this table is $1 \leqq n \leqq 1500$ and the largest prime is purposely omitted in the majority of cases; hence a prime is indicated, for the most part, by a blank space. Gauss gave a factor table of n^2+1 (*Werke*, Göttingen, v. 2, second ed., 1876, p. 478–481), where the numbers are selected according to the criterion that the largest prime factor should not exceed 197. He has given an enumeration of 657 such integers (from $n=2$ to $n=14\ 033\ 378\ 718$) with their complete factorizations—the factor 2 being omitted throughout. Gauss constructed similar factor tables for integers of the form n^2+k^2, where k assumes the values 2, 3, 4, . . . , 9, inclusive.

In his *Binomial Factorisations*, v. 1, London, 1923, p. xxvi, the late A. J. C. CUNNINGHAM mentions an unpublished ms. table of his which gives the complete factorization of n^2+1 for $1 \leqq n \leqq 15{,}000$. On the basis of this table he enumerates (p. 238–239) the positive values of $n < 15{,}000$ for which n^2+1 is prime. (I have discovered no errata in his list.) He has also tabulated (p. 240–244) in separate groups those values of n for which $(1/2)(n^2+1)$, $(1/5)(n^2+1)$, $(1/10)(n^2+1)$, $(1/13)(n^2+1)$, $(1/17)(n^2+1)$, are prime. For errors in these, and in Euler's table, see MTE **3**.

As Gauss first indicated (*Werke*, v. 2, second ed., p. 497–500), one of the most important uses of such a table is the derivation of arctangent relations for the calculation of π. I have elaborated this theme in "On the derivation of arctangent equalities," *Amer. Math. Mo.*, v. 45 (1938), p. 108–109. In the same volume, in an expository article "On arccotangent relations for π" (p. 657–664) D. H. Lehmer compared the relations with regard to ease in application to the calculation of π to a large number of decimal places.

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MECHANICAL AIDS TO COMPUTATION

1. *Seventeenth Century Calculating Machines*.—In *Nature*, v. 150, 31 Oct. 1942, p. 508–509, is an address delivered at a memorial luncheon held in London on 19 October, by the president of the Royal Astronomical Society, S. Chapman, "Blaise Pascal (1623–1662) tercentenary of the calculating machine." A report of this luncheon organized by a small committee, of which L. J. Comrie was chairman, appeared in this same issue of *Nature*, p. 527, "Pascal tercentenary celebration." The 120 guests included many distinguished French scientists, as well as an official deputation from General de Gaulle's headquarters. At the age of nineteen Pascal invented the first calculating machine, made in 1642. By this means he hoped to assist his father, Etienne Pascal (d. 1651), discoverer of the limaçon, in statistical work involving additions and subtractions of sums of money. Such operations were those to which its applications were confined. During the following decade he made improvements, and one of his machines of 1652, bearing his signature, is preserved in the Conservatoire des Arts et Métiers. A replica of this is in the Science Museum, South Kensington, London. A detailed account of this machine by Denis Diderot (1713–1784), with illustrations, was published in *Encyclopédie, ou Dictionnaire Raisonné des Sciences des Arts et des Métiers*, Paris, v. 1, 1751, p. 680–684, Planches, v. 5, 1767, algèbre et trigonométrie, plate II. Also in 3rd ed., Geneva, and Neuchâtel, 1779, v. 3, p. 381–388, plates, v. 1. See also *Encyclopédie Méthodique. Mathématiques*, . . . , Paris, v. 1, 1784, p. 136–142; and *Oeuvres Complètes de Blaise Pascal*, v. 2, Paris, Hachette, 1860, p. 368–380. Chapman notes, "In 1652 he [Pascal] presented one of the last of his fifty models, with a famous letter, to Queen Christina of Sweden. He also wrote a prospectus of his invention that would do credit to a modern school of salesmanship." This

letter and prospectus are given on p. 359–368 of the volume of the *Oeuvres* here cited. The account of the "Machine Arithmétique de M. Pascal" in *Machines et Inventions approuvées, par l'Académie Royale des Sciences, depuis son Établissement jusqu'à present*, v. 4, Paris, 1735, p. 137–140 is accompanied by two fine plates. An account of Pascal's machine is also given in M. d'Ocagne, *Le Calcul Simplifié par les Procédés Mécaniques et Graphiques*, second ed., Paris, 1905, p. 24–29.

In the luncheon exhibition room were replicas of Pascal's machine, and also of one of the first three English calculating machines made in 1663–1666 by the diplomatist, mathematician, and inventor, Sir Samuel Morland (1625–1695). Morland's Trigonometrical Calculating Machine was invented in 1663 and constructed in 1664. It "provides a means of rapidly constructing triangles to scale from given data by using graduated bars and circles. Any ordinary problem in plane trigonometry which can be solved by plotting on paper may be quickly solved by this instrument. The sin, cos, tan, etc., of any angle may be read off at once. Multiplication and division may also be performed by employing the graphical method of similar triangles." Morland constructed also another type of multiplying instrument, as well as a calculating machine which he invented in 1666 and which he described as a "new and most useful instrument for addition and subtraction of pounds, shillings, pence, and farthings; without charging the memory, disturbing the mind, or exposing the operator to any uncertainty which no method hitherto published can justly pretend to." See D. Baxandall, *Catalogue of the Collections in the Science Museum, South Kensington . . . Mathematics*, I. *Calculating Machines and Instruments*, London, 1926, p. 8, 14–16, and plate.

In 1671 G. W. Leibniz (1646–1716) conceived the idea of a calculating machine which would perform multiplication by rapidly repeated addition. It was not until 1694 that his first complete machine was actually constructed. It was preserved in the former Royal Library at Hannover. Leibniz describes his machine in "Brevis descriptio machinae arithmeticae," *Miscellanea Berolinensia*, v. 1, 1710, p. 317–319, with a plate exhibiting the appearance of the instrument. Cf. Louis Couturat, *La Logique de Leibniz, d'après des documents inédits*, Paris, 1901, p. 115–116, 295–296.

Attention may be drawn to interesting articles on Calculating Machines by David Baxandall in *Encyclopædia Britannica*, fourteenth ed., v. 4, 1929, p. 548–553; and by E. M. Horsburgh in (a) *A Dictionary of Applied Physics*, ed. by R. Glazebrook, v. 3, London, 1923, p. 193–201, and (b) *Modern Instruments and Methods of Calculation. A Handbook of the Napier Tercentenary Exhibition*, London, 1914, p. 69–277. See also Antonino Asta's "Calcolatrici Macchine," *Enciclopedia Italiana*, Milan and Rome, v. 8, 1930, p. 352–358.

R. C. A.