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CORRECTIONS TO "INVOLUTIONS FIXING $\mathbb{RP}^{\mathbf{odd}} \sqcup P(h,i)$, II"

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The purpose of this note is to correct statements of some assertions in [1]. The mistake occurs in the argument of the case in which the normal bundle ν^k over $P(h,i)$ is nonstandard. Specifically, some incorrect calculations first happen in the arguments of the cases $u = 0$ and $u > 1$ of page 1309 (in the proof of Lemma 3.4 of [1]). This leads to the loss of the existence of some involutions with nonstandard normal bundle ν^k in those two cases, so that the statements of Lemma 3.4 and Proposition 3.4 are incorrect, and so is part of the statement of Theorem 2.3 in [1].

Following the notations of [1], Lemma 3.4 in [1] should be corrected as follows.

Lemma. *If ν^k is nonstandard, then $h = 2$ with the following possible cases:*

(A) *for $u = 0$, one has that $(k,a) = (2,2)$, $\nu^2 = \tau \otimes \eta$ and $i \equiv 3 \mod 4$;*

(B) *for $u = 1$, one has that $(k,a) = (4,1)$ and $\nu^4 = \tau \oplus (\tau \otimes \eta)$;*

(C) *for $u > 1$, one has that either $a = 1$ and $6 \leq k$ ($\equiv 2 \mod 4$) with ν^k stably cobordant to $3\xi \oplus (\tau \otimes \eta)$ or $a = 3$ and $4 \leq k$ ($\equiv 0 \mod 4$) with ν^k stably cobordant to $\xi \oplus (\tau \otimes \eta)$, where ξ is a 1-plane bundle over $P(2, 2^u(2v+1))$, η is a 2-plane bundle over $P(2, 2^u(2v+1))$, and τ is the 2-plane bundle (the tangent bundle of $\mathbb{RP}^2$ pulled back to $P(2, 2^u(2v+1))$).*

Note. Stong in [3] found the strange tensor product $\tau \otimes \eta$ over the Dold manifold with the total class $1 + c + c^2 + d$.

Proof. Since the mistake in the proof of Lemma 3.4 of [1] only occurs in the cases $u = 0$ and $u > 1$ of page 1309 but other arguments are true, one needs to merely show (A) and (C).

If $u = 0$, then a is even and $k = 2$ by Lemma 3.1 in [1]. To ensure $k = 2$, from (3.2) in [1] one must have $a = 2$, so the total class $w(\nu^2) = 1 + c + c^2 + d$. Thus, $\nu^2 = \tau \otimes \eta$. By direct computation, one has that

$$w[0]_1 = \begin{cases} c & \text{on } P(2,i), \\ \alpha & \text{on } \mathbb{RP}^2, \end{cases} \quad \text{and} \quad w[0]_2 = \begin{cases} ce + c^2 + d + \binom{i+3}{2}c^2 & \text{on } P(2,i), \\ \alpha^2 + e^2 & \text{on } \mathbb{RP}^2. \end{cases}$$

Form the class

$$\hat{w}_2 = w[0]_2 + e^2 + w[0]_1^2 = \begin{cases} ce + e^2 + d + \binom{i+3}{2}c^2 & \text{on } P(2,i), \\ 0 & \text{on } \mathbb{RP}^2. \end{cases}$$

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One has that the value of $\hat{w}_2 e^{2i+1}$ on $\mathbb{RP}^2$ is zero, so the value of this on $P(2,i)$ is zero, too. Thus

$$0 = \hat{w}_2 e^{2i+1}[\mathbb{RP}(\nu^2)] = \frac{1+c+d+\binom{i+3}{2}c^2}{1+c+c^2+d}[P(2,i)] = 1 + \binom{i+3}{2}$$

and so $i \equiv 3 \mod 4$.

