{iq\cdot x}, \quad H_I = pe^{iq\cdot x}, \tag{2.3}$$

incident on S. Here $q = (\alpha_1, \alpha_2, -\beta) = \omega\sqrt{\epsilon_0\mu_0}(\cos\theta_1 \cos\theta_2, \cos\theta_1 \sin\theta_2, -\sin\theta_1)$ is the incident wave vector whose direction is specified by θ_1 and θ_2, with $0 < \theta_1 < \pi$ and $0 < \theta_2 \leq 2\pi$. The vectors s and p satisfy

$$s = \frac{1}{\omega\epsilon_0}(p \times q)\ ,\ q \cdot q = \omega^2\epsilon_0\mu_0\ ,\ p \cdot q = 0\ .$$

From the Maxwell equations (2.1) and (2.2), it is straightforward to deduce the following vector Helmholtz equation:

$$(\triangle + k^2)E = 0 \ \text{ in } \Omega, \tag{2.4}$$

where $k^2 = \omega^2\epsilon_0\mu_0$. We are interested in quasiperiodic solutions, *i.e.*, solutions E and H such that, for an $\alpha = (\alpha_1, \alpha_2, 0)$, $[E(x_1, x_2, x_3) - E_I(x_1, x_2, x_3)]e^{-i\alpha\cdot x}$ and $[H(x_1, x_2, x_3) - H_I(x_1, x_2, x_3)]e^{-i\alpha\cdot x}$ are periodic in the x_1 direction of period Λ_1 and in the x_2 direction of period Λ_2. Since the region below S is a perfect conductor, only reflected waves exist. Hence the boundary condition is

$$n \times E = 0 \quad \text{on } S, \tag{2.5}$$

where n is the outward normal to the surface. It is evident that to completely specify the boundary value problem, we also need to impose a radiation condition in the x_3 direction. The radiation condition that we impose is the boundedness of E as x_3 tends to infinity. More precisely, we insist that E is composed of bounded outgoing plane waves plus the incident wave E_I.

Since $Ee^{-i\alpha\cdot x}$ is Λ-periodic, we can expand it in a Fourier series:

$$E(x) = E_I + \sum_{n\in Z^2} E^{(n)}(x_3)e^{i(\alpha_n+\alpha)\cdot x}, \tag{2.6}$$

where

$$E^{(n)}(x_3) = \Lambda_1^{-1}\Lambda_2^{-1}\int_0^{\Lambda_1}\int_0^{\Lambda_2}(E(x) - E_I)e^{-i(\alpha_n+\alpha)\cdot x}dx_1dx_2$$

and

$$\alpha_n = (2\pi n_1/\Lambda_1, 2\pi n_2/\Lambda_2, 0).$$

Denote

$$\Gamma = \{x \in \mathbf{R}^3 : x_3 = b\}\ ,$$

where $b > \max\{f(x_1, x_2)\}$.

Define

$$\beta^{(n)}(\alpha) = \begin{cases} \sqrt{k^2 - |\alpha_n + \alpha|^2}, & k^2 > |\alpha_n + \alpha|^2, \\ i\sqrt{|\alpha_n + \alpha|^2 - k^2}, & k^2 < |\alpha_n + \alpha|^2 . \end{cases}$$

Throughout, we assume that $k^2 \neq |\alpha_n + \alpha|^2$ for all $n \in Z^2$.

It follows from knowledge of the fundamental solution to the periodic Helmholtz equation, see for example [9], that E can be expressed as a sum of plane waves in $\Omega_1 = \{x_3 > b\}$:

$$E = E_I + \sum_{n \in Z^2} A^{(n)} e^{i\beta^{(n)}(\alpha)x_3 + i(\alpha_n + \alpha)\cdot x}, \tag{2.7}$$

where the $A^{(n)}$ are constant (complex) vectors.

By matching the two expansions (2.6) and (2.7) at $x_3 = b$, we get

$$A^{(n)} = E^{(n)}(b) e^{-i\beta^{(n)}(\alpha)b} . \tag{2.8}$$

Further, since

$$\nabla \cdot E = 0 \ , \ \ \nabla \cdot E_I = 0 \ ,$$

we have from (2.7) that

$$(\alpha_n + \alpha) \cdot E^{(n)} + \beta^{(n)} E_3^{(n)} = 0 \ . \tag{2.9}$$

Lemma 2.1. *There exists a boundary pseudo-differential operator B of order one, such that*

$$e_3 \times (\nabla \times (E - E_I)) = B(P(E - E_I)) \ \text{ on } \Gamma, \tag{2.10}$$

where $e_3 = (0, 0, 1)$ and the operator B is defined by

$$Bf = -i \sum_{n \in Z^2} \frac{1}{\beta^{(n)}} \{(\beta^{(n)})^2 (f_1^{(n)}, f_2^{(n)}, 0) + ((\alpha + \alpha_n) \cdot f^{(n)})(\alpha + \alpha_n)\} e^{i(\alpha + \alpha_n)\cdot x},$$

where P is the projection onto the plane orthogonal to e_3, i.e.,

$$Pf = -e_3 \times (e_3 \times f),$$

and

$$f^{(n)} = \Lambda_1^{-1} \Lambda_2^{-1} \int_0^{\Lambda_1} \int_0^{\Lambda_2} f(x) e^{-i(\alpha_n + \alpha)\cdot x} dx_1 dx_2 \ .$$

The proof may be given by using the expansion (2.7) together with (2.8), (2.9), and some simple calculation.

