RECENT MATHEMATICAL TABLES

Under this heading Tables published currently, and within the past ten years, will be reviewed. During the years 1934–1936 twelve articles on "Mathematical Tables" appeared in the quarterly journal, *Scripta Mathematica*, New York, v. 2–4. These articles included reviews of tables, numbered consecutively 1 to 74. Since it is not planned that any of these tables shall be reviewed here, a list of them is assembled below. The new series of articles is to assume quite a different character from the earlier series in that the reviews will be longer and no notice will be taken of certain types of elementary tables previously considered. In spite of this, it has been suggested by more than one correspondent that convenience of reference would be enhanced by having the new series of reviews numbered consecutively, beginning with number 75. Hence this is to be our plan, and we list below the publications reviewed in numbers **1–74.**

The previous reviews appeared in *Scripta Mathematica* as follows: v. 2, 1934 (nos. 1–12, p. 91–93, 297; nos. 13–24, p. 193–197; nos. 25–32, p. 297–299; nos. 33–39, p. 379–380); v. 3, 1935 (nos. 40–41, p. 97–98; nos. 42–44, p. 192–193; nos. 45–48, p. 282–283; nos. 49–52, p. 364–366); v. 4, 1936 (nos. 53–58, p. 101–104; nos. 59–65, p. 198–201; nos. 66–70, p. 294–295; nos. 71–74, p. 338–340).

1. *Noordhoff's Schooltafel*, Groningen, P. Noordhoff, 1933, 112 p. 14.9×23.6 cm.

2. W. L. Hart, compiler, *Logarithmic and Trigonometric Tables*, Boston, New York, and Chicago, D. C. Heath and Co., [1933], iv, 124 p. 15×22 cm.

3. R. S. Burington, *Handbook of Mathematical Tables and Formulas*, Sandusky, Ohio, Handbook Publishers, Inc., 1933, 8, 251 p. 13.2×19.7 cm.

4. P. Herget, "A table of sines and cosines to eight decimal places," *Astr. Jn.*, v. 42, 28 Jan. 1933, cols. 123–125. 23.7×31.3 cm.

5. B. V. Numerov, *Tablit͡sy natural'nykh znacheniĭ trigonometricheskikh funkt͡siĭ s 5-'i͡u desi͡atichnymi znakami* [Tables of the natural values of the trigonometric functions to five places of decimals], Leningrad-Moscow, Gosudarstvennoe Tekhniko-teoreticheskoe Izdatel'stvo, 1933, 58 p. 16.9×25.2 cm.

6. B. V. Numerov, *Tablit͡sy dli͡a vychisleniia geograficheskikh i pri͡amougol'nykh koordinat Gaussa-Kriugera, dli͡a shirot ot 36° do 72° s tochnost'i͡u do 0.1 metra i 0″.01 (dli͡a raboty s arifmometrom). Tables for calculation of geographic and rectangular coordinates of Gauss-Krüger for latitudes from 36° to 72° with accuracy to 0.1 m. and 0″.01 (for work with a calculating machine).* Leningrad, Astronomical Institute, 1933, 80 p. 17.5×26.1 cm.

7. J. Plassmann, *Tafel der Viertel-Quadrate aller Zahlen von 1 bis 20009, zur Erleichterung des Multiplizierens vierstelliger Zahlen. Mit vielen Ratschlägen für das praktische Rechnen in Handel, Gewerbe and Wissenschaft.* Leipzig, Max Jänecke, 1933, 26, 200 p. 15×21.6 cm.

8. F. J. Duarte, *Nouvelles Tables Logarithmiques à 36 décimales*, Paris, Gauthier-Villars, 1933, xxviii, 128 p. 17.1×25.5 cm.

9. J. Boccardi, *Tables logarithmiques des factorielles jusqu'à 10000!*, Cavaillon, 1932, 36 p. +1 sheet errata. 14×20.9 cm.

10. H. Tallquist, "Tafel der 24 ersten Kugelfunktionen $P_n(\cos\theta)$," Societas Scientiarum Fennica, *Commentationes Physico-Mathematicae*, v. 6, no. 3, Mar. 1932, 11 p. The title page for volume 6 is dated 1933. 16.5×23.5 cm.
11. A. Dinnik, "Tafeln der Besselschen Funktionen von der gebrochenen Ordnung" [title also in Ukrainian], Allukrainische Akademie der Wissenschaften, *Naturwissenschaftlich-technische Klasse*, 1933, 29 p. 17.3 ×26.5 cm.
12. K. Hayashi, *Tafeln für die Differenzenrechnung sowie für die Hyperbel-, Besselschen, elliptischen und anderen Funktionen*, Berlin, Springer, 1933. 21×27.7 cm.
13. A. Gérardin, (a) "Liste inédite des 1001 nombres premiers de la forme $2x^2+2x+1$ pour x compris entre 15,800 et 23,239, $f(x)$ de 499,311,601 à 1,008,148,721"; (b) "Liste inédite de 1035 nombres premiers de 8 et 9 chiffres extraits de diverses séries quadratiques," *Sphinx Oedipe*, v. 27, May 1932, 4, 4 p. 13.8×22.1 cm.
14. M. Kraitchik and S. Hoppenot, "Les grands nombres premiers," *Sphinx*, v. 3, Oct. 1933, p. 145–146; Nov. 1933, p. 161–162. 15.5×24 cm.
15. H. W. Weigel, *$x^n+y^n=z^n$? Die elementare Lösung des Fermat-Problems* . . . , Leipzig, [1933?]; preface dated November, 1932. 45.3×28.7 cm.
16. *Minimum Decompositions into Fifth Powers*, prepared under the direction of L. E. Dickson (Br. Assoc. Adv. Sci., *Mathematical Tables*, v. 3) London, 1933, 4to, 8, 368 unnumbered pages. 21.3×27.8 cm.
17. N. G. W. H. Beeger, *Additions and corrections to "Binomial Factorisation" by . . . A. J. C. Cunningham . . . 1923–29*, Amsterdam, 1933, [ii], 12 p. Hectograph print by the author at Nicolaas Witsenkade 10. 17×21 cm.
18. J. Sherman, "A four place table of $(\sin x)/x$," *Z.f.Kristallographie*, Leipzig, v. A85, p. 404–419, June 1933. 15.3×23 cm.
19. A. Umansky, "$A=\cosh\xi\cos\xi$, $B=\frac{1}{2}(\cosh\xi\sin\xi+\sinh\xi\cos\xi)$, $C=\frac{1}{2}\sinh\xi\sin\xi$, $D=\frac{1}{4}(\cosh\xi\sin\xi-\sinh\xi\cos\xi)$; Tafeln hyperbolisch-trigonometrischer Funktionen" [title also in Ukrainian], Académie des Sciences d'Ukraine, *Classe des sciences naturelles et techniques*, *Jn. du Cycle Industriel et Technique*, Kiev, nos. 2–3, 1932, p. 97–117 of Umansky's paper. 18×25.5 cm.
20. J. B. Russell, "A table of Hermite functions," *Jn. of Math. and Physics*, Mass. Inst. Tech., v. 12, May 1933, p. 291–297. 17×25.2 cm.
21. G. Prévost, *Tables de Fonctions Sphériques et de leurs Intégrales pour calculer les coefficients du développement en série de Polynomes de Laplace d'une fonction de deux variables indépendantes.* Bordeaux and Paris, 1933, xxxii, 96, viii*, 163* p. 22×27.5 cm.
22. E. L. Ince, (a) "Tables of the elliptic-cylinder functions"; (b) "Zeros and turning points of the elliptic-cylinder functions," Royal So. Edinburgh, *Proc.*, v. 52, 1932, (a) p. 355–423; (b) p. 424–433. 17.3×25.5 cm.
23. A. J. Thompson, *Logarithmetica Britannica being a Standard Table of Logarithms to Twenty Decimal Places . . . Part VI . . . Numbers 60,000 to 70,000.* (*Tracts for Computers*, no. XVIII.) London, Cambridge Univ. Press, 1933, 102 p.+a plate. 22×27.7 cm.
24. *Tables of the Higher Mathematical Functions*, computed and compiled under the direction of H. T. Davis with the cooperation of 20 computers and assistants. Bloomington, Ind., v. 1, 1933, 14, 377 p.+2 plates. 17.5×24.9 cm.
25. E. W. Brown and D. Brouwer, *Tables for the Development of the Disturbing Function with Schedules for Harmonic Analysis*, Cambridge, Engl.,

1933, 2+p. 69–157. Reprinted from Yale University Observatory, *Trans.*, v. 6, pt. 5. 23.5×31.8 cm.

26. H. J. LUCKERT, *Über die Integration der Differentialgleichung einer Gleitschicht in zäher Flüssigkeit* (Diss. Berlin), Leipzig, 1933; reprint from Berlin, Universität, Institut f. angewandte Mathem., *Schriften d. mathem. Seminars*, v. 1, p. 245–274. 16×24 cm.

27. L. BENDERSKY, "Sur la fonction gamma généralisée," *Acta Math.*, v. 61, Nov. 1933. 22×29.5 cm.

28. P. HARZER, "Tabellen für alle statistischen Zwecke in Wissenschaft und Praxis, insbesondere zur Ableitung des Korrelations-Koeffizienten und seines mittleren Fehlers," Bayer. Akad. d. Wiss., *Abh., math.-natw. Abt.*, neue Folge, Heft 21, 1933, iv, 91 p. 23×28.5 cm.

29. A. S. PERCIVAL, *Mathematical Facts and Formulae*, London and Glasgow, Blackie, 1933, vi, 125 p. 12×18.3 cm.

30. H. B. DWIGHT, *Tables of Integrals and Other Mathematical Data*, New York, Macmillan, 1934, x, 222 p. 14.4×21.5 cm.

31. F. E. FOWLE, ed., *Smithsonian Physical Tables*, eighth rev. ed., Washington, D. C., 1933, liv+682 p. 15.3×23.1 cm.

32. E. JAHNKE and F. EMDE, *Tables of Functions with Formulae and Curves. Funktionentafeln mit Formeln und Kurven.* Second rev. ed., with 171 figures. Leipzig, Teubner, 1933, xviii, 330 p. 16×24 cm.

33. W. J. SEELEY, *Table for the Rapid Evaluation of the Square-Root-of-the-Sum-of-the-Squares of two Numbers*, Published by the author, Duke Univ., Durham, N. C., 1933, 8 unnumbered p. rotograph print. 13.5 ×20.3 cm.