If $u > 1$, then, by Lemma 3.1 in [1], a is odd and k is even. Further, $a = 1$ or 3 since $h = 2$. Now by direct calculations,

$$\begin{aligned}
\frac{1}{w(\nu^k)}[P(2, 2^u(2v+1))] &= \frac{1}{(1+c)^a(1+c+d)(1+\frac{c^2d}{1+d})}[P(2,2^u(2v+1))] \\
&= \frac{d^{2^u(2v+1)}}{(1+c)^{a+1}}[P(2,2^u(2v+1))] \\
&= \begin{cases} 1 & \text{if } a = 1, \\ 0 & \text{if } a = 3, \end{cases}
\end{aligned}$$

and

$$\frac{1}{w(\nu^{j+1})}[\mathbb{RP}^2] = \frac{1}{(1+\alpha)^k}[\mathbb{RP}^2] = \begin{cases} 1 & \text{if } k \equiv 2 \mod 4, \\ 0 & \text{if } k \equiv 0 \mod 4. \end{cases}$$

Thus, $a = 1$ if and only if $k \equiv 2 \mod 4$, and $a = 3$ if and only if $k \equiv 0 \mod 4$. When $a = 1$, one has $w(\nu^k) = (1+c)^3(1+c+c^2+d)$ so $k > 4$ and ν^k is stably cobordant to $3\xi \oplus (\tau \otimes \eta)$; when $a = 3$, one has $w(\nu^k) = (1+c)(1+c+c^2+d)$ so $k > 2$ and ν^k is stably cobordant to $\xi \oplus (\tau \otimes \eta)$. □

Next, Proposition 3.4 in [1] should be corrected as follows.

Proposition. *The involution* $(M^{2^{u+1}(2v+1)+k+h}, T)$ *fixing* $\mathbb{RP}^{2^{u+1}(2v+1)+k-1} \sqcup P(h, 2^u(2v+1))$ *with* ν^k *nonstandard exists only for the following four cases:*

(i) $(h,u,k,a) = (2,0,2,2)$, $\nu^2 = \tau \otimes \eta$ *and* v *is odd;*

(ii) $(h,u,k,a) = (2,1,4,1)$ *and* $\nu^2 = \tau \oplus (\tau \otimes \eta)$*;*

(iii) $(h,a) = (2,1)$ *with* $u > 1$, $k \equiv 2 \mod 4$ *is in the range* $6 \leq k \leq Y_1$ *and* ν^k *is stably cobordant to* $3\xi \oplus (\tau \otimes \eta)$*, where* $Y_1 \leq 2^{u+1} - 2$*;*

(iv) $(h,a) = (2,3)$ *with* $u > 1$, $k \equiv 0 \mod 4$ *is in the range* $4 \leq k \leq Y_2$ *and* ν^k *is stably cobordant to* $\xi \oplus (\tau \otimes \eta)$*, where* $Y_2 \leq 2^{u+1}$.

Note. Proposition 3.4 in [1] only indicates the existence of the involution of case (ii) in the above proposition, and its proof is true. However, as stated in the above proposition, actually there are also other cases in which the involutions with ν^k nonstandard exist.

Proof. First, by the above lemma, one has $h = 2$. As stated in the introduction of [1], it suffices to discuss the existence of involutions $(\bar{M}^{2^{u+1}(2v+1)+k+2}, \bar{T})$ fixing $\mathbb{RP}^2$ with normal bundle $\nu^{2^{u+1}(2v+1)+k}$ having $w(\nu^{2^{u+1}(2v+1)+k}) = (1+\alpha)^{2^{u+1}+k}$ and $P(h, 2^u(2v+1))$ with ν^k nonstandard. In a similar way to the argument of case (ii) as shown in the proof of Proposition 3.4 of [1], one can easily prove that the involution with ν^k nonstandard exists for the following cases:

(a) $(h,u,k,a) = (2,0,2,2)$, $\nu^2 = \tau \otimes \eta$ and v is odd;