Remark 2.1. The significance of this result is that the Dirichlet to Neumann operator B carries the information on radiation condition in an explicit form. Here it is crucial to assume that $\beta^{(n)}$ is nonzero. The present form of the result is equivalent to the one in Abboud [1]. Another equivalent form was independently derived by Dobson [8].

Therefore, the direct scattering problem can be formulated as follows: Find a quasiperiodic solution that solves the problem (2.4), (2.5), and (2.10). Questions on existence and uniqueness have been studied by several authors [1], [9], [8], [5] by using the method of integral equations or variational approaches. The result in the most general setting, where $\epsilon = \epsilon(x)$ is only a bounded measurable function in $\mathbf{R}^3$, indicates that the problem attains a unique weak quasiperiodic solution for all but possibly a discrete set of frequencies. The results can be greatly improved when additional smoothness is imposed on ϵ, hence on the structure. In fact, following an approach developed in Abboud [1], Ammari [3] proved the following result.

Lemma 2.2. *The direct scattering problem has a unique quasiperiodic solution*

$$E \in C^2(\Omega) \cap C^1(\overline{\Omega}) \ .$$

We next describe an inverse (scattering) problem. Suppose that E (quasiperiodic) solves the scattering problem (2.4), (2.5), and (2.10) for a given incident plane wave E_I as in (2.3). The inverse problem can be stated as follows: Determine $f(x_1, x_2)$ from the knowledge of $n \times E|_\Gamma$. Note that it makes sense physically to measure the tangential electric field rather than the electric field, since the total energy through the boundary Γ is

$$\mathrm{Re} \int_\Gamma n \cdot (E \times \overline{H}) ds = \mathrm{Re} \int_\Gamma (n \times E) \cdot \overline{H} ds \ .$$

The following unique continuation result is crucial in the study of the inverse problem.

Lemma 2.3. *Let Ω be a domain of $\mathbf{R}^3$ with a smooth boundary $\partial\Omega$. Assume that (E, H) is a solution of Maxwell's equations* (2.1), (2.2) *with analytic ϵ and μ. Let $I \subset \Omega$ be an analytic surface. Suppose also that*

$$n \times E = 0 \ , \ n \times H = 0 \ \ on\ I.$$

Then $E = 0$, $H = 0$ in $\overline{\Omega}$.

The result may be proved by an application of Holmgren's uniqueness theorem. See Abboud and Nédélec [2] for a proof.

Remark 2.2. By analyzing the proof in [2], it appears that the regularity assumptions on ϵ, μ, and $\partial\Omega$ may be weakened. In particular, the result remains valid if $\partial\Omega$ is piecewise smooth and I is of C^2.

We next state a useful imbedding result. The reader is referred to Girault and Raviart [10] for a proof.

Lemma 2.4. *Let $\Omega \subset \mathbf{R}^3$ be a bounded and convex open subset with a C^2 boundary $\partial\Omega$. Assume that $\phi \in H^1(\Omega)^3$ with $\phi \cdot n = 0$ on $\partial\Omega$. Then*

$$|\phi|^2_{1,\Omega} = \sum_{j=1}^{3} \int_\Omega |\nabla \phi_j|^2 dx \le \int_\Omega |\nabla \times \phi|^2 dx + \int_\Omega |\nabla \cdot \phi|^2 dx.$$

3. Eigenvalue problem

In this section, we study the first eigenvalue $\lambda_1 = \lambda_1(\Omega)$ of the following eigenvalue problem:

$$\left\{ \begin{array}{l} -\triangle u = \lambda u \quad \text{in} \quad \Omega, \\ \nabla \cdot u = 0 \quad \text{in} \quad \Omega, \\ n \times u = 0 \quad \text{on} \quad \partial\Omega, \end{array} \right. \tag{3.1}$$

where Ω is a bounded and smooth domain in $\mathbf{R}^3$ and n is the outward normal on $\partial\Omega$. We establish

Theorem 3.1. *If Ω is bounded, smooth and convex in $\mathbf{R}^3$, then*

$$\lambda_1(\Omega) \geq \mu_2(\Omega),$$

where $\mu_2(\Omega)$ is the first nontrivial eigenvalue of the Neumann eigenvalue problem

$$\begin{cases} -\triangle v = \mu v & \textit{in} \quad \Omega, \\ \frac{\partial v}{\partial n} = 0 & \textit{on} \quad \partial\Omega. \end{cases} \tag{3.2}$$

From a well-known result of Payne and Weinberger [11],

$$\mu_2(\Omega) \geq (\frac{\pi}{\mathrm{diam}(\Omega)})^2$$

when Ω is convex. Consequently

Corollary 3.2. *If Ω is bounded, smooth and convex in $\mathbf{R}^3$, then*

$$\lambda_1(\Omega) \to \infty$$

as $\mathrm{diam}(\Omega) \to 0$.