34. D. KATZ, *Pocket Tables for Cubics. A Systematic Method for Algebraic Treatment of Cubic Equations*, South Milwaukee, Wis., 1933, 6 p. in folded sheet 11×8$\frac{3}{4}$ in.

35. M. KRAÏTCHIK and S. HOPPENOT, "Les grands nombres premiers," *Sphinx*, v. 4, June 1934, p. 81–82. 15.5×24 cm.

36. M. KRAÏTCHIK, "Les grands nombres premiers," *Mathematica*, Cluj, v. 7, 1933, p. 92–94. 17×24.1 cm.

37. H. T. DAVIS, "Polynomial approximation by the method of least squares," *Annals of Math. Statistics*, v. 4, Aug. 1933, p. 155–195. 17.5×25.2 cm.

38. F. W. SPARKS, *Universal Quadratic Zero Forms in Four Variables*, Chicago, Ill., private edition distributed by the Univ. of Chicago libraries, 1933. Facsimile prints, except for cover and title page. 17×24 cm.

39. E. McC. CHANDLER, *Waring's Theorem for Fourth Powers*. Chicago, Ill., private edition distributed by the Univ. of Chicago libraries, 1933, iv, 62 p. Pages ii and 1–62 are facsimile prints. 17×24 cm.

40. M. KRAÏTCHIK and S. HOPPENOT, "Les grands nombres premiers," *Sphinx*, v. 4, Aug. 1934, p. 113–114. 15.5×24 cm.

41. E. L. INCE, *Cycles of Reduced Ideals in Quadratic Fields*. (Br. Ass. Adv. Sci., *Mathematical Tables*, v. 4.) London, 1934, 4to, xvi, 80 p. 20.8×28 cm.

42. A. J. THOMPSON, *Logarithmetica Britannica being a standard Table of Logarithms to Twenty Decimal Places . . . Part I . . . Numbers 10,000 to 20,000.* (*Tracts for Computers*, no. 19.) Cambridge, Univ. Press, 1934, 107 p. (unnumbered)+2 photogravure plates giving a facsimile of the will of Henry Briggs. 22×27.7 cm.

43. P. POULET, "De nouveaux amicables," *Sphinx*, v. 4, Sept. 1934, p. 134–135. 15.5×24 cm.

44. M. KRAÏTCHIK and S. HOPPENOT, "Les grands nombres premiers," *Sphinx*, v. 4, Nov. 1934, p. 161–162. 15.5×24 cm.

45. Astronomicheskiĭ Institut, *Ephemerides for the Determination of Time-corrections by equal Altitudes (Zinger's method) for 1935*, Leningrad, 1934. 17.2×26.2 cm.

46. J. Bouman and W. F. De Jong, "Grafische bepaling van buigingsfiguren," Akad. v. Wetenschappen, Amsterdam, *Verhandelingen, afd. Natuurkunde*, v. 4, no. 4, 1931. 17.5×25.5 cm.

47. O. Lohse, *Tafeln für numerisches Rechnen mit Maschinen*. Zweite Auflage neubearbeitet von P. V. Neugebauer. Leipzig, Engelmann, 1935. vi, 113 p. 17×24.2 cm.

48. S. Hoppenot, *Table des Solutions de la Congruence* $x^4 \equiv -1$ (mod. N) *pour* $100000 < N < 200000$, Brussels, Librairie du "Sphinx," 1935, 18 p. 15.6×24.1 cm.

49. E. S. Allen, *Six-Place Tables. A Selection of Tables of Squares, Cubes, Square Roots, Cube Roots, Fifth Roots and Powers, Circumferences and Areas of Circles, Common Logarithms of Numbers and of the Trigonometric Functions, the Natural Trigonometric Functions, Natural Logarithms, Exponential and Hyperbolic Functions and Integrals*. Fifth edition. New York and London, 1935, xxiii, 175 p. 11×18 cm.

50. N. Samoilova-Îakhontova, *Tablit͡sy Ellipticheskikh Integralov* [Tables of Elliptic Integrals]. Moscow and Leningrad, 1935, 108 p.+errata slip. 17×24.5 cm.

51. A. M. Legendre, *Tables of the Complete and Incomplete Elliptic Integrals, reissued from Tome II of Legendre's Traité des Fonctions Elliptiques, Paris, 1825, with an Introduction* by Karl Pearson. *With autographed Portrait of Legendre*. Cambridge, University Press, 1934, xliii, 94 p. 20.5×25.5 cm.

52. H. Gupta, "Decompositions into squares of primes," Indian Acad. Sci. *Proc., s.A*, v. 1, p. 789–794, May 1935. 18×24.6 cm.

53. J. R. Airey, "The circular and hyperbolic functions, argument $x/\sqrt{2}$," *Phil. Mag.*, s. 7, v. 20, Oct. 1935, p. 721–731. 17×25.3 cm.

54. J. R. Airey, "The circular and cosine functions, argument $\log_e x$," *Phil. Mag.*, s. 7, v. 20, Oct. 1935, p. 731–738. 17×25.3 cm.

55. L. E. Dickson, *Researches on Waring's Problem* (Carnegie Institution of Washington, *Publication* no. 464), Washington, 1935, v, 257 p. 17×25 cm.

56. R. C. Shook, *Concerning Waring's Problem for Sixth Powers*. Diss., Univ. Chicago, Chicago, Ill., 1934, iv, 38 p. 17×24 cm.

57. B. W. Jones, *A Table of Eisenstein-reduced Positive Ternary-Quadratic Forms of Determinant* $\leqq 200$ (National Research Council, *Bulletin*, no. 97), Washington, National Acad. Sci., 1935, 51 p. 17×24.5 cm.

58. H. Roussilhe and Brandicourt, *8 Place Tables of the Natural Values of Sines, Cosines and Tangents according to the Centesimal System, for each Centigrade from 0 to 100 Grades . . . followed by 20 Place Tables of the Natural Values of the Six Trigonometrical Functions according to the Centesimal System for each Grade from 0 to 100 Grades. Taken from the Tables of M. Andoyer*, New revised edition (International Geodetic and Geophysical Union Association of Geodesy, *Special Publication*, no. 1), Paris, 1933, 23, 99, 18 p. 18×26.7 cm.

59. V. R. Bursian, "Tablit͡sy znacheniĭ funkt͡sii $I_{1/3}$" [Tables of values of the function $I_{1/3}$], Leningradskiĭ Gosudarstvennyĭ Universitet imeni A. S. Bubnova, *Uchenyi͡e Zapiski, . . . serii͡a Fizicheskikh Nauk* [Leningrad State University in the name of A. S. Bubnov. Annals, . . . series of physical sciences], Leningrad, v. 1, no. 1, 1935, p. 4–9; German abstract, p. 8–9. 22.5×29 cm.

60. F. Triebel, *Rechen-Resultate. Tabellen zum Ablesen der Resultate von Multiplikationen und Divisionen bis 100×1 000 = 100 000 in Bruchteilen und ganzen Zahlen. Zum praktischen Gebrauch für Stückzahl-, Lohn-, und Prozentberechnungen. Rechenhilfsmittel für alle Arten des Rechnens mit Zahlen jeder Grösse. Radizieren (Wurzelsuchen) nach vereinfachtem Verfahren.* Sixth ed. Berlin, M. Krayn, 1934, 285 p. 19×26.5 cm.

61. L. Silberstein, "On complex primes," *Phil. Mag.*, s. 7, v. 19, June 1935, p. 1097–1107. 17×25.3 cm.

62. J. Peters, A. Lodge and E. J. Ternouth, E. Gifford, *Factor Table giving the Complete Decomposition of all Numbers less than 100,000.* (British Association for the Advancement of Science, *Mathematical Tables*, v. 5.) London, B. A. A. S., 1935, xv, 291 p. 22.5×28 cm.

63. J. R. Airey, "Toroidal functions and complete elliptic integrals," *Phil. Mag.*, s. 7, v. 19, Jan. 1935, p. 177–188. 17×25.3 cm.

64. J. P. Möller and H. Q. Rasmusen, "Tafel der Funktion $x^{2/3}$ zur Verwendung bei parabolischer Bahnbestimmung nach der Methode von B. Strömgren," *Astron. Nach.*, v. 258, 4 Jan. 1936, cols. 9–10.

65. A. J. Thompson, *Logarithmetica Britannica being a standard Table of Logarithms to Twenty Decimal Places . . . Part VII . . . Numbers 70,000 to 80,000.* (*Tracts for Computers*, no. 20.) Cambridge, University Press, 1935, 106 p. (unnumbered)+3 p. of facsimiles. 22×28 cm.

66. S. Sakamoto, *Tables of Gudermannian Angles and Hyperbolic Functions.* Tokyo, 1934, 157 p. 12.6×18.6 cm.

67. L. I͡a. Neĭshuler, *Tablit͡sy priblizhennykh Vychisleniĭ: Delenie, Umnozhenie, desi͡atichnye i naturalnye logarifmy, polnye Kvadraty chetyrekhznachnykh Chisel* [Tables of approximate Computations: Division, Multiplication, decimal and natural Logarithms, complete Squares of four-place Numbers], second edition enlarged, Moscow and Leningrad, 1933, 139 p. 19.6×26.3 cm.+1 plate 66×43.5 cm.

68. L. I͡a. Neĭshuler, *Tablit͡sy Proizvedeniĭ Pi͡atiznachnykh Chisel na dvukhznachnye. Umnozhenie li͡ubykh Chisel, Delenie i Prot͡sentirovanie s Tochnymi 4 i 6 Znakami* [Tables of products of five-figure numbers by two-figure numbers. Multiplication of any numbers, division and percentage-making, correct to 4 and 6 places]. *Posobie dli͡a Statistikov, Ekonomistov, Inzhenerov, Bukhgalterov i Proch.* [Aid for statisticians, economists, engineers, bookkeepers, etc.] Novocherkassk, Izdatelstvo Donskogo Politekhnicheskogo Instituta, 1930, 201 p. 25.7×35 cm.

69. J. R. Airey, "Bessel functions of nearly equal order and argument," *Phil. Mag.*, s. 7, v. 19, Feb. 1935, p. 230–235. 17×25.3 cm.

