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ERRATUM TO "DISTANCE BETWEEN TOROIDAL SURGERIES ON HYPERBOLIC KNOTS IN THE 3-SPHERE"

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Jose Angel Frías and Enrique Ramírez-Losada pointed out that the proof of Lemma 5.9 in my paper [1] is false. This is caused by the improper application of the argument in the proof of Lemma 5.3. The purpose of this note is to amend the proof.

Proof of Lemma 5.9. The proof remains valid until the end of the second paragraph. Assume $t > 4$. For simplicity, let $\sigma(1) = r$. Then G_S has two S-cycles with label pairs $\{t/2, t/2+1\}$ and $\{t/2+r-1, t/2+r\}$. Since $r \geq 3$, these pairs are disjoint. Thus the edges of these S-cycles form two essential cycles on $\widehat{T}$, which split $\widehat{T}$ into two annuli F_1 and F_2.

Although the families A, B and D correspond to the same permutation σ, C corresponds to another permutation σ' such that an edge in C has label j at u_1 and label $\sigma'(j) \equiv j+r-3 \pmod t$ at u_2. Note that A contains a $\{t/2, t/2+r-1\}$-edge a_1 and a $\{t/2+1, t/2+r\}$-edge a_2. They lie in the same annulus, F_1 say, or distinct annuli on $\widehat{T}$. In either case, it implies that σ has exactly two orbits. Thus the edges of A form two disjoint cycles L_1 and L_2 on $\widehat{T}$, where L_1 contains $v_{t/2}$ and L_2 contains $v_{t/2+1}$. If a_1 and a_2 lie in F_1, then there is no vertex inside F_2, so we have a contradiction as in the proof of Lemma 2.7(2) by constructing a twice-punctured Klein bottle. Hence we may assume that a_i lies in F_i for $i = 1, 2$. Then all vertices on L_1 but $v_{t/2}$ and $v_{t/2+r-1}$ lie inside F_2, and all vertices on L_2 but $v_{t/2+1}$ and $v_{t/2+r}$ lie inside F_1.

Assume $t \geq 8$. Among the positive loops at u_1, there is an edge e whose labels are not the labels of those S-cycles. In fact, if $r \neq 3, t-1$, then choose the $\{t/2-1, t/2+2\}$-edge as e. Otherwise, choose the $\{t/2+4, t/2-3\}$-edge as e. However, we cannot locate the edge e on $\widehat{T}$.

Assume $t = 6$. Then $r = 3$ or 5. Let a, b and d be the edges of A, B and D with label 1 at u_1. Then these three edges, connecting v_1 with v_r, are mutually parallel on $\widehat{T}$. Hence there are at least five parallel edges between v_1 and v_r. This means

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that G_S contains at least two edges with label r at u_1 and label 1 at u_2. However, there is no such edge when $r = 3$, or only one such edge when $r = 5$. □

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References

1. M. Teragaito, *Distance between toroidal surgeries on hyperbolic knots in the 3-sphere*, Trans. Amer. Math. Soc. **358** (2006), 1051–1075. MR2187645 (2006h:57005)

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