Remark 3.1. The above results might not be true if Ω is not convex. For example, let $\Omega_\epsilon = B_{2\epsilon} - B_\epsilon$, where B_d is the ball with radius d and center 0, and let g be the solution of

$$\begin{cases} \triangle g = 0 & \text{in} \quad \Omega_\epsilon, \\ g = 1 & \text{on} \quad |X| = \epsilon, \\ g = 0 & \text{on} \quad |X| = 2\epsilon. \end{cases}$$

Set $u = \nabla g$; then

$$\begin{cases} -\triangle u = 0 = 0u & \text{in} \quad \Omega_\epsilon, \\ \nabla \cdot u = \triangle g = 0 & \text{in} \quad \Omega_\epsilon, \\ n \times u = 0 & \text{on} \quad \partial\Omega_\epsilon. \end{cases}$$

That is, 0 is an eigenvalue of (3.1).

Remark 3.2. In the two-dimensional case, the two eigenvalue problems (3.1) and (3.2) are equivalent. Thus $\lambda_1(\Omega) = \mu_2(\Omega)$. In fact, let μ be a nontrivial eigenvalue of (3.2) and v be an associated eigenfunction. It is easy to verify that $u = (-v_y, v_x)$ is an eigenfunction of (3.1) corresponding to the eigenvalue μ. Conversely, if λ is an eigenvalue of (3.1) and u is an eigenvector, we define

$$v(\overline{x}, \overline{y}) = \int_{(x_0,y_0)}^{(\overline{x},\overline{y})} u_2 dx - u_1 dy \ , \tag{3.3}$$

where (x_0, y_0) is a fixed point in Ω. This function is well defined since the integral in (3.3) is path independent from $\nabla \cdot u = 0$. It is easy to check that $v_x = u_2$, $v_y = -u_1$, and $\frac{\partial v}{\partial n}|_{\partial\Omega} = n \times u|_{\partial\Omega} = 0$. Define

$$h = \triangle v + \lambda v \ .$$

We have

$$\begin{aligned} h_x &= \triangle v_x + \lambda v_x = \triangle u_2 + \lambda u_2 = 0, \\ h_y &= \triangle v_y + \lambda v_y = -(\triangle u_1 + \lambda u_1) = 0. \end{aligned}$$

Thus h is a fixed constant, say C_0. It is evident that $\lambda \neq 0$, since otherwise $\triangle v = C_0$ and $\frac{\partial v}{\partial n}|_{\partial\Omega} = 0$, which implies that $C_0 = 0$ and v is a constant. Hence $u = 0$, which contradicts the assumption that u is an eigenfunction. Next choose $\tilde{v} = v - C_0/\lambda$.

It follows from a simple calculation that $\tilde{v}$ is an eigenfunction of (3.2) with respect to the eigenvalue λ.

We comment that in the three-dimensional case, the two eigenvalue problems may not be equivalent. In fact, if Ω is strictly convex, we have $\lambda_1(\Omega) > \mu_2(\Omega)$.

In order to prove Theorem 3.1, we need the following technical lemmas.

Lemma 3.3. *If $u \in L^2(\Omega)$ and $\nabla \cdot u = 0$ in Ω, then u has a divergence free extension on $\mathbf{R}^3$. That is, there exists a function $v \in L^2(\mathbf{R}^3)$ such that $\nabla \cdot v = 0$ in $\mathbf{R}^3$ and $v = u$ in Ω.*

Lemma 3.4. *If $u \in L^2(\Omega)$ and $\nabla \cdot u = 0$ in Ω, then there exists a function $\phi \in H^1(\mathbf{R}^3)$ such that $\nabla \cdot \phi = 0$ in $\mathbf{R}^3$ and $u = \nabla \times \phi$ in Ω.*

The proofs of Lemmas 3.3 and 3.4 may be found in [10], and are omitted here.

Lemma 3.5. *If λ is an eigenvalue of* (3.1) *and u is a corresponding eigenfunction, then*

$$\lambda \geq \frac{\int_\Omega |\nabla \times \phi|^2 dx}{\int_\Omega |\phi|^2 dx} \tag{3.4}$$

for any ϕ such that $u = \nabla \times \phi$ and $\nabla \cdot \phi = 0$.

Proof. Using the divergence theorem, we have

$$\begin{aligned} \lambda &= \frac{\int_\Omega (\nabla \times \nabla \times u, u) dx}{\int_\Omega |u|^2 dx} \\ &= \frac{\int_\Omega |\nabla \times u|^2 dx - \int_{\partial\Omega} (\nabla \times u) \cdot (n \times u) ds}{\int_\Omega |u|^2 dx} \\ &= \frac{\int_\Omega |\nabla \times u|^2 dx}{\int_\Omega |u|^2 dx} = \frac{\int_\Omega |\nabla \times \nabla \times \phi|^2 dx}{\int_\Omega |\nabla \times \phi|^2 dx} \\ &\geq \frac{[\int_\Omega (\nabla \times \nabla \times \phi, \phi) dx]^2}{\int_\Omega |\phi|^2 dx \ \int_\Omega |\nabla \times \phi|^2 dx} \\ &= \frac{[\int_\Omega |\nabla \times \phi|^2 dx]^2}{\int_\Omega |\phi|^2 dx \ \int_\Omega |\nabla \times \phi|^2 dx} = \frac{\int_\Omega |\nabla \times \phi|^2 dx}{\int_\Omega |\phi|^2 dx}, \end{aligned}$$

which completes the proof. □

Remark 3.3. It is important to observe that ϕ is not unique in Lemma 3.4. In fact, if ϕ_1 and ϕ_2 satisfy $u = \nabla \times \phi_1 = \nabla \times \phi_2$, then

$$\nabla \times (\phi_1 - \phi_2) = 0 \quad \text{in} \quad \Omega.$$

Hence there is a function f such that

$$\phi_1 - \phi_2 = \nabla f, \qquad \triangle f = 0.$$

Conversely if $u = \nabla \times \phi$, then $u = \nabla \times \phi_1$ for $\phi_1 = \phi + \nabla f$, where f is any harmonic function on Ω.