70. J. R. Airey, "The Bessel function derivatives $\frac{\partial}{\partial\nu}\cdot J_\nu(x)$ and $\frac{\partial^2}{\partial\nu^2}\cdot J_\nu(x)$," *Phil. Mag.*, s. 7, v. 19, Feb. 1935, p. 236–243. 17×25.3 cm.

71. Jiří Kaván, *Rozklad všetkých čisel celých od 2 do 256000 v Prvočinitele. Tabula Omnibus a 2 usque ad 256000 numeris integris omnes divisores primos praebens. Editio stereotypa.* (Publikácie štátneho astrofyzikálneho Observatoria, Stará Ďala, Ceskoslovensko.) Prague, Typis B. Stýblo, 1934, xi, 514 p. 29×30 cm. [This above title is taken from parts of two title pages.]

72. K. Karas, "Tabellen für Besellsche Funktionen erster und zweiter Art mit den Parametern $\nu = \pm\frac{2}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}$," *Z. f. angew. Math. u. Mechanik,* v. 16, Aug. 1936, p. 248–252. 20.2×28.6 cm.

73. H. Tallquist, "Sechsstellige Tafeln der 16 ersten Kugelfunktionen $P_n(x)$," Finska Vetenskaps-Societeten, Helsingfors, *Acta* n.s. A, v. 2, no. 4, 1937, 43 p. 22.3×28.4 cm.

74. F. Tölke, *Besselsche und Hankelsche Zylinderfunktionen nullter bis dritter Ordnung vom Argument* $r\sqrt{i}$, Stuttgart, Konrad Wittwer, 1936, 92 p. 17.6×25 cm.

J. R. Airey, Nos. **53, 54, 63, 69, 70**; E. S. Allen, No. **49**; Astronomicheskiĭ Institut, No. **45**; N. G. W. H. Beeger, No. **17**; L. Bendersky, No. **27**; J. Boccardi, No. **9**; J. Bouman and W. F. DeJong, No. **46**; E. W. Brown and D. Brouwer, No. **25**; R. S. Burington, No. **3**; V. R. Bursian, No. **59**; E. McC. Chandler, No. **39**; H. T. Davis, Nos. **24, 37**; L. E. Dickson, Nos. **16, 55**; A. Dinnik, No. **11**; F. J. Duarte, No. **8**; H. B. Dwight, No. **30**; F. E. Fowle, No. **31**; A. Gérardin, No. **13**; H. Gupta, No. **52**; W. L. Hart, No. **2**; P. Harzer, No. **28**; K. Hayashi, No. **12**; P. Herget, No. **4**; S. Hoppenot, No. **48**; E. L. Ince, Nos. **22, 41**; E. Jahnke and F. Emde, No. **32**; B. W. Jones, No. **57**; K. Karas, No. **72**; D. Katz, No. **34**; Jiří Kaván, No. **71**; M. Kraïtchik, No. **36**; M. Kraïtchik and S. Hoppenot, Nos. **14, 35, 40, 44**; A. M. Legendre, No. **51**; O. Lohse, No. **47**; H. J. Luckert, No. **26**; J. P. Möller and H. Q. Rasmusen, No. **64**; L. I͡a. Neĭshuler, Nos. **67, 68**; Noordhoff, No. **1**; B. V. Numerov, Nos. **5, 6**; A. S. Percival, No. **29**; J. Peters, A. Lodge, and E. J. Ternouth, E. Gifford, No. **62**; J. Plassmann, No. **7**; P. Poulet, No. **43**; G. Prévost, No. **21**; H. Roussilhe and Brandicourt, No. **58**; J. B. Russell, No. **20**; S. Sakamoto, No. **66**; N. Samoilova-I͡akhontova, No. 50; W. J. Seeley, No. **33**; J. Sherman, No. **18**; R. C. Shook, No. **56**; L. Silberstein, No. **61**; F. W. Sparks, No. **38**; H. Tallquist, Nos. **10, 73**; A. J. Thompson, Nos. **23, 42, 65**; F. Tölke, No. **74**; F. Triebel, No. **60**; A. Umansky, No. **19**; H. W. Weigel, No. **15**.

R. C. A.

75[D].—Robert Elmo Benson (1903–), *Natural Trigonometric Functions Containing the Natural Sine, Cosine, Tangent and Cotangent to Seven Decimal Places for every Ten Seconds of Arc from* 0° *to* 90° *Semi-Quadrantally arranged.* Third edition, Los Angeles, California, Mac Printing Co , 133 East Third St., 1941. viii, 181 p. 15.2×23.2 cm. The first edition of this book appeared in 1927, and the second edition in 1929.

As calculating machines came into use, the need for seven-place tables of the natural values of the trigonometric functions stimulated a number of authors to prepare them. The first was by C. L. H. M. Jurisch, *Tables Containing Natural Sines and Cosines to Seven Decimal Figures of all Angles between 0° and 90° to every ten Seconds* . . . , Cape Town, 1884. 18×25.5 cm.; second ed. 1898; third, 1904; fourth (last) 1923. Then W. Jordan's work, prepared mainly for geodesists, *Opus Palatinum Sinus- und Cosinus-Tafeln von 10″ zu 10″*. Hannover, 1897. 16×24.7 cm.; second ed. 1913; third, 1923; sixth, 1936. In 1942 L. J. Comrie reported to me that *New Manual of Natural Trigonometrical Functions to Seven Places of Decimals of Sines and Cosines of Angles from* 0 *to* 10000 *Seconds,* published at New York by Wm. Chas. Mueller, 1907, is simply a pirated issue of the first edition of this Jordan work. And then the first table containing four functions, by H. Brandenburg, *Siebenstellige trigonometrische Tafel für Berechnungen mit der Rechenmaschine enthaltend die unmittelbaren, natürlichen Werte der vier Winkellinien—Verhältnisse, Sinus, Tangens, Cotangens und Cosinus* . . . *von* 10 *zu* 10 *Sekunden* . . . Leipzig, 1923. 20×27.8 cm. This edition contained, according to Mr. Comrie, over 400 errors. But the second edition, 1931, with prefaces in German, English, French, Spanish, and Japanese, "is practically free from error." And finally New Zealand, Department of Lands and Survey, *Natural Sines and Cosines for Every Ten Seconds of Arc to Seven Places of Decimals,* Wellington, 1927. 20.6×32.7 cm.

All four of these tables are directly or indirectly based on the two following extraordinary works:

I. o Georg Joachim von Lauchen, called Rhaeticus (1514–1576) and Valentin Otho, *Opus Palatinum de Triangulis,* [2 v.]. Neustadt, 1596. 21.6×35.8 cm. The second folio volume of 540 pages contains the complete ten-decimal trigonometric canon for every ten seconds of the quadrant, semiquadrantally arranged, with differences for all of the tabular results throughout. At the time of his death Rhaeticus "left this canon all but complete; and the trigonometry was finished and the whole, edited by Valentin Otho under the title 'Opus Palatinum,' in honor of the Elector Palatine Frederick IV, who bore the expense of publication." At the end of the first volume is an excessively inaccurate canon (180 p.) of cosecants and cotangents to 7 places for every 10 seconds. In the ten-decimal canon August Gernerth found 598 errors, 130 of them being in the differences. These are all listed in *Z. f. d. österr. Gymnasien*, Heft VI, 1863, p. 414–425.

II. Bartholomäus Pitiscus (1561–1613), *Thesaurus Mathematicus sive Canon Sinuum ad radium* 1.00000.00000.00000 . . . Frankfurt, 1613. 24×35 cm. This work contains (a) natural sines for every ten seconds throughout the quadrant, to 15D, with first, second, and third differences; and (b) natural sines for every second, 0° to 1° and from 89° to 90°, to 15D, with first and second differences. Both of these tables were calculated by Rhaeticus. Pitiscus added, to 25D, the following: sin 30°, sin 15°, sin 5°, sin 1°, sin 30′, sin 10′, sin 5′, sin 1′, sin 30″, sin 10″, sin 5″, sin 1″; and sines for every 20″ for the first 35′ (beginning with 10′), to 22D, with differences. Such are the works on which the volumes listed above were based. Gernerth lists also, *l.c.*, p. 426–428, 110 errors which he found in this canon of sines.

The first edition (1927) of Mr. Benson's work was published before the second edition of the Brandenburg volume, but after the appearance of such publications as

M. H. Andoyer, *Nouvelles Tables Trigonométriques Fondamentales*, 3 v. Paris, 1915–1918. 24×31.3 cm. The complete trigonometric canon to 15D for every 10″ of the quadrant; and for [0°(9′)45°; 17D]. And

E. Gifford, *Natural Sines to Every Second of Arc and Eight Places of Decimals*, Manchester, 1914. 15.2×24.3 cm.; second edition, 1926. Also *Natural Tangents* . . . 1920 and 1927. Some of the errors of one of these volumes are dealt with in MTE **1**.

Mr. Benson does not mention the source of his tables, but L. J. Comrie has reported on the first edition (Royal Astr. So., *Mo. Notices*, v. 92, 1932, p. 340), "A thorough checking of the table revealed 150 errors, which, apart from a few copying or proof-reading errors, can be found in Rhaeticus, and are also in Brandenburg, who openly stated the source of his values [Rhaeticus]." The preface of the second edition (1929) contains the statement that "the few errors which were found existing in the first edition have been carefully corrected by the printer and it is now believed that these tables are absolutely correct." In the present edition this sentence starts "The few errors which were found existing in the first and second editions . . . ," but otherwise the two sentences and prefaces are identical. A rearrangement of pages of the first edition brought successive half-degree ranges of values of sines and cosines opposite the corresponding values for tangents and cotangents. In response to inquiries Mr. Benson wrote to me as follows in December, 1942:

"My table of *Natural Trigonometric Functions* was made from original calculations. The functions were calculated to 12 decimal places at various intervals, averaging about every 10 minutes. The intermediate functions were then derived by interpolation from these 12 decimal functions. Upon completion, the sines and cosines were compared with the *Opus Palatinum* of Jordan. The same procedure was followed in compiling the tangents, except that due to my anxiety to complete a very monotonous labor I succumbed to the temptation to take most of the intervening functions from Brandenburg [1923]. I discovered quite a number of errors most of which were obvious transpositions. These, of course, were corrected in my manuscript.