(b) $(h,k,a) = (2,6,1)$ with $u > 1$ and $w(\nu^6) = (1+c)^3(1+c+c^2+d)$;

(c) $(h,k,a) = (2,4,3)$ with $u > 1$ and $w(\nu^4) = (1+c)(1+c+c^2+d)$, which means that the above proposition holds for case (i), case (iii) with $k = 6$, and case (iv) with $k = 4$. In particular, the same argument as above can also show that $\bar{M}^{2^{u+1}(2v+1)+k+2}$ is cobordant to zero in case (b) with $u > 2$ and case (c). Furthermore, one can apply the Γ-operation to $(\bar{M}^{2^{u+1}(2v+1)+k+2}, \bar{T})$ to obtain more involutions with nonstandard ν^k. Thus, it remains to estimate the upper bound of k in cases (iii) and (iv). If $u > 1$, by direct computations, one has that

$$w[0]_4 = \begin{cases} c^2d + cde + de^2 + d^2 + \binom{a+1}{2}c^2e^2 & \text{on } P(2, 2^u(2v+1)), \\ \binom{a+1}{2}\alpha^2 e^2 & \text{on } \mathbb{RP}^2. \end{cases}$$

Form the class

$$\hat{w}_4 = w[0]_4 + \binom{a+1}{2} w[0]_1^2 e^2 = \begin{cases} c^2d + cde + de^2 + d^2 & \text{on } P(2, 2^u(2v+1)), \\ 0 & \text{on } \mathbb{RP}^2. \end{cases}$$

For case (iii), if $k > 2^{u+1} - 2$, one has that the value of

$$\hat{w}_4^{2^u(v+1)} e^{1+k-2^{u+1}}$$

on $\mathbb{RP}^2$ is zero, but the value of this on $P(2, 2^u(2v+1))$ is

$$\begin{aligned} \hat{w}_4^{2^u(v+1)} e^{1+k-2^{u+1}} [\mathbb{RP}(\nu^k)] &= \frac{d^{2^u(v+1)}(1+c+c^2+d)^{2^u(v+1)}}{w(\nu^k)} [P(2, 2^u(2v+1))] \\ &= \frac{d^{2^u(v+1)}(1+c+c^2+d)^{2^u(v+1)}}{(1+c)^3(1+c+c^2+d)} [P(2, 2^u(2v+1))] \\ &= (1+c)d^{2^u(v+1)}(1+c+c^2+d)^{2^u(v+1)-1}[P(2, 2^u(2v+1))] \\ &= d^{2^u(2v+1)} \binom{2^u(v+1)-1}{2^u v} (1+c)^{3(2^u-1)+1}[P(2, 2^u(2v+1))] \\ &= c^2 d^{2^u(2v+1)} [P(2, 2^u(2v+1))] \\ &= 1, \end{aligned}$$

which leads to a contradiction. Thus, one has that $k \leq 2^{u+1} - 2$ so $Y_1 \leq 2^{u+1} - 2$.

For case (iv), if $k > 2^{u+1}$, one has that the value of

$$\hat{w}_4^{2^u(v+1)} (1 + w[0]_1)^2 e^{k-2^{u+1}-1}$$

on $\mathbb{RP}^2$ is zero, but the value of this on $P(2, 2^u(2v+1))$ is

$$\begin{aligned} &\hat{w}_4^{2^u(v+1)} e^{1+k-2^{u+1}} [\mathbb{RP}(\nu^k)] \\ &= \frac{d^{2^u(v+1)}(1+c+c^2+d)^{2^u(v+1)}(1+c)^2}{w(\nu^k)} [P(2, 2^u(2v+1))] \\ &= \frac{d^{2^u(v+1)}(1+c+c^2+d)^{2^u(v+1)}(1+c)^2}{(1+c)(1+c+c^2+d)} [P(2, 2^u(2v+1))] \\ &= (1+c)d^{2^u(v+1)}(1+c+c^2+d)^{2^u(v+1)-1}[P(2, 2^u(2v+1))] \\ &= 1. \end{aligned}$$

This is impossible. Thus, one has that $k \leq 2^{u+1}$ so $Y_2 \leq 2^{u+1}$. □

Finally, combining the above lemma and proposition, the correct statement of Theorem 2.3 in [1] should be the following.