The next result provides a characterization of the first eigenvalue of (3.1).

Lemma 3.6. *If $\lambda_1 = \lambda_1(\Omega)$ is the first eigenvalue of (3.1), then*

$$\lambda_1 = \min_{\phi \in S} \frac{\int_\Omega |\nabla \times \phi|^2 dx}{\int_\Omega |\phi|^2 dx},$$

where

$$S = \{\phi \mid \nabla \cdot \phi = 0 \quad \mathit{in} \quad \Omega, \;\; \phi \cdot n = 0 \quad \mathit{on} \quad \partial\Omega\}.$$

Proof. Let u be an eigenfunction of the eigenvalue λ_1. According to Lemma 3.4, for a fixed divergence free function ϕ satisfying $u = \nabla \times \phi$, we choose $\phi_1 = \phi - \nabla f$, where f solves

$$\begin{aligned} \triangle f &= 0 \quad \text{in} \quad \Omega, \\ \frac{\partial f}{\partial n} &= \phi \cdot n \quad \text{on} \quad \partial\Omega \,. \end{aligned}$$

The existence of f follows from the fact that

$$\int_{\partial\Omega} \phi \cdot n ds = \int_\Omega \nabla \cdot \phi dx = 0 \,.$$

Therefore from Lemma 3.5

$$\lambda_1 \geq \frac{\int_\Omega |\nabla \times \phi_1|^2 dx}{\int_\Omega |\phi_1|^2 dx}, \tag{3.5}$$

where

$$\phi_1 \cdot n = \phi \cdot n - \frac{\partial f}{\partial n} = 0 \quad \text{on} \quad \partial\Omega$$

and $\nabla \cdot \phi_1 = 0$. Hence

$$\lambda_1 \geq \min_{\phi \in S} \frac{\int_\Omega |\nabla \times \phi|^2 dx}{\int_\Omega |\phi|^2 dx} \,. \tag{3.6}$$

Denote

$$\gamma = \min_{\phi \in S} \frac{\int_\Omega |\nabla \times \phi|^2 dx}{\int_\Omega |\phi|^2 dx}.$$

We next wish to show that $\gamma \geq \lambda_1$. Let $\phi \in S$ be the minimum; then

$$\int_\Omega (\nabla \times \phi, \nabla \times \psi) dx = \gamma \int_\Omega (\phi, \psi) dx$$

for any $\psi \in S$. For any $h \in C^\infty(\Omega)$, let f be a solution of

$$\begin{aligned} \triangle f &= \nabla \cdot h \quad \text{in} \quad \Omega, \\ \frac{\partial f}{\partial n} &= h \cdot n \quad \text{on} \quad \partial\Omega. \end{aligned}$$

Consider $\psi = h - \nabla f$; then $\nabla \cdot \psi = 0$ and $\psi \cdot n = 0$ on $\partial\Omega$. It follows that

$$\begin{aligned} J &= \int_\Omega (\nabla \times \phi, \nabla \times h) dx - \gamma \int_\Omega (\phi, h) dx \\ &= \int_\Omega (\nabla \times \phi, \nabla \times \psi) dx - \gamma \int_\Omega (\phi, \psi) dx - \gamma \int_\Omega (\phi, \nabla f) dx \\ &= -\gamma \int_\Omega \nabla \cdot (f\phi) dx = -\gamma \int_{\partial\Omega} (f\phi) \cdot n ds = 0. \end{aligned}$$

A direct computation using the divergence theorem shows that

$$J = \int_\Omega (\nabla \times \nabla \times \phi - \gamma\phi, h) dx + \int_{\partial\Omega} ([\nabla \times \phi] \times n, h) ds,$$

which implies that $[\nabla \times \phi] \times n = 0$ on $\partial\Omega$ and $\nabla \times \nabla \times \phi = \gamma\phi$. Set $u = \nabla \times \phi$. We see immediately that u is an eigenfunction of (3.1) with eigenvalue γ. Therefore $\gamma \geq \lambda_1$. □

Remark 3.4. The reason we choose $\phi \in S$ is that such a choice makes $\int_\Omega |\phi|^2 dx$ the smallest in the family

$$\{\phi \mid u = \nabla \times \phi, \ \ \nabla \cdot \phi = 0\}.$$

Now we are ready to prove Theorem 3.1.

