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CORRECTIONS AND IMPROVEMENTS TO: "ON THE STOCHASTIC HEAT EQUATION WITH SPATIALLY-COLORED RANDOM FORCING"

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Theorems 1.8 and 1.11 of [1] state that if L_σ is sufficiently large (that is, if there is enough noise in the system), then the Lyapunov exponents are positive. In this sense, both theorems are correct. However, the quantitative bounds on the Lyapunov exponents are not correctly derived and need to be adjusted. We do this here, and also describe how to slightly improve Condition 1.10 of that paper in this particular context.

Let us replace Condition 1.10 of [1] with the following (extremely weak) condition.

Condition 1 (Replacement for Condition 1.10 of [1]). There exists $\boldsymbol{a} \in \mathbf{R}^d_+$ such that

$$\int_{[\boldsymbol{a},2\boldsymbol{a}]} \frac{\hat{f}(z)}{\mathrm{Re}\Psi(z)}\,\mathrm{d}z > 0,$$

where $[\boldsymbol{a}\,,2\boldsymbol{a}] := [a_1\,,2a_1] \times \cdots \times [a_d\,,2a_d]$.

The preceding hypothesis is extremely weak, and amounts to non-triviality of the function f, equivalently non-triviality of the underlying noise $\dot{F}$. Theorem 1.8 of [1] can now be stated in the following corrected form.

Theorem 2 (Replacement for Theorem 1.8 of [1]). *Suppose $b \equiv 0$, the preceding Condition 1 and Conditions 1.1 and 1.7 (parts (1) and (2) only; part (3) is no longer needed) of [1] hold, and $\eta := \inf_{x\in\mathbf{R}^d} u_0(x) > 0$. Then there exists a sufficiently-large number $\mathrm{L}_\sigma \in (0\,,\infty)$ such that if $\sigma(z) \geq \mathrm{L}_\sigma|z|$ for all $z \in \mathbf{R}$, then*

$$\inf_{x\in\mathbf{R}^d} \overline{\gamma}_x(2) > 0.$$

To prove this, we correct the proof of Theorem 1.8 [1] as follows: Everything is correct up to and including (5.56), except $\mathrm{d}z_d$ should be written as $\mathrm{d}z_\ell$. However, (5.58) is faulty. We can fix it by observing that, thanks to (5.56), if $z_1,\ldots,z_\ell \in [\boldsymbol{a}\,,2\boldsymbol{a}]$, then the absolute value of the kth coordinate of $z_j - z_{j-1}$ is less than or equal to the absolute value of the kth coordinate of z_j for every $k \in \{1\,,\ldots,d\}$ and $j \in \{1\,,\ldots,\ell\}$. Therefore, (5.59) holds for such $z_1,\ldots,z_\ell$ and hence the following

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corrected form of (5.60) is valid:

$$\inf_{x\in\mathbf{R}^d}\int_0^\infty \mathrm{e}^{-\beta t}\mathrm{E}\left(|u_t(x)|^2\right)\mathrm{d}t \geq \frac{\eta^2}{\beta}\sum_{\ell=0}^\infty\left(\frac{\mathrm{L}_\sigma^2}{(2\pi)^d}\int_{[\boldsymbol{a},2\boldsymbol{a}]}\frac{\hat{f}(z)}{\beta+2\mathrm{Re}\Psi(z)}\,\mathrm{d}z\right)^\ell.$$

If, in particular, L_σ is large enough and β is sufficiently small, then the preceding sum diverges; that is, (5.62) of [1] is valid. The remainder of the proof of Theorem 1.8 of [1] works without further problems. See, in particular, the argument that starts 2 lines after (5.64) and concludes the proof of Theorem 1.8 in [1].

Similarly, Theorem 1.11 can be corrected in the following form.

Theorem 3 (Replacement for Theorem 1.11 of [1])**.** *Suppose $b \equiv 0$, the preceding Condition 1 and Condition 1.7 (parts (1) and (2) only; part (3) is no longer needed) of [1] hold, and $\eta := \inf_{x\in\mathbf{R}^d} u_0(x) > 0$. Suppose, in addition, that $\sigma \geq 0$ pointwise, and $q := \liminf_{|z|\to\infty}\sigma(z)/|z| > 0$. If $u_0(x) > 0$ and $\mathrm{P}\{u_t(x) > 0\} = 1$ for all $t > 0$ and $x \in \mathbf{R}^d$, then $\overline{\gamma}_x(2) > 0$ for every $x \in \mathbf{R}^d$ provided that $\eta := \inf_{x\in\mathbf{R}^d} u_0(x)$ and q are both sufficiently large.*

Because of these corrections, a few remarks and comments need to be adjusted suitably in order to accommodate the corrected lower bounds for the Lyapunov exponents; see, in particular, (1.22), Theorem 5.7, and Example 5.8 of [1].

References

[1] Mohammud Foondun and Davar Khoshnevisan, *On the stochastic heat equation with spatially-colored random forcing*, Trans. Amer. Math. Soc. **365** (2013), no. 1, 409–458, DOI 10.1090/S0002-9947-2012-05616-9. MR2984063

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