"Now as to the accuracy of my tables in their present form. In the first edition there were found and reported to me, seventeen errors. These were all corrected before the second edition was printed in 1929. When Mr. Ives compiled his table [first edition, 1931; see RMT **76**] he very carefully compared them with mine and found the 150 errors that you mentioned. These errors are all in the tangent table and are in every case an error of 1 in the seventh figure. Mr. Ives and I discussed this at some length and agreed that the value derived would not justify the expense of making so many corrections. Consequently these errors still exist in the present edition. I realize that to any mathematical purist this is unforgivable, but to a practising engineer it is of no consequence. To the best of my knowledge and belief there are no errors in the table of sines and cosines."

R. C. A.

76[D].—Howard Chapman Ives, *Natural Trigonometric Functions to Seven Decimal Places for Every Ten Seconds of Arc Together with Miscellaneous Tables*, Second Edition, New York, John Wiley & Sons, 1942, vi, 351 p. 17×24.9 cm.

The first edition of this book was published in 1931 and the present edition has been enlarged by 22 pages, with new tables, to 7D, for every 10″, for cotangents from 0° to 2° and for tangents 88° to 90°. These are cut down to 3 to 6 places in the following main table (semi quadrantal) of sines, cosines, tangents, and cotangents (p. 23–292), and indeed the cotangent is given to only 6D on to 5°42′30″. Each page is devoted to 10 minutes of the functions, with differences and proportional parts (not found in Benson's volume).

Then follow eleven other miscellaneous tables, namely: Length of arc to radius unity; Coefficient K for central angles of certain curves; Radii from are definition; Radii from chord definition; Curves with even foot radii; Functions of a 1° curve; Corrections to tangent distances; Corrections to external distances; Trigonometric functions, formulas, and solution of triangles; Minutes in decimals of a degree; Units of length and of surface.

In the preface to the new edition it is stated that all errors found in the previous edition have been corrected for this edition. There is nothing in the prefaces of either edition which makes definite claim that the tables were compiled from original calculations. Indeed there is a passage which suggests rather the reverse; that, at least the table of tangents and cotangents are based on "fifteen place tables" computed by others. Of fifteen-place sexagesimal tables published before 1931 there are, in addition to the works of Rhaeticus and Andoyer (see RMT **75**), the following:

H. Briggs, *Trigonometria Britannica* [the trigonometry by H. Gellibrand, but the tables calculated by Briggs and published after his death]. Gouda, 1633. 21.4×23.9 cm. Natural sines to 15 D, and tangents and secants to 10D, for every 36″, or .01°, of the quadrant. See RMT **79.**

A. M. Legendre, *Traité des Fonctions Elliptiques et des Intégrales Eulériennes, avec des Tables pour en faciliter le Calcul numérique*, v. 2, Paris, 1826, p. 252–255. 21×27 cm. Sines for every 15′ of the quadrant.

J. Peters, "Einundzwanzigstellige Werte der Funktionen Sinus und Cosinus . . . " Preussische Akademie der Wissenschaften, *Abhandlungen, mathem. physik. Classe*, 1911, Anhang; also separately printed, 54 p. 22.7×26.4 cm. Sines and Cosines to 21 places of decimals for every 10′ of the quadrant and for every second of the first ten minutes with the first three differences.

C. E. Van Orstrand and M. A. Shoultes, "Values of sine θ and cosine θ to 33 places of decimals for various values of θ expressed in sexagesimal seconds," Washington Academy of Sciences, *Journal*, v. 12, p. 423–436, 1922. 17.3×25.8 cm. The range is 0″(1″)100″ (100″)1000″(1000″)45°.

The authors of printed sexagesimal tables of natural trigonometric functions, before 1931, to more than seven, but less than fifteen, places of decimals were G. J. Rhaeticus (1596); Pitiscus (1599, second ed. 1608, third ed. 1612); E. Gifford, Sines (first ed. 1914, second ed. 1926), Tangents (1920 and 1927). DeMorgan has told us that "Pitiscus will always be remarkable as the priest who wished that all his bretheren were mathematicians, to make them manageable and benevolent."

After the publication of the first edition of Ives's work, L. J. Comrie stated (Royal Ast. So., *Mo. Notices*, v. 92, 1932, p. 340–341) that the fifteen-place tables used by Ives, are "not those of Andoyer as one might imagine, but those of Pitiscus [Rhaeticus] as is shown by the 50 errors found by comparison with Andoyer. Most of the errors in Ives are also in Benson and the first edition of Brandenburg—all derived from the same source. From internal evidence it is clear that neither Benson nor Ives has printed from plates, thus introducing the grave risks associated with movable type." Since there were no early fifteen-place tables of tan and cot, possibly Mr. Comrie implies that for these functions the ten-decimal canon of Rhaeticus (1596) was used. On the other hand in response to my inquiry Mr. Ives made the following statement in a letter of 30 Nov. 1942: "My tables were not based on anything special. Almost all of the common books were used in checking, Andoyer, . . . , Benson, Trautwine, Brandenburg, Gifford, etc. For the most part Andoyer's 15 place tables were consulted in checking the last place . . . There were some 13 typewritten errors in my first edition. . . . As for the tables of Rhaeticus, while most of the later tables might be based on that, I never saw a copy of the book." Mr. Ives states also that his book was printed "from plates."

It may be noted that Mr. Ives published also

Seven Place Natural Trigonometrical Functions . . . , New York, Wiley, 1929, 10.3×17.4 cm. The six trigonometric functions, and versed sines, and external secants (secant-1), are given for every sexagesimal minute.

R. C. A.

77[D].—U. S. Coast and Geodetic Survey, *Special Publication*, no. 231, *Natural Sines and Cosines to Eight Decimal Places*, Washington, Government Printing Office, Washington, D. C., 1942, 541 p. 19×27.3 cm. Copies are sold by the Superintendent of Documents, Washington, D. C.

There is not one word of preface, text, or explanation in the volume, and the first page of the table is on the back of the titlepage. The table is for every second of the quadrant, and each page contains the results for five minutes of arc. Recalling recent tables covering exactly this same range one wonders why it should ever have been prepared and published. There was Emma Gifford's *Natural Sines to Every Second of Arc, and Eight Places of Decimals, computed . . . from Rheticus*, Manchester, 1914, iv, 544 p. 15.2×24.2 cm. Second edition, type reset, slightly smaller pages, 1926; and also the volume of Peters and Comrie (see RMT **78**), *Achtstellige Tafel der trigonometrischen Funktionen für jede Sexagesimalsekunde des Quadranten*, Berlin, 1939. Being curious as to the possible relation of the new table to these or to other tables, in August 1942 I made application to the Survey for information, and received from its Acting Director, Mr. J. H. Hawley, the following facts: "The values in our *Special Publication* no. 231 were largely taken from the second edition of *Natural Sines to Every Second of Arc and Eight Places of Decimals* by E. Gifford. Numerous checks were made by comparison with the values given in *Nouvelles Tables Trigonométriques Fondamentales* by H. Andoyer [v. 1, Paris, 1915; 15 place table of sine and cosine for every ten seconds of arc]. Our tables were very carefully checked by differencing and by making horizontal and vertical summations according to patterns which we found workable. Our volume was not compared with the Peters table. This would undoubtedly be a desirable thing to do but we do not have the personnel available to undertake it at the present time. We believe that our table is reasonably free of errors and in much more convenient form to use than the Gifford table" [that is, semiquadrantal rather than the quadrantal display]. Before the volume was distributed the publisher had corrected the errors in sin 41°21′49″ and 54″. L. J. Comrie compared the entries of the Survey volume for five full degrees with the corresponding Peters entries. It was thus shown that for one ninth of all the entries in the volume there were only last-figure unit errors. Hence Mr. Comrie believes that the table may be regarded as reliable.

The manuscript for this work was prepared in 1941 as a Project of the Work Projects Administration in Philadelphia, under the sponsorship of the U. S. Coast and Geodetic Survey, L. G. Simmons, senior geodetic engineer, in charge. Nearly 1280 errors were found in Gifford's table and Mr. Hawley kindly placed these at our disposal for publication (see MTE **1**).

R. C. A.

78[D].—Johann Theodor Peters (1869–), *Achtstellige Tafel der trigonometrischen Funktionen für jede Sexagesimalsekunde des Quadranten*, Berlin, Verlag des Reichsamts für Landesaufnahme, 1939, xii, 901 p. 20.9×29.7 cm.

In the year that this volume was published Mr. Peters was 70 years of age and was already an emeritus "Observator" and "Professor" (appointed 1901) of the Astronomisches Recheninstitut of the University of Berlin. In 1909 he started the publication of the 15 volumes of remarkable mathematical tables which have made his name famous. In recognition of this great contribution to science the Prussian Academy of Sciences awarded him the Leibnitz Medal. By 1911 he had published his 7-and 8-place tables of the logarithms of the trigonometric functions for every second of arc. At the meeting of the Astronomische Gesellschaft in Budapest in August 1930 Mr. Comrie, then director of the Nautical Almanac Office in Greenwich, discussed with Dr. Peters the calculation of twelve-figure values of the six trigonometric functions for each second of arc. By 1935 Mr. Comrie had completed the calculations for the cotangent and cosecant, and the copy for 7-figure and 8-figure tables of the four principal functions. Mr. Peters made independent calculations of all six functions. It had been originally planned to print the calculated values to seven, eight, and ten figures in the three sets of three volumes each. The first volume of each set was to contain the sin and cos functions, the second the tan and cot functions, and the third the sec and csc functions. But of these 9 volumes planned a publisher (the German Government) was found for only two, in the volume before us, for sin, tan, cot, cos. On each page are the values for 3′. An introductory table (p. vi–xi) contains eight-figure values of w'' cot w for every 10″ for $w=0°$ to $w=2°$; whence cot $w= (w''$ cot $w)/w''$ is readily found. So also for tan w near 90°.

The calculations of Peters were based on Andoyer's *Nouvelles Tables Trigonométriques Fondamentales*, v. 1–3, Paris, 1915–1918, in which the values of the six trigonometric functions are given for every ten seconds of the quadrant, divided into 90°. For the functions sin, cos, tan, sec, every fifth value, with twelve decimals, was extracted from these tables. The values were then checked by differences up to the fourth order, and 49 new values interpolated between every two consecutive values. (The method is described in detail in J. T. Peters, *Zehnstellige Logarithmentafel*, v. 1, Berlin, 1922, p. xi–xii.) The great care taken in proofreading included the checking by Mr. D. H. Sadler, the present director of the Nautical Almanac Office, of the accuracy of abbreviation of twelve-figure to eight-figure values.