Proof of Theorem 3.1. Note that for any $\phi \in S$ and any constant vector $c \in \mathbf{R}^3$, we have

$$\begin{aligned}
\int_\Omega |\phi + c|^2 dx &= \int_\Omega (|\phi|^2 + |c|^2) dx + 2\int_\Omega \phi \cdot c dx \\
&= \int_\Omega (|\phi|^2 + |c|^2) dx + 2\int_\Omega \nabla \cdot [(c \cdot x)\phi] dx \\
&= \int_\Omega (|\phi|^2 + |c|^2) dx + 2\int_{\partial\Omega} (c \cdot x)\phi \cdot n ds \\
&= \int_\Omega (|\phi|^2 + |c|^2) dx \geq \int_\Omega |\phi|^2 dx.
\end{aligned}$$

For any $\phi \in S$, let $\psi = \phi + c$, with c chosen so that $\int_\Omega \psi dx = 0$. An application of Lemma 2.4 gives

$$\begin{aligned}
\frac{\int_\Omega |\nabla \times \phi|^2 dx}{\int_\Omega |\phi|^2 dx} &= \frac{\int_\Omega [|\nabla \times \phi|^2 + |\nabla \cdot \phi|^2] dx}{\int_\Omega |\phi|^2 dx} \\
\geq \quad & \frac{\int_\Omega [|\nabla \times \phi|^2 + |\nabla \cdot \phi|^2] dx}{\int_\Omega |\psi|^2 dx} \geq \frac{|\psi|_{1,\Omega}^2}{\int_\Omega |\psi|^2 dx} \geq \mu_2(\Omega),
\end{aligned}$$

where we have used the well-known characterization

$$\mu_2(\Omega) \leq \frac{\int_\Omega |\nabla g|^2 dx}{\int_\Omega g^2 dx}$$

for any g such that $\int_\Omega g dx = 0$. Finally, Lemma 3.6 yields that

$$\lambda_1(\Omega) \geq \min_{\phi \in S} \frac{\int_\Omega |\nabla \times \phi|^2 dx}{\int_\Omega |\phi|^2 dx} \geq \mu_2(\Omega),$$

which completes the proof of Theorem 3.1. □

4. Uniqueness of the inverse problem

Suppose that for a given incident plane wave E_I, $E_{f_j}(x)$ $(j = 1, 2)$ are Λ-quasiperiodic and solve the scattering problem (2.4), (2.5), and (2.10) with respect to the profiles $f_j(x_1, x_2)$, where the functions f_j are Λ-periodic. Let $b > \max\{f_1(x_1, x_2), f_2(x_1, x_2)\}$ be a fixed constant. Denote $D_j = \{f_j < x_3 < b\}$.

Definition. Two profiles Γ_1 and Γ_2 are said to satisfy Property (A) if there is a simply connected bounded domain U such that the following three conditions are satisfied:

- U is convex,
- $\partial U = \partial U_1 \cup \partial U_2$, $\partial U_1 \subset \Gamma_1$ and $\partial U_2 \subset \Gamma_2$,
- ∂U is C^2.

For a bounded set U, we denote its radius by $|U|$. Now, we are ready to state the main uniqueness result for the inverse problem.

Theorem 4.1. *Assume that f_1, f_2 are Λ-periodic C^2 functions and that the profiles $S_1 = \{x_3 = f_1(x_1, x_2)\}$ and $S_2 = \{x_3 = f_2(x_1, x_2)\}$ satisfy Property (A). Then there is a constant $\delta(k) > 0$ such that if $|U| \le \delta(k)$, then $n\times E_{f_1}|_{x_3=b} = n\times E_{f_2}|_{x_3=b}$ implies $f_1(x_1, x_2) = f_2(x_1, x_2)$.*

Proof. We prove this theorem by contradiction. Assume $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ are different functions. Let $E = E_{f_1} - E_{f_2}$, $f(x_1, x_2) = \max\{f_1(x_1, x_2), f_2(x_1, x_2)\}$, and $D = \{x : f(x_1, x_2) < x_3 < b\}$.

Then

(4.1)
$$n \times E|_{x_3=f(x_1,x_2)} = \begin{cases} 0 & \text{for } f_1(x_1, x_2) = f_2(x_1, x_2), \\ -n \times E_{f_2}(x_1, x_2, f_1(x_1, x_2)) & \text{for } f_1(x_1, x_2) > f_2(x_1, x_2), \\ n \times E_{f_1}(x_1, x_2, f_2(x_1, x_2)) & \text{for } f_1(x_1, x_2) < f_2(x_1, x_2)\ . \end{cases}$$

It follows from Lemma 2.1 and $n \times E_{f_1}|_{x_3=b} = n \times E_{f_2}|_{x_3=b}$ that

$$n \times (\nabla \times E_{f_1}|_{x_3=b}) = n \times (\nabla \times E_{f_2}|_{x_3=b})$$

or, since $n = e_3$ on Γ,

$$n \times H_{f_1}|_{x_3=b} = n \times H_{f_2}|_{x_3=b}\ ,$$

where H_{f_1}, H_{f_2} are the corresponding magnetic fields, respectively. Here a simple application of Lemma 2.3 yields that

$$E = 0 \ \text{ in } D\ .$$

In particular,

$$n \times E_{f_2}(x_1, x_2, f_1(x_1, x_2)) = 0 \qquad \text{for } \ f_1 > f_2\ , \tag{4.2}$$

$$n \times E_{f_1}(x_1, x_2, f_2(x_1, x_2)) = 0 \qquad \text{for } \ f_1 < f_2\ . \tag{4.3}$$

Because of the assumption, there is a bounded and convex domain U whose boundary $\partial U \subset S_1 \cup S_2$. Thus the set U is either in D_1 or in D_2. We shall only consider the case $U \subset D_1$; the same argument may be used to treat the other case. It is easily seen from the boundary condition on E_{f_1} and (4.3) that

$$n \times E_{f_1}|_{\partial U} = 0\ . \tag{4.4}$$

Moreover, since E_{f_1} satisfies Maxwell's equations (2.1), (2.2) in U, $\nabla \cdot E_{f_1} = 0$.