Theorem. *Suppose that* (M^{j+q}, T) *fixes* $\mathbb{RP}^j$ *with normal bundle* ν^q *having* $w(\nu^q) = (1+\alpha)^q$ *with odd* $q > 1$, *and* $P(h,i)$ *with normal bundle* ν^k *having* $w(\nu^k) = (1+c)^a(1+c+d)^b w(\rho)^\varepsilon$. *Let* $2^A \leq h \leq 2^{A+1}$ *and write* $i = 2^u(2v+1)$. *Then* $(b,q,j) = (1, h+1, 2i+k-1)$, k *is even with* $2 \leq k \leq \begin{cases} 2^{u+1}+2 & \text{if } u = 1 \\ 2^{u+1} & \text{if } u \neq 1 \end{cases}$, *and* $i+a$ *is odd.*

(I) *When* $\varepsilon = 0$ *(i.e.,* ν^k *is standard), one has that*

(1) $a < 2^u$;

(2) $j+1 \equiv i+a+1 \mod 2^{A+1}$ *and* $i+k \equiv a+1 \mod 2^{A+1}$. *In particular,*

(a) *for* $u \leq A$, $k = 2^u+a+1$ *and* $2^{u+1}(v+1) \equiv 0 \mod 2^{A+1}$;

(b) *for* $u > A$, $k \equiv a+1 \mod 2^{A+1}$.

Further, (M^{j+q}, T) *with standard* ν^k *exists for* k *in a range* $X_1 \leq k \leq X_2$, *and is cobordant to*

$$\Gamma^{k-2a-2}(P(h, N^{i+a+1}), T_{N^{i+a+1}}) \sqcup (\mathbb{RP}^{j+h+1}, T_{h+1})$$

where $2 \leq X_1, X_2 \leq 2k_0 = \begin{cases} 2^{u+1}+2 & \text{if } u = 1 \\ 2^{u+1} & \text{if } u \neq 1 \end{cases}$, *and more precisely*

(c) *for* $u \leq A$, $X_1 = a+2$ *and* $X_2 \leq 2^u+a+1$;

(d) *for* $u > A$, $2 \leq X_1 \leq h+2$ *and* $X_2 \leq 2^{u+1} - (h - \text{common}(h,a))$ *where* $\text{common}(h,a)$ *is the common part of the 2-adic expansions of* h *and* a.

(II) *When* $\varepsilon \neq 0$ *(i.e.,* ν^k *is nonstandard), one has* $h = 2$. *Further,* (M^{j+q}, T) *with nonstandard* ν^k *exists only for the following cases:*

(1) $(u,k,a) = (0,2,2)$, $\nu^2 = \tau \otimes \eta$ *and* v *is odd;*

(2) $(u,k,a) = (1,4,1)$ *and* $\nu^2 = \tau \oplus (\tau \otimes \eta)$;

(3) $a = 1$ *with* $u > 1$, $k \equiv 2 \mod 4$ *is in the range* $6 \leq k \leq Y_1$ *and* ν^k *is stably cobordant to* $3\xi \oplus (\tau \otimes \eta)$, *where* $Y_1 \leq 2^{u+1} - 2$;

(4) $a = 3$ *with* $u > 1$, $k \equiv 0 \mod 4$ *is in the range* $4 \leq k \leq Y_2$ *and* ν^k *is stably cobordant to* $\xi \oplus (\tau \otimes \eta)$, *where* $Y_2 \leq 2^{u+1}$.

In concluding this note, it should be pointed out that there is an additional number 384 in line 18 of page 4555 in [2], which should be omitted.

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