CORRIGENDUM TO “WEST’S PROBLEM ON EQUIVARIANT HYPERSONES AND BANACH-MAZUR COMPACTA”

SERGEY ANTONYAN

In our article [1], on p. 3389, the definition of the weak topology of the $G$-nerve $\mathcal{N}(U)$ contains a gap. Namely, it is claimed there that the topology on $\mathcal{N}(U)$ induced from $\mathcal{J}$ is the weak one, which is false. The author apologizes for this mistake.

Nevertheless, in the proofs of Lemmas 4.2, 4.4 and 5.2, where the topology of $\mathcal{N}(U)$ is essential, in fact the right weak topology of $\mathcal{N}(U)$ was applied. Thus, all of the proofs given in [1] are correct and complete.

Using the notation and references adopted in [1], the above-mentioned gap in the definition of the topology of the $G$-nerve $\mathcal{N}(U)$ may be filled by replacing the text on p. 3389 starting in line 26 and ending in line 35, by the following.

“For every simplex $L = (\mu_0, \ldots, \mu_n) \subset \tilde{\mathcal{N}}(U)$, set 

$$\Delta(L) = \bigcup \{ \Delta(S, F_S) | S \text{ is a subsimplex of } L \}.$$  

Clearly, $\Delta(L)$ is an invariant subset of the finite join $G/H_{\mu_0} \ast \cdots \ast G/H_{\mu_n}$. We always will consider the induced topology and $G$-action on $\Delta(L)$. Observe that, if $N$ is a subsimplex of $L$, then $\Delta(N)$ is a closed invariant subset of $\Delta(L)$. Indeed, let $\xi : G/H_{\mu_0} \ast \cdots \ast G/H_{\mu_n} \to L$ be the continuous map sending the point $\sum_{i=0}^n t_{\mu_i} g_{\mu_i} H_{\mu_i} \in G/H_{\mu_0} \ast \cdots \ast G/H_{\mu_n}$ to the point $\sum_{i=0}^n t_{\mu_i} \mu_i \in L$. Since $P_{LN} : \prod_{\mu \in L} G/H_{H_\mu} \to \prod_{\mu \in N} G/H_{H_\mu}$ is the Cartesian projection, we see that the preimage $\xi^{-1}(N)$ is just $\Delta(N)$. Since $N$ is closed in $L$, this yields that $\Delta(N)$ is closed in $\Delta(L)$, as required. Invariance of $\Delta(N)$ is evident.

It is clear that, if $K \subset \tilde{\mathcal{N}}(U)$ is yet another simplex, then $\Delta(L) \cap \Delta(K) = \Delta(L \cap K)$. Consequently, $\Delta(L) \cap \Delta(K)$ is closed in both $\Delta(L)$ and $\Delta(K)$.

Consider the following invariant subset of $\mathcal{J}$:

$$\mathcal{N}(U) = \bigcup \{ \Delta(L) | L \in \tilde{\mathcal{N}}(U) \}.$$  

We consider the weak topology on $\mathcal{N}(U)$ determined by the family

$$\{ \Delta(L) | L \in \tilde{\mathcal{N}}(U) \}.$$  

Namely, a set $U \subset \mathcal{N}(U)$ is, by definition, open in $\mathcal{N}(U)$ if and only if $U \cap \Delta(L)$ is open in $\Delta(L)$ for every simplex $L \subset \tilde{\mathcal{N}}(U)$.

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The $G$-action on $\mathcal{N}(U)$, defined by the following formula, makes $\mathcal{N}(U)$ a $G$-space, called the $G$-nerve of $U$:

$$g \ast \left( \sum_{\mu \in M} t_\mu g_\mu' H_\mu \right) = \sum_{\mu \in M} t_\mu g g_\mu H_\mu, \quad g \in G.$$ 

Since the intersection $\Delta(L) \cap \Delta(K)$ is closed in both $\Delta(L)$ and $\Delta(K)$, we see that each space $\Delta(L)$ retains its original topology and is a closed invariant subset of $\mathcal{N}(U)$ (see, e.g., [2] Ch. VI, §8). We call $\Delta(L)$ a $G$-n-simplex over the n-simplex $L$.

In the proofs of Lemmas 4.4 and 5.2 the following well-known and easily proved property of the weak topology is used: a map $f: \mathcal{N}(U) \to Z$ is continuous if and only if each restriction $f|_{\Delta(L)}$ is continuous."

As is defined on page 3389, lines 9–10, the elements of a $G$-normal cover $U$ are tubular slice-sets $gS_\mu$ with companion groups $H_\mu$. However, in order to emphasize the role of $H_\mu$, we have used the denotation $(gS_\mu, H_\mu)$ instead of $gS_\mu$, which in some occasions may cause confusion. Thus, in Lemmas 4.1, 4.2 and 5.2 the denotation $U = \{ (gS_\mu, H_\mu) | g \in G, \mu \in M \}$ should be replaced by $U = \{ gS_\mu | g \in G, \mu \in M \}$, where $S_\mu$ is an $H_\mu$-slice.

For the proof of Lemma 5.2 it is important to formulate Lemma 4.1 in the following more precise form.

**Lemma 4.1.** Let $X$ be a paracompact $G$-space and $V$ an open cover of $X$. Then $X$ admits a $G$-normal cover $U = \{ gS_\lambda | g \in G, \lambda \in \Lambda \}$ with the companion groups $\{ H_\lambda \}_{\lambda \in \Lambda}$ such that each $H_\lambda$ is the stabilizer of a point $x_\lambda \in S_\lambda$ and $U$ is a star-refinement of $V$.

In Lemma 5.2 under the term “$\varepsilon$-cover” we mean the family of all open balls in $L_0(n)$ which have radius $\varepsilon$.

In the formulation and in the proof of Lemma 5.2, always $G = O(n)$.

Also, one should correct the following misprints:

1. page 3389, line 14: “$O \in U_1$” should be “$O \in U_1$”.
2. page 3389, line 20: “of $U$” should be “of $U$”.
3. page 3390, line 27: “$(gS_\lambda, G_x)$” should be “$gS_\lambda$”.
4. page 3390, line 30: “an open $G$-normal cover” should be “a $G$-normal cover”.
5. page 3391, line 28: “$\Delta(L, F_\mu)$” should be “$\Delta(L)$”.
6. page 3392, line 2: “$R(x) \in F_{3n-1}(s^1)$” should be “$R(x) \in F_{3n-1}(s^1)$”, where $s^1$ is the 1-dimensional skeleton of $s$”.
7. page 3392, line 7: “$\Delta(L, F_\mu)$” should be “$\Delta(L)$”.
8. page 3392, line 13: “$F_{3n-1}(s^1)$” should be “$F_{3n-1}(s^1)$”.
9. page 3394, line 30: “$q'(g_1A_\mu) = q'(g_1A_\lambda)$” should be “$q'(g_1H_\mu) = q'(g_1H_\lambda)$”.
10. page 3394, line 36: “of $g_0A_\lambda$” should be “of $g_0A_\lambda$ $g_1A_\mu$”.
11. page 3394, line 41: “of $g_1A_\mu$” should be “of $g_0A_\lambda$ $g_1A_\mu$”.
12. page 3398, line 22: “domain. Since” should be “domain V. Since”.
References


**Department of Mathematics, Faculty of Sciences, Universidad Nacional Autónoma de México, Mexico D.F. 04510, Mexico**

*E-mail address: antonyan@servidor.unam.mx*