For the four natural functions sin, tan, cot, and cos, this table undoubtedly supersedes all other existing eight-place tables, in point of view of accuracy, arrangement, and printing.

Comparatively few copies of this table had been sent out of Germany when the second world war had started. Since it was of value for various phases of the war effort, the British War Office made for its own use, but not for sale, even to British scientists, a facsimile edition of the Peters Comrie work, with an English title page, foreword, and introduction. The title page is as follows:

Eight-figure Table of the Trigonometrical Functions for every Sexagesimal Second of the Quadrant by Professor Dr. J. Peters. *By order of the Reich Minister for the Interior, published by the Reich Survey Office.* Berlin, Publishers of the Reich Survey Office, 1939. 20×28 cm. Thus the paper pages (but not the tables) are somewhat smaller. The paper is not of as good quality as the original. On page 902 we find "Printed under the authority of His Majesty's Stationery Office," December, 1939. There was a second printing in 1940.

For certain information about the seven-place table for every second of arc, see the report of L. J. Comrie, president of Commission 4 (Ephemerides) of the International Astronomical Union, *Transactions*, v. 6, 1939, p. 359–361. Portraits of Mr. Peters and of Mr. Comrie, as well as brief biographical notes, may be seen in *Porträtgallerie der Astronomischen Gesellschaft*, Budapest, 1931.

R. C. A.

79[D].—J. T. Peters, *Seven-Place Values of Trigonometric Functions for every Thousandth of a Degree. Published and distributed in the Public Interest by authority of the Alien Property Custodian under License #1.* New York, D. Van Nostrand Co., 1942, [vi, 368] p. 17.7×23.7 cm.

This is a work for which copyright is "vested in the Alien Property Custodian, 1942, pursuant to law," and it is a reproduction of *Siebenstellige Werte der Trigonometrischen Funktionen von Tausendstel zu Tausendstel des Grades*, Berlin-Friedenau, 1918; reprinted, Leipzig, Teubner, 1930. The dimensions of the pages of this original volume are each 7–8 cm. larger, but the type pages of both volumes are identical in size. The Introduction of the new table is now in English as also are the headings of the tables on the last five pages, "Conversion of minutes and seconds to decimal parts of a degree," "Conversion of decimal parts of a degree to minutes and seconds," "Conversion of degrees to time," Conversion of time to degrees." The main tables give natural sines for every 0°.001 throughout the quadrant, followed by tangents at the same interval. Proportional parts are also given. This very inconvenient arrangement was dictated by the optical company for which they were prepared. There is also a one-page table of sin and tan [0°.00(0°.01)0°.58; 8 or 9D], and therefore of cos, cot [89°.42(0°.01)90°; 8 or 9D].

It is of special interest that the basis of the preparation of these tables was a work prepared more than three hundred years ago by Henry Briggs (1561–1631), professor of geometry at the University of Oxford from 1619. His tables were published posthumously at Gouda in 1633, at Vlacq's expense, in *Trigonometria Britannica.* They were of natural sines (to 15D) and tangents and secants (to 10D), also log sines (to 14D) and tangents (to 10D), at intervals of a hundredth of a degree from 0° to 90°, with interlined differences foı all of the functions. These were all but completed by Briggs at the time of his death. They were published, with a work on trigonometry by Henry Gellibrand (1597–1637), professor of astronomy at Gresham College, London. In the work before us these tables are thus incorrectly referred to as Gellibrand's.

It is indeed extraordinary not only that Briggs should have conceived such tables as of high importance, but also that they should be of such remarkable accuracy. Peters found only two errors in all the sines and tangents—Briggs was correct where he listed a third error—but he overlooked other errors noted by Amelia de Lella (see MTE **2**). Peters tested the accuracy of the Briggs table by differencing. Nine new values were inserted between each two of his values. "As in doing this no effort was made to obtain an accuracy of one-half of a unit in the seventh place, but simply not to overstep an accuracy of one unit in that position, which is sufficient accuracy for the needs of practical calculators, consideration of first differences only was satisfactory for the interpolations throughout the table of sines. The larger part of the table of tangents from 0° to 72° was dealt with in the same way. From 72° to 89°, second differences were also taken strictly into account in the interpolations." For the tangents from 89° to 90° Peters first calculated the logarithms of the tangents from 89° to 90° by the method explained in the introduction of J. Bauschinger and J. T. Peters, *Logarithmisch-Trigonometrische Tafeln mit acht Dezimalstellen,* Leipzig, 1910, and then found their antilogarithms. "The possible error does not amount to one unit of the seventh decimal place at any place in the tables; in the entire table of sines the greatest possible error is ±0.65 units of the seventh decimal; in the table of tangents the error may rise to ±0.85 in very rare cases." The Introduction includes a dozen illustrative examples of the use of the tables, primarily prepared for the machine computer.

The tables having been reproduced by some sort of photolithographed process, are naturally not as clear as in the original work, but such defects are of no great consequence. Since they are the only tables of their kind,[1] they will surely continue to serve a very useful purpose in varied types of research.

In conclusion the reader is reminded of Wilhelm Oswald Lohse (1845–1915), *Tafeln für numerisches Rechnen mit Maschinen,* Leipzig, 1909, vi, 123 p. 20.8×29.7 cm.; second newly rev. ed. by P. V. Neugebauer, 1935, vi, 113 p. 17×24.2 cm. See RMT **47**. This volume contains the natural values of the six trigonometric functions for every 0°.01, with differences. Mr. Comrie has reported that this table is "especially convenient for orbital work." He has also warned that the similar table based on Briggs and compiled by Amelia De Lella, "should be avoided."

R. C. A.

80[D].—Leo Hudson and E. S. Mills, *Natural Trigonometric Functions. Tables. Sine, Cosine, Tangent, Cotangent, Secant and Cosecant to Eight Decimal Places with Second differences to ten Decimal Places Semi-quadrantally arranged. Instructions for use with the Monroe Adding Calculator.* Orange, N. J., Monroe Calculating Machine Co., 1941. 20+[47] p. 17.5×25.5 cm.

On pages up to 13 is material of elementary trigonometry, and then through page 20 are the "Instructions." The Tables occupy the remaining unnumbered pages. The title-page might easily mislead one who had not seen the book. Each page is devoted to one degree, and sines, cosines, tangents, and secants, are given for every minute, 0°–45°, to 8D; cosecants, cotangents, sines, cosines, are similarly presented for the range 45°–90°. Thus tangents and secants for this latter range, as well as cotangents and cosecants for the range 0°–45° cannot be read directly from the table. It is explained that these may be found by machine calculations of reciprocals. Beside each

[1] Gellibrand gave in his trigonometry (*l.c.* p. 43–44) sin x for $x=0^\circ.000(0^\circ.625)90^\circ$; 19D. Thus 144 values of the table under review are here carried to many more places of decimals.

of the columns tabulated is a column headed "Diff. per second"; e.g. between 10′ and 11′ we find 484.82 for sin and tan, and 1.48 for cos and sec; between 44°59′ and 45° the "diff" for sin and tan mount to 342.87 and 969.35 respectively. For the true differences these numbers are each to be multiplied by 10^{-8}. This is the interpretation of the phrase on the title-page, "with second differences to ten decimal places."

We are told that the sines were calculated from the formula

$$\sin x = x - x^3/3! + x^5/5! - x^7/7! + \cdots.$$

"The sine values were calculated until the ninth decimal place became stable and when it was 5 or greater the eighth decimal was increased by one; where it was less than 5, it was dropped. The Difference-per-Second values were calculated by dividing the difference between the minute values to nine decimal places by 60, carrying the calculation to ten decimal places, and equating the tenth decimal place to the nearest integer." The values of tan and sec were computed from $\tan x = \sin x/\cos x$; and $\sec x = 1/\cos x$, respectively.

Two errata are on a slip pasted in the book opposite the first page of the table. The volume is strongly bound, and clearly printed, but it is very expensive—about 9 cents a page. If a computer desires to use an 8-place table of the natural functions, it is obvious that the volume by Peters (see RMT **78**) would be decidedly preferable to the present volume. But if it turns out that one may depend upon the accuracy of this table, it may be well for the computer to know of this table when the Peters volume is not available. Comparison of the two volumes in a score of random angles showed constant agreement.

R. C. A.

81[D].—Project for Computation of Mathematical Tables, *Tables of Sines and Cosines for Radian Argument.* Prepared by the Federal Works Agency, Work Projects Administration for the State of New York, conducted under the sponsorship of the National Bureau of Standards. New York, 1940, xxii, 275 p. 20.9×27.1 cm. Reproduced by a photo offset process. Sold by the U. S. Bureau of Standards, Washington, D. C.

Elsewhere in this issue is a reference to an account of the organization and achievements of the New York Mathematical Tables Project, of which Arnold N. Lowan is the technical director. Among its publications the volume under review is the third.

One of the main purposes of tables of sines and cosines with radian argument is to facilitate rapid calculation of transcendental functions from asymptotic expansions. Interpolation in calculating

$J_{1/2}(x) = (2/\pi x)^{1/2} \sin x$, and $J_{-1/2}(x) = (2/\pi x)^{1/2} \cos x$, *e.g.*, requires such tables.

It seems desirable that we should sometime survey tables of circular functions of angles other than those in sexagesimal units. Hence we shall here begin by listing tables with radian arguments, with their ranges, commencing with the one under review. This is a table of sines and cosines for the ranges [0.000(0.001)25; 8D], p. 1–250, [0(1)100; 8D], [$0(10^{-5})0.01$; 12D], [$10^{-t}(10^{-t})9\times10^{-t}$; $t = 5, 4, 3, 2, 1$; 15D]. There are also a table of $p(1-p)$, for $p = 0.000(0.001)0.500$; and conversion tables of radians to degrees, and of degrees to radians.

L. J. Comrie recently suggested to A. N. Lowan the addition to this volume of two more pages (250A and 250B) printed so as to take the table to 25.2, i.e. just beyond 8π. "It is a great pity, when using a progressing argument to have to sacrifice a lot of the book by subtracting 6π."

The description of the self-checking feature of the iterated process of computation, and the double-differencing test of the final manuscript, which was then reproduced in facsimile, tend to inspire belief in the view of the authors "that this table is free from error."

The Project has also prepared tables of $\tan x$, $\cot x$ for the range [0(0.001)2; 8D or 8S]; these are soon to be published by the Columbia University Press.