Multiplying both sides of the equation

$$\nabla \times \nabla \times E_{f_1} - k^2 E_{f_1} = 0 \text{ in } \ U$$

by $\overline{E_{f_1}}$ and integrating over U lead to

$$\int_U |\nabla \times E_{f_1}|^2 dx - k^2 \int_U |E_{f_1}|^2 dx = 0\ . \tag{4.5}$$

Here we have used the boundary condition $n \times E_{f_1} = 0$ on ∂U.

Since $\lambda_1(U)$ is the first eigenvalue, we have

$$\lambda_1^2(U) \int_U |E_{f_1}|^2 dx \le k^2 \int_U |E_{f_1}|^2 dx\ . \tag{4.6}$$

From Corollary 3.2, $\lambda_1(U) \to \infty$ as $|U| \to 0$. Thus there is a constant $\delta(k)$ such that if $|U| < \delta(k)$ then $\lambda_1(U) > k^2$. Therefore, using (4.6), for $|U| < \delta(k)$

$$E_{f_1} = 0 \text{ in } U .$$

Now an application of Lemma 2.3 again gives that

$$E_{f_1} = 0 \text{ in } D_1 . \tag{4.7}$$

But this contradicts the identity (2.7), since $\beta \neq 0$ and E_I is a downward incoming nonzero plane wave.

Hence $f_1(x_1, x_2) = f_2(x_1, x_2)$. □

Remark 4.1. When ϵ_0 has a nonzero imaginary part, a global uniqueness result has been proved in [3]. In that case, the hypotheses on Property (A) and sufficient smallness of $|U|$ will no longer be needed. Indeed $E_{f_1} = 0$ in U follows directly from (4.5). However, in general, global uniqueness may not be possible when ϵ_0 is real. This is evident in the simplest case with a plane wave incident on a flat surface. In this case, the solution of the scattering problem can be written down explicitly. The nonuniqueness is obvious since the scattering fields will remain the same when one moves the flat surface up or down in certain multiples of the wavelength.

5. Stability of the inverse problem

In applications, it is impossible to make exact measurements. Thus stability results are crucial in the reconstruction of profiles.

For any two domains D_1 and D_2 in $\mathbf{R}^3$, define

$$d(D_1, D_2) = \max\{\rho(D_1, D_2), \rho(D_2, D_1)\},$$

where

$$\rho(D_j, D_l) = \sup_{x \in D_j} \inf_{y \in D_l} |x - y| .$$

That is, $d(D_1, D_2)$ is the Hausdorff distance between D_1 and D_2. Denote $D = \{x;\ f(x_1, x_2) < x_3 < b\}$. Let D_h be the domain between the plane $x_3 = b$ and a periodic surface S_h with the same period as the function f. Assume that S_h is given by the function $x = F(s) + h\sigma_h(s)n(s)$, where $x = F(s_1, s_2)$ $(0 \leq s_1 \leq s_0$, $0 \leq s_2 \leq \tilde{s}_0)$ is a parametric representation of the interface S, $F \in C^2$, $n(s)$ is again the outward normal, and assume that

$$\begin{aligned} &\sigma_h \in C^2, \quad \|\sigma_h\|_{C^2} \leq C \ \ (C \text{ independent of } h)\,, \\ &\sigma_h \to \sigma \text{ uniformly as } h \to 0 . \end{aligned} \tag{5.1}$$

Clearly

$$C_1 h \leq d(D, D_h) \leq C_2 h ,$$

where C_1 and C_2 are positive constants. Let $\mathcal{H}(x)$ be the mean curvature of S. In addition, assume that

$$\sigma\, \mathcal{H} \not\equiv 0 . \tag{5.2}$$

For the fixed incident plane wave E_I, consider the scattering problem

$$\begin{aligned} \nabla \times \nabla \times E_h - k^2 E_h &= 0 \quad \text{in } D_h \,, \\ n \times E_h &= 0 \quad \text{on } S_h \,, \\ e_3 \times (\nabla \times E_h) &= e_3 \times (\nabla \times E_I) + B(P(E_h - E_I)) \text{ on } \Gamma \,, \end{aligned}$$

where P is again the projection operator defined in Lemma 2.1.

We have the following local stability result.

Theorem 5.1. *Under the assumptions* (5.1) *and* (5.2),

$$d(D_h, D) \le C \|n \times (E_h - E)|_{x_3=b}\|_{H^{1/2}} \ , \tag{5.3}$$

where the constant C is independent of h.

Remark 5.1. The constant C in (5.3) may depend on the family $\{\sigma_h\}$. The result indicates that for small h, if the boundary measurements are $O(h)$ close to the scattered fields in the $H^{1/2}$ norm, then D_h is $O(h)$ close to D in the Hausdorff distance.

Remark 5.2. The result extends Theorem 2.2 in [6] to the doubly periodic case. We will use the method and ideas developed in [6]. However, the situation here is more complicated because

- the direct problem is now in a vector form;
- the data employed is the tangential electric field rather than the electric field.

Construct a surface $\tilde{S}_h: \ x = F(s) + h\tilde{\sigma}_h(s)n(s)$ with $\tilde{\sigma}_h(s) \in C^2$ lie above both S_h and S. Then the quotient difference function $e_h = (E_h - E)/h$ satisfies

$$\nabla \times \nabla e_h - k^2 e_h = 0 \quad \text{in } \tilde{D}_h \ , \tag{5.4}$$

$$e_3 \times (\nabla \times e_h) = B(Pe_h) \quad \text{on } \Gamma, \tag{5.5}$$

$$|n \times e_h| \le C \quad \text{on } \tilde{S}_h \ , \tag{5.6}$$

where $\tilde{D}_h$ denotes the region between Γ and $\tilde{S}_h$ in one period $(x_1,\, x_2)$.

For a domain $\Omega \subset \mathbf{R}^3$ which is bounded in x_3 and is periodic in x_1, x_2 of period $\Lambda = (\Lambda_1, \Lambda_2)$, we denote by $H_p^m(\Omega)$ the subset of all functions that are the restrictions to Ω of the periodic functions in $H_{loc}^m(\mathbf{R}^3)$. Recall that a function u is quasiperiodic if $ue^{-i\alpha\cdot x}$ is periodic in x_1 of period Λ_1 and in x_2 of period Λ_2. A function u is said to be in $H_{qp}^m(\Omega)$ if it is quasiperiodic and $ue^{-i\alpha\cdot x} \in H_p^m(\Omega)$.

We first establish

Lemma 5.2.

$$\int_{\tilde{D}_h} |e_h|^2 dx \le C \ , \tag{5.7}$$

where the constant C is independent of h.

Proof. Introduce a scattering problem

$$\nabla \times \nabla \times w_h - k^2 w_h = e_h \quad \text{in } \tilde{D}_h \ , \tag{5.8}$$

$$e_3 \times (\nabla \times w_h) = B^*(Pw_h) \quad \text{on } \Gamma \ , \tag{5.9}$$

$$n \times w_h = 0 \quad \text{on } \tilde{S}_h \ , \tag{5.10}$$

where B^* is the adjoint of B in the sense that

$$\int_{\Gamma_p} (Bf)\overline{g} ds = \int_{\Gamma_p} f\overline{B^* g} ds,$$

where $\Gamma_p = \{(x_1, x_2, b) \mid 0 < x_1 < \Lambda_1, 0 < x_2 < \Lambda_2\}$. We use the L^2 scalar product

$$(f, g) = \int f\overline{g} dx \ .$$

This problem is the adjoint problem of the original scattering problem. The fact that it has a unique solution follows from the existence and uniqueness of solutions to the original problem and the Fredholm theory.

For any $\psi \in H^1_{qp}(\tilde{D}_h)$, we then have

$$\begin{aligned}(e_h, \psi) &= \int_{\tilde{D}_h} e_h \overline{\psi} dx = \int_{\tilde{D}_h} \nabla \times w_h \cdot \nabla \times \overline{\psi} dx \\ &\quad - \int_{\tilde{D}_h} k^2 w_h \overline{\psi} dx + \int_{\Gamma \cup \tilde{S}_h} n \cdot ((\nabla \times w_h) \times \overline{\psi}) ds\end{aligned}$$

or

(5.11)

$$(\overline{e_h}, \overline{\psi}) = \int_{\tilde{D}_h} \overline{\nabla \times w_h} \cdot \nabla \times \psi dx - \int_{\tilde{D}_h} k^2 \overline{w_h} \psi dx + \int_{\Gamma \cup \tilde{S}_h} n \cdot (\overline{(\nabla \times w_h)} \times \psi) ds \; .$$

From the scattering problem for e_h (5.4–5.6), we get similarly that

$$\int_{\tilde{D}_h} \nabla \times e_h \cdot \nabla \times \overline{\phi} dx - k^2 \int_{\tilde{D}_h} e_h \cdot \overline{\phi} dx + \int_{\Gamma \cup \tilde{S}_h} n \cdot ((\nabla \times e_h) \times \overline{\phi}) ds = 0 \; . \tag{5.12}$$

By choosing $\psi = e_h$ in (5.11) and $\phi = w_h$ in (5.12), we obtain after some simple calculation that

$$||e_h||^2 = \int_{\Gamma \cup \tilde{S}_h} n \cdot (\overline{(\nabla \times w_h)} \times e_h) ds - \int_{\Gamma \cup \tilde{S}_h} n \cdot ((\nabla \times e_h) \times \overline{w_h}) ds \; .$$

Using (5.9) and (5.5), it is easy to see that

$$\int_{\Gamma} n \cdot (\overline{(\nabla \times w_h)} \times e_h) ds - \int_{\Gamma} n \cdot ((\nabla \times e_h) \times \overline{w_h}) ds = 0 \; .$$

Thus by observing (5.10) and (5.6),

$$\begin{aligned}||e_h||^2 &= \int_{\tilde{S}_h} n \cdot (\overline{(\nabla \times w_h)} \times e_h) ds \\ &\leq C \int_{\tilde{S}_h} |n \times \overline{(\nabla \times w_h)}| ds \\ &\leq C ||e_h|| \; , \end{aligned} \tag{5.13}$$

which completes the proof. □

Proof of Theorem 5.1. We prove it by contradiction. Suppose that (5.3) is not true. Then

$$||\frac{n \times E_h - n \times E}{h}||_{H^{1/2}(\Gamma)} \to 0 \; ,$$

that is,

$$e_3 \times e_h \to 0 \; , \;\; e_3 \times (\nabla \times e_h) \to 0 \;\text{ in }\; H^{1/2}(\Gamma) \; . \tag{5.14}$$

Using Lemma 5.2 and the elliptic theory, we deduce that e_h converges to a function $\tilde{e}$ uniformly in compact subsets of $\overline{D} \backslash S$, $\nabla \times \nabla \times \tilde{e} - k^2 \tilde{e} = 0$ in $D \backslash S$, and $n \times \tilde{e} = n \times (\nabla \times \tilde{e}) = 0$ on Γ. By the unique continuation result Lemma 2.3, $\tilde{e} = 0$ in $\overline{D} \backslash S$. Thus

$$e_h \to 0 \;\text{ uniformly in compact subsets of }\; D \backslash S \; . \tag{5.15}$$

Since $\tilde{S}_h$ is uniformly in C^2, for any smooth function Φ we can construct a family of functions ψ_h such that $\psi_h \in H^2_{qp}(\tilde{D}_h)$ and

$$\psi_h = 0 \text{ on } \tilde{S}_h\,,$$
$$\|\psi_h\|_{C^2(\tilde{D}_h)} \le C\,,$$
$$\left\|\tfrac{\partial \psi_h}{\partial n} - \Phi\right\|_{L^\infty(\tilde{S}_h)} \to 0\,.$$

From (5.5), integration by parts yields that

$$\int_{\tilde{D}_h} e_h \cdot (\Delta + k^2)\overline{\phi} dx + \int_{\Gamma\cup\tilde{S}_h} [\frac{\partial e_h}{\partial n}\overline{\phi} - \frac{\partial \overline{\phi}}{\partial n} e_h] ds = 0\,. \tag{5.16}$$

Choosing $\phi = \psi_h$ in (5.16) and using (5.15), (5.14), we obtain, as $h \to 0$,

$$\lim_{h\to 0}\int_{\tilde{S}_h} e_h \cdot (n \times \nabla \times \overline{\psi}_h) ds = \int_S \lambda\sigma \frac{\partial E}{\partial n}\cdot \Phi = 0\,, \tag{5.17}$$

where λ is a positive valued function. Since $\sigma\mathcal{H} \not\equiv 0$ on S, we assume, by using continuity, that both σ and $\mathcal{H}$ are nonzero on an open subset S_0 of S, and $\mathcal{H}$ does not change sign on S_0. The identity (5.17) holds for arbitrary Φ. Thus

$$\frac{\partial E}{\partial n} = 0 \;\text{ on } S_0\,. \tag{5.18}$$

Using an identity in differential geometry given by Abboud and Nédélec (Lemma 3.1 in [2]), we have

$$\nabla \cdot E = \nabla_{S_0}\cdot (n \times (E \times n)) + 2\mathcal{H} E \cdot n + \frac{\partial E \cdot n}{\partial n} \;\text{ on } S_0\,. \tag{5.19}$$

Using (5.18), (5.19), $\nabla \cdot E = 0$, and $n \times E = 0$ on S_0, we get by some simple calculation that

$$\begin{aligned}\int_{S_0} \frac{\partial E}{\partial n}\overline{E} ds &= \int_{S_0} \frac{\partial E\cdot n}{\partial n} n \cdot \overline{E} ds \\ &= -2\int_{S_0} \mathcal{H} n\cdot E\; n\cdot \overline{E} ds = -2\int_{S_0} \mathcal{H}|n\cdot E|^2 ds\,.\end{aligned}$$

Thus

$$n \cdot E = 0 \;\text{ on } S_0\,.$$

Therefore $E = 0$ and $\frac{\partial E}{\partial n} = 0$ on S_0. We infer by unique continuation that $E = 0$ in D, which once again is a contradiction. □

Remark 5.3. It remains to see whether or not the hypothesis on $\mathcal{H}$ may be dropped, that is, replace Hypothesis (5.2) with $\sigma \not\equiv 0$. In the case $H \equiv 0$, S is a plane from Bernstein's theorem on minimum surfaces, and so the stability result remains valid. Actually, without loss of generality, we may assume that $S = \{x_3 = 0\}$. Let $\sigma \neq 0$ on S_0 (an open set of S); then $E = (0, 0, E_3(x))$ from unique continuation. Because $\nabla \cdot E = 0$, we get $\frac{\partial E_3}{\partial x_3} = 0$ in D, which is a contradiction to (2.7) because $q_3 = -\beta \neq 0$. The above argument works also in the case where there is an open set S_0 of S such that $\sigma \neq 0$ on S_0 and S_0 is flat.

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