Other radian tables are as follows:

Carl Burrau (1867–), *Tables of Cosine and Sine of Real and Imaginary Angles expressed in Radians* (*Circular and Hyperbolic Functions*). Berlin, 1907. 16.5×23 cm. The volume has also

German and French title pages and prefaces. The table of sines and cosines [0.000(0.001)1.609; 6D] occupies p. 1–9.

George Ferdinand Becker (1847–1919) and Charles Edwin Van Orstrand (1870–), *Smithsonian Mathematical Tables, Hyperbolic Functions*, Washington, Smithsonian Institution, 1909. 15×22.8 cm. The table of sin x, cos x, log sin x, log cos x, $x=$ [0.0000(0.0001)0.100(.001)1.600; 5D] is on p. 173–223.

John Robinson Airey (1868–1937), Tables I–II, and Arthur Thomas Doodson (1890–), Table III, "Sines and cosines of angles in circular measure," Br. Ass. Adv. Sci., *Report*, 1916, p. 60–91. 13.7×21 cm. Table I for sines and cosines [0.000(0.001)1.600; 11D]. Table II for $\theta-\sin\theta$, $1-\cos\theta$, with first differences [0.00001(.00001)0.00100; 11D]. Table III for sines and cosines [0.0(0.1)10.0; 15D]. For second editions of Tables I and III see below.

C. E. Van Orstrand, "Tables of the exponential function and of the circular sine and cosine to radian argument," National Academy of Sciences, *Memoirs*, v. 14, no. 5, 1921. 24.2×30.3 cm. Tables X–XIII, p. 47–78, give sin x and cos x for the following ranges: [0(1)100; 34D], [0.0(0.1) 10.0; 23D], [0.000(0.001)1.600; 23D], [1.10^{-t}, $2\cdot10^{-t}$, $\cdots$, $9\cdot10^{-t}$, where $t=4, 5, \cdots, 10$; 25D]. In Table XIV sin 0.1 and cos 0.1 are each given to 120D, and sin 1.0 and cos 1.0 to 105D.

J. R. Airey, Br. Ass. Adv. Sci., *Report* (a) *1923*, p. 287–289, sin x and cos x [0(1)100; 15D]; (b) *1924*, p. 276–278, sin x and cos x [10.0(0.1)20.0(0.5)50.0; 15D]; (c) *1928*, p. 305–307, sin x and cos x [20.0(0.2)40; 15D]. The calculations of table (a) were based on the values of sin 1^r and cos 1^r, each to 105D, given by C. A. Bretschneider in *Archiv d. Math. u. Phys.*, v. 3, 1843, p. 29. The values to 52D, given in the Anhang, p. 60, of Peters, *Zehnstellige Logarithmentafel*, v. 1, 1922, are in agreement with these. The values of sin 1^r, cos 1^r to 25D, given by Christoph Gudermann, *Jn. f. d. reine u. angew. Math.*, v. 6, 1830, p. 11, are each a unit too large in the 25th place, while the value of cos 1^r in its 13th place should be "1," not "0." But H. S. Uhler has calculated sin 1^r, cos 1^r to 477D, Conn. Acad. Arts and Sciences, *Trans.*, v. 32, 1937, p. 433–434. On the latter page is also given sin 10^r, cos 10^r, cos 20^r to 212D; sin 100^r, cos 100^r, sin 200^r, cos 200^r to 72D. From Mr. Uhler's computations it may be checked that the corresponding results of Bretschneider, Airey, and Van Orstrand are wholly correct.

Keiichi Hayashi (1879–), *Sieben- und mehrstellige Tafeln der Kreis- und Hyperbelfunktionen und deren Produkte sowie der Gammafunktion* . . . , Berlin, Springer, 1926. 21×27.3 cm. There are tables of sines and cosines (p. 2–204) [0.00001(.00001)0.00100; 20D], [0.010(.0001)0.0999; 10D], [0.100(.001)0.999; 12D], [1.000(.001)2.999; 12D], [3.00(.01)9.99; 18D], [10.0(.1)20(1)50; 15D]. L. J. Comrie has told us that in this volume are more than 1500 errors (other than last figure errors).

K. Hayashi, *Fünfstellige Funktionentafeln, Kreis-, zyklometrische, . . . Funktionen, . . .* , Berlin, Springer, 1930, 16.5×24.7 cm. On p. 2–40, 44–47, 162–164, are sin x, cos x, tan x, for $x=$ [0(0.01)10.00; 5D]; tan x, sec x, $x=$ [1.560(0.001)1.590; 5D]; sin x, cos x for $x=$ [0(0.0001) 0.0100; and 10(1)40; 10D]; $\sin\frac{x\pi}{2}$, $\cos\frac{x\pi}{2}$ for $x=$ [0.001 (.001)0.500; 5D]. This volume was based on the previous one. Hayashi is professor of engineering at the Kyushu Imperial University.

Br. Ass. Adv. Sci., Committee for the Calculation of Mathematical Tables, *Mathematical Tables*, v. 1, London, Br. Ass., 1931. 21.5×27.9 cm. On p. 3–7 sin x and cos x are given for x = [0.0(0.1)50.0; 15D] by A. T. Doodson, J. R. Airey, L. J. Comrie. This is followed (p. 8–23, by the Airey table of sin x and cos x for $x=$ [0.000(0.001)1.600; 11D], first printed with the Doodson table in 1916, as we noted above; similarly for other Airey tables above (1923–1924).

It will thus be seen that the values of sin x and cos x in the New York volume for the range from 0 to 1.600 are contained in tables of Van Orstrand, and of the Br. Ass. Adv. Sci. volume. The same may be said of values for key arguments at intervals of 0.1, for the rest of the range.

Among other tables, useful for certain kinds of work, the following may be mentioned:

Frederick Eugene Fowle (1869–), *Smithsonian Physical Tables*. Eighth rev. ed., (*Smithsonian Miscellaneous Collections*, v. 88), Washington, D. C., 1933. 15×23 cm. Table 15, p. 37–40, Sin, Cos, Tan, Cot, and their logs. [0.00(.01)1.60; 5D].

N. H. KOLKMEIJER, "Trigonometric and exponential functions," *International Tables for the Determination of Crystal Structures*, v. 2, *Mathematical and Physical Tables*, Berlin, Gebrüder Borntraeger, 1935. 18.5×27.7 cm. On p. 546–550 is a table of sin $2\pi x$ and cos $2\pi x$ for $x=$ [0.000(0.001) 1.000; 4D]. But this table was extended by M. J. Buerger, *Numerical Structure Factor Tables* (Geological So. Amer., *Special Papers* no. 33) New York, 1941, 15.4×23.9 cm. Table 1, p. 12–111, is a double entry table for the values of cos $2\pi hx$, sin $2\pi hx$ for $x=$ [0.000(0.001)0.999; 3D], and for $h=1(1)30$. This is one of five tables for use in computation of the x-ray diffraction intensities connected with a given crystal structure. The computation involves the products of two or more sines or cosines of this form. If $y=(1+\cos^2 2x)/\sin 2\theta$, the last four tables are of y, $y^{1/2}$, y^{-1}, $y^{-1/2}$, to 3D, with argument $\sin\theta=0.000(0.001)0.999$.

ULFILAS MEYER (1885–) and ADALBERT DECKERT, *Tafeln der Hyperbelfunktionen Formeln*, [Berlin, 1924]. 16.8×24.1 cm. On p. 22–30, 50–58, sin x, cos x, tan x, log sin x, log cos x, log tan x, are given for the range [0.000(0.001)1.569; 5D or S]. The scope of the volume was primarily determined by the needs of German telegraph engineers.

LOUIS MELVILLE MILNE-THOMSON (1891–) and L. J. COMRIE, *Standard Four-figure Mathematical Tables, including many New Tables, Trigonometrical Functions for Radians, . . . , Edition A, with positive Characteristics in the logarithms*, London, Macmillan, 1931. 19.2×26.5 cm. In Edition B the logarithms of numbers less than unity are printed with negative characteristics. Table IX (p. 96–131): (a) csc x, cot x, log sin x, log tan x, with differences, for the range [0.0000(0.0001) 0.0400; 4S]. (b) sin x, csc x, tan x, cot x, sec x, cos x, log sin x, log tan x, log cot x, log cos x, with differences, for the range [0.000(0.001) 1.570; 4S]. (c) sin x, cos x, log sin x, log cos x, for the range [0.0(0.1)7.9; 4D].

HENRI CHRÉTIEN, *Nouvelles Tables des Sinus Naturels Spécialement adaptées au Calcul des Combinaisons Optiques donnant les sinus, ou des arcs, directement, sans interpolation avec six ou cinq décimales et avec une décimale supplémentaire par interpolation ordinaire, suivies de tables de Tangentes Naturelles et de Conversion des Angles.* Paris, *Revue d'Optique Théorique et Instrumentale*, 1932. 16.5×26.5 cm. Table I (p. 9–14) is of $\Delta=x-\sin x$, for each $\Delta=0.000001(0.000001)0.001000$, x and sin x being given to 6D, so that x ranges from $0.^r014423$ to $0.^r181842$, i.e. to 10°25′. Table II (p. 15–27), $\Delta=0.00001(0.00001)0.02400$, to 5D, x ranging from $0.^r03107$ to $0.^r52661$, i.e. to 30°10′. Compare the table by Airey given above. Table III (p. 29–34) sin x, $x=$[0.500(0.001)1.600; 5D], 28°39′ to 91°41′. Table IV (p. 35–39) tan x, $x=$[0.000(0.001)1,000; 5D], to 57°18′. Tables V–VII (p. 40–42) are for converting radians, grades, and degrees, into one another. Chrétien was professor at the Institute of Optics of the Sorbonne. The advantages, in machine calculation, of the use of this table in comparison with others published before the volume under review, are set forth by L. J. Comrie, "The use of calculating machines in ray tracing," Physical So., *Proc.*, v. 52, 1940, p. 246–249; discussions, p. 250–252.

J. R. AIREY, "The circular and hyperbolic functions, argument $x/\sqrt{2}$" *Philosophical Mag.* s. 7, v. 20, 1935, p. 721–726. Tables of sin $(x/\sqrt{2})$ and cos $(x/\sqrt{2})$ for $x=$[0.0(0.1)20.0; 12D]; sin $x/\sqrt{2}$ and cox $x/\sqrt{2}$ appear in the asymptotic expansion of $J_0(z)$ in the calculation of ber x and bei x, the argument z of the Bessel function being taken along the "semi-imaginary" axis $z=xe^{-i\pi/4}$. In preparing the tables some of the calculations were based on sin $1/\sqrt{2}$ and cos $1/\sqrt{2}$ each given to 20D, and sin $0.1/\sqrt{2}$ and cos $0.1/\sqrt{2}$ each to 18D. See the Note on p. 31 of this issue.

R. C. A.

82[A, B].—*Barlow's Tables of Squares, Cubes, Square Roots, Cube Roots, and Reciprocals, of all Integer Numbers up to 12,500, edited by* Leslie John Comrie (1893–). Fourth edition, London, E. & F. N. Spoon, Ltd., and Brooklyn, N. Y., Chemical Publishing Co., Inc., 234 King St., 1941. xii, 258 p. 14×21.7 cm.

In the early part of the nineteenth century Peter Barlow (1776–1862), professor of mathematics at the Royal Military Academy, Woolwich (1806–1847), and fellow of the Royal Society (1823), was one of the leading mathematicians of England. He published a volume entitled *An Elementary Investigation of the Theory of Numbers* (London, 1811), and his *A New Mathematical and Philosophical Dictionary* (London, 1814) is still a valuable source of information. His contributions to the magnetic theory, optics, and strength of materials were also notable. For other details about his numerous papers and books see *Royal Soc. Catalogue of Scientific Papers*, and R. Ast. So., *Mo. Notices*, v. 23, 1862, p. 127–128.

A second volume which he published in 1814 was

New Mathematical Tables, containing the Factors, Squares, Cubes, Square Roots, Cube Roots, Recriprocals, and Hyperbolic Logarithms, of all numbers from 1 to 10000; Tables of Powers and Prime Numbers; an extensive Table of Formulae, or General Synopsis of the most important particulars relating to the Doctrines of Equations, series, fluxions, fluents, etc. lxiv, 336 p., [**A, B, C, D, E, F, H, M**]. There are in the volume, X Tables, and a long introduction. Errata in the fourth and higher powers of integers, in Table III, are given by A. J. C. Cunningham in *Messenger Math.*, v. 35, 1905, p. 18–19. In 1840, on the suggestion of the Society for the Diffusion of Useful Knowledge, Augustus De Morgan (1806–1871) brought out a new edition of Table I, with the "factors" omitted, and with the following title, *Barlow's Tables of Squares, Cubes, Square Roots, Cube Roots, Reciprocals, of all Integer Numbers up to 10,000.* In preparing this edition De Morgan secured the services of Mr. Farley, of the Nautical Almanac Office, who discovered 14 errors in the squares, 33 in the cubes, 14 in the square roots, 7 in the cube roots, 22 in the reciprocals. These errors are listed in the stereotype reprints at least as late as 1866; they do not appear in the printing of 1897, for example.

The third edition of Table I, brought out by L. J. Comrie in 1930, was a great advance on De Morgan's edition. The page was enlarged to that of the original Barlow, and an additional column was introduced, showing the square root of (a) the reciprocal of a number up to 1000, and then (b) 10 times the number in the argument column, in other words giving the square root of every 10th number between 10,000 and 100,000. Differences are also introduced in the four columns after $n=1000$. In the preface Mr. Comrie gives most satisfying information as to his exceedingly thorough checking of various details of the De Morgan edition; there were no errors in the squares and cubes, but many errors in the other columns. He exhibits the extent to which portions of a number of other tables may be trusted. In this edition Mr. Comrie gives also the fourth to the tenth powers of all numbers from 1 to 100 (and n^4 to 1000), the 11th–20th powers of the integers 1–10, and two pages of binomial coefficients, constants, etc.

The Comrie edition of this work has never been allowed to go out of print, and there has al, ways been an American agent for the publication. In spite of this, in 1935 G. E. Stechert and Co.- of New York, brought out a pirated facsimile (14.6×23 cm.) of this third edition; it was made in Shanghai, China. Two changes occur on the title page, "New impression" is added under "Third Edition" and Stechert's name has been substituted for that of the American agent of 1930 and later. The page is somewhat larger but the printing is appreciably inferior to that of the English edition. The Russians perpetrated a similar piracy of the third edition in 1933.

In the present real fourth edition 50 pages have been added to the third edition, through extending the table from 10,000 to 12,500, "particularly in order to avoid discontinuities when working with numbers just below and just above unity." This edition is one of the indispensable tables for the computer to have at hand. The square roots and cube roots are to seven places, and the reciprocals to seven significant figures, i.e. nine places to 1000, and above this, ten. See MTE **4**.

We understand that Mr. Comrie has in ms. a natural extension to this volume, namely: a table of $n^{-1/2}$, $(10n)^{-1/2}$ for $n=1000(1)12500$. We hope that its publication may not be long delayed.

R. C. A.

83[N].—*Financial Compound Interest and Annuity Tables computed by* Financial Publishing Company *under editorial supervision of* Charles H. Gushee. Boston, Financial Publishing Co.; London, Geo. Routledge & Sons, Ltd., 1942, 884 p. 13×23.3 cm.

This volume contains six principal tables, p. 2–733, arranged side by side, three on each of two facing pages. These tables give, for different rates and periods, the amount of 1, that is $s=(1+i)^n$; the amount of 1 per period, $s_{\overline{n}|}=[(1+i)^n-1]/i$; periodic deposit that will grow to 1 at a future date, $1/s_{\overline{n}|}$; present worth of 1, $v^n=1/s$; present worth of 1 per period, $a_{\overline{n}|}=(1-v^n)/i$; periodic payments necessary to pay off a loan of 1, $1/a_{\overline{n}|}=i/(1-v^n)=i+1/s_{\overline{n}|}$.

The rates are 1% (1/4%) 7% (1/2%) 10%. These nominal annual rates are shown for annual, semiannual, quarterly or monthly compounding. There are 360 periods for monthly compounding, or 30 years; 240 periods for quarterly compounding; 240 periods for semiannual compounding; 120 periods for annual compounding. The results in the tables are to ten places of decimals, which means that 16 significant figures are given in some places.

Pages 735–779 are devoted to Auxiliary Tables for fractions $(1/p)$ and multiples (n) of a unit period. For $p=365, 360, 180, 52, 26, 13, 12, 6, 5, 4, 3, 2, 1$, $n=2, 3, 4, 6, 12, 13, 26, 52, 180, 360, 365$, we are given, mostly, to ten places of decimals, $s=(1+i)^{1/p}=1+i'$; $s_{\overline{p}|i'}=i/[(1+i)^{1/p}-1]=[(1+i')^p-1]/i'$; $1/s_{\overline{p}|i'}$, i' indicating the interest for a fraction of the period. For multiples of the unit period $s=(1+i)^n$; $s_{\overline{n}|i}=[(1+i)^n-1]/i$. The percentages tabulated are, per period, as follows: 1/12% (1/48%) 7/12% (1/24%) 7/8% (1/16%) 1 3/4% (1/8%) 3 1/2% (1/4%) 7% (1/2%) 10%.

A wide range of examples (78) illustrating the application of the tables is clearly set forth on pages 781–869. They include those for which adjustments or extensions of the tables are necessary to fit the problems. Interpolation within the tables (including example 70), pages 842–861, with tables of interpolation factors, would certainly guide the non-mathematical specialist in the right direction. Probably the same remark may be made about the section on "Non-financial applications," where the compound interest law for continuous compounding comes up, even though we do find there "Instantaneous compounding means that we add an infinitely tiny amount of interest an infinitely large number of times."

The authors are probably correct in stating that "The main virtue of this edition is that it shows these tables for a greater number of rates, and for a greater number of periods than have ever before been published in a single book." The whole arrangement seems eminently practical and convenient. Of course certain things not given in this volume could be found in other tables; for example, in P. A. Violeine, *Nouvelles Tables pour les Calculs d'Intérêts Composés d'Annuités et d'Amortissement*, neuvième edition, entièrement refondue par A. Arnaudeau, *nouveau tirage avec un supplément contenant les tables de* 8% *à* 15%, Paris, Gauthier Villars, 1924, 21.5×27 cm. We here find tables, to ten places of decimals, for s, $s_{\overline{n}|}$, and $1/a_{\overline{n}|}$, for $10\frac{1}{2}$% ($\frac{1}{2}$%) 15%, for $n=1, \cdots, 100$.

R. C. A.

84[K, O, P, S].—EDWARD CHARLES DIXON MOLINA (1877–), *Poisson's Exponential Binomial Limit. Table I—Individual Terms, Table II—Cumulated Terms.* New York, D. Van Nostrand Co., 1942. viii, 46, ii, 47 p. 21×27.9 cm.

The "Limit" to which reference is here made occurs in *Recherches sur la Probabilité des Jugements en matière criminelle et en matière civile, précédées des règles générales du Calcul des Probabilités* by Siméon Denis Poisson (1781–1840), Paris, 1837, p. 206. Jacques (James) Bernoulli (1654–1705) in his *Ars Conjectandi*, Basle, 1713, p. 38–42, states, in effect, that if the probability of an event is p, the probability of exactly x successes in n independent trials is

$$P = C_x^n p^x (1-p)^{n-x},$$

the $x+1$st term in the expansion of the binomial $(\overline{1-p}+p)^n$. In $P(c, n, p)=\sum_{x=c}^{n} C_x^n p^x(1-p)^{n-x}$, which is the probability of an event happening at least c times in n trials, we have a series of frequencies, in which Poisson allowed $n\to\infty$, and $p\to 0$, while $np=a$ is kept constant, and found as "limit"

$$P(c, a) = \sum_{x=c}^{\infty} \left(\frac{a^x e^{-a}}{x!}\right).$$

Mr. Molina's Table I is of $(a^x e^{-a}/x!)$, for $a=0.001(0.001)0.01(0.01)0.3(0.1)15(1)100$; $x=0(1)150$; to 6D. For $a=.001$, the only values of x are 0, 1, 2; but for $a=100$, $x=56(1)150$. Table II of $P(c, a)$ also to 6D is for the same range of a, and $c=0(1)153$. The volume was lithoprinted by Edwards

Brothers, Inc. and leaves a great deal to be desired. On very many pages there are numbers which are much blacker, and less distinct, than the other numbers, and in some places one cannot be sure what numbers are intended; for example, Table I, $a=14.4$, $x=14$; $a=19$, $x=21$; $a=81$, $x=68$. There are a number of cases of only parts of numbers being printed.

A table of $(a^x e^{-a}/x!)$, to 4D, was given by the Russian statistician Ladislaus von Bortkiewicz (1868–1931), in his *Das Gesetz der kleinen Zahlen*, Leipzig, 1898, p. 49–52, 16×23.7 cm., $a=0.1(0.1)10.0$; $x=0(1)24$. The fourth figure is inaccurate in many instances; also there are objections to the author's use of the term "law of small numbers," as applied to $P(c, a)$. H. E. Soper gave a six-place table of $(a^x e^{-a}/x!)$ in *Biometrika*, v. 10, 1914, p. 27–35, 19×27 cm., for $a=0.1(0.1)15.0$; $x=0(1)37$. This was also printed as Table LI in *Tables for Statisticians and Biometricians*, ed. K. Pearson, Cambridge, University Press, 1914; second ed. 1924.

Tables for $P(c, a)=.0001$, .001, and .01 respectively, with a running from .0001 to 928, appeared in Mr. Molina's article "Computation formula for the probability of an event happening at least c times in n trials," *Amer. Math. Mo.*, v. 20, 1913, p. 193. A similar table, with much more extensive values assigned to $P(c, a)$ was given by G. A. Campbell in *Bell System Technical Jl.*, v. 2, 1923, p. 108–110; also in *The Collected Papers of George Ashley Campbell*, New York, 1937.

Mr. Molina has here published independently prepared tables of great importance in many fields. Only small parts of Tables I and II have appeared in print earlier. He tells us how "The work of computing and checking which these tables represent is that of many individual members of the Bell System over the course of almost forty years." We are informed that these tables have been found especially useful in quality engineering work in finding the most advantageous plan for a given set of conditions. The table $P(c, a)$ is in a form directly usable for solving problems of "single, double, and multiple sampling which require the determination of compound probabilities relating to the occurrence of c or more, or c or less defective units in the first, second, etc., samples of stated sizes."

In H. Levy, *Elements of Probability*, Oxford, Clarendon Press, 1936, "The telephone problem," p. 144–145, it is remarked that the telephone service in operation presents an enormous number of practical problems in probability. These are, however, necessarily so technical that a simple case only, involving $P(c, a)$, is given in illustration. Extensive and interesting discussion of "the Poisson Law" and its applications, is to be found in T. C. Fry's *Probability and its Engineering Uses*, London, Macmillan, 1928; on p. 458–463 are tables for $a^x e^{-a}/x!$ for $a=0.1(0.1)1(1)20$, $x=0(1)44$; and for $P(c, a)$, a as before, and $x=0(1)45$.

R. C. A.

85[L, M].—J. C. Jaeger and Martha Jaeger, "A short table of

$$\int_0^\infty \frac{e^{-xu^2}}{J_0^2(u)+Y_0^2(u)}\frac{du}{u}\text{,"}$$ Royal So. Edinburgh,

Proc., s.A, v. 61, no. 19, 1942, p. 229–230. 17.5×25.4 cm.

This table, from the University of Tasmania, follows a paper, no. 18, by Mr. Jaeger, "Heat flow in the region bounded internally by a circular cylinder" (p. 223–228). The object of the paper was "to give some numerical results for the cooling of the region bounded internally by a circular cylinder, with constant initial temperature, and various boundary conditions at the surface. Problems of this nature are of importance in connection with the cooling of mines, and in various physical questions." The discussion leads to integrals of the type

$$I(p, q; x) = \int_0^\infty \frac{e^{-xu^2}}{[puJ_1(u)+qJ_0(u)]^2+[puY_1(u)+qY_0(u)]^2}\frac{du}{u}.$$

When x is large, an approximation given in the paper readily yields numerical results. But when x is small we are led to $I(0,1; x)$ which is tabulated for x, [0.00(0.01)10(0.1)100(1)1000; 3D]. In all cases the values were "calculated to at least one more place than shown so it is hoped that the final figures are correct."

R. C. A.

86[C].—HORACE SCUDDER UHLER (1872–), *Original Tables to 137 Decimal Places of Natural Logarithms for Factors of the Form* $1 \pm n \cdot 10^{-p}$. *Enhanced by Auxiliary Tables of Logarithms of Small Integers.* New Haven, Conn. The Author, Sloane Physics Laboratory, 1942. [120] p. 21.3×27.7 cm. Published in an edition of 800 copies.

These tables will be of very little interest to the "practical computer." Occasionally one sees statements about the maximum accuracy necessary in tabulating logarithms for practical purposes. Although most of these statements have exceptions, it is absurd to expect any practical use for a table to 137 decimal places. It would be more correct to say that these tables are of practical value only in solving impractical problems. It is for just such a problem that the author has prepared these tables. He has made them available to the public in the hope that they may be used effectively on other problems of the same kind.

These tables give the natural logarithms of numbers of the forms $1+n \cdot 10^{-p}$ and $1-n \cdot 10^{-q}$ where n, p, and q are positive integers not exceeding 9, 21, and 69 respectively, together with the logarithms of certain small integers (2–11, 13, 17, 19, 23, 29, 31, 47, 53, 71, 97, 101, 103, 107, 109, 113) and of 10^k for $k=20, 30, 40, \cdots, 90$, and 110, all to 137 or more decimal places.

At first one might think that in publishing natural instead of common logarithms, the author has failed to recognize the fact that numbers are usually written to the base 10. On the contrary, the author is exploiting the base 10 to the utmost in choosing the natural system of logarithms. In fact, the natural logarithm of a number like $1+3 \cdot 10^{-8}$ is equal to

$$0.00000\ 00299\ 99999\ 55000\ 00089\ 999 \cdots$$

whereas its common logarithm is

$$0.00000\ 00130\ 28834\ 26166\ 47418 \cdots ,$$

an unpredictable sequence of digits. In short since

$$\ln (1 \pm n \cdot 10^{-p}) = \pm n \cdot 10^{-p} - \tfrac{1}{2}n^2 \cdot 10^{-2p} \pm \tfrac{1}{3}n^3 \cdot 10^{-3p} + \cdots ,$$

the first six terms of this series are rational numbers whose decimal expressions either terminate quickly or are periodic of period one so that the digits of this logarithm follow simple general patterns (given in Tables 5 and 6) until the term $\frac{1}{7}n^7 \cdot 10^{-7}$ is reached. This fact renders it unnecessary to give tables of natural logarithms of $1 \pm n \cdot 10^{-p}$ for p slightly more than $N/7$ where N is the number of decimal places desired. This is why p, as mentioned above, does not exceed 21. The last two-thirds of Table 2, giving $\ln(1-n \cdot 10^{-q})$ for $21 \leqq q \leqq 69$, is, in the opinion of the reviewer, largely a waste of printing. It is interesting to note that the decimal system, for once, is superior in this respect to the duodecimal system, since in the latter system the earlier term $\frac{1}{5}n^5 \cdot 10^{-5}$ gives trouble.

The general method of using a table of this sort to compute natural logarithms and exponentials is the same as in the familiar tables for common logarithms and anti-logarithms. (The only table of common logarithms comparable with these under review is an obscure table of Parkhurst[1] to 100 decimals.) In explaining the use of these tables the author chooses problems having about 15 decimals. The reader is left to discover for himself the technique of dealing with problems having over 100 decimal places.

The elaborate mechanical, semi-mechanical, and photographic methods used in calculating and checking the tables make it quite unlikely that a single error exists in the thousands of digits printed.

D. H. L.

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1. Henry Martyn Parkhurst (1825–1908), *Astronomical Tables comprising Logarithms from 3 to 100 Decimal Places and other Useful Tables*, New York, 1868; other editions 1869, 1871, 1873, 1875, 1876, 1881, and 1889 (final). 11.3×15.1 cm. The volume was set up and electrotyped by the author himself, a "law stenographer." Some of the later editions have more than twice as many pages as the first edition. Tables II, III, IX contain, with 19 exceptions, log N to 102D, $N=1(1)$ 109. Memoirs by Parkhurst may be found in Harvard College, Astronomical Observatory, *Annals*, v. 18 (1890), 29 (1893). *The Catalogue of the Astor Library, Authors and Books L-R*, Cambridge, Mass., 1887, lists an "incomplete" copy of the first section of Parkhurst's *Tables*, as

87[A, C, P].—Ray M. Page, *14000 Gear Ratios. Tabulated Ratios presented in Common Fractional and Decimal Forms and in Differently arranged Sections to facilitate the Solution of all Classes of Gear-Ratio Problems.* New York, The Industrial Press, 148 Lafayette St.; and Machinery Publishing Co., 17 Marine Parade, Brighton, England, 1942. iv, 404 p. 21.5×27.7 cm.

There are four tables in this volume, p. 23–403. In Table I are the decimal equivalents of rational fractions arranged in 119 tables, each table being complete on one page with the denominator of the fraction serving as an index. These denominators range from 2 to 120 inclusive. The decimal equivalents are given to eleven places when the ratio is less than unity, and to nine places when greater. Such gear ratios are of importance in Machine Shops and Mechanical Engineering, where the gears generally used do not have more than 120 teeth. The tables here include any ratio from 1/120 to 120/2 resulting in 14,280 ratios which decimally start at 0.008333 . . . and continue to 60.000 . . . . As an illustrative example of the use of such ratios (*Machinery's Handbook*, eleventh ed., New York and London, 1941, p. 671) it may be required that the speeds of the driving and driven gears are to be as near as possible to 1149 and 473 revolutions per minute. It may be stipulated, however, that the number of teeth in the larger gear must not exceed 60. Dividing 473 by 1149 we find 0.41166 . . . . By referring to the tables, the nearest fractional value to this ratio with a denominator less than 60, is found to be 7/17. "Thus, the nearest number of teeth in the gears can be 14 and 34, or 21 and 51. This will give speeds of 1149 and 473.118 revolutions per minute, which introduces a very small error. In the absence of such tables, the method of obtaining the approximate fraction 7/17 would be very cumbersome." In the first 22 pages of the work under review are many examples of gear ratio and speed problems.

The decimal equivalents of the ratios in Table I are in Table II (p. 143–258) arranged, in somewhat abbreviated form, in order of magnitude, and each is followed by its logarithm, to 7D, and by equivalent ratios, sometimes very numerous. For example, .66667 . . . is followed by 40 ratios from $\frac{2}{3}$ to 80/120, all equivalent to $\frac{2}{3}$.

In Table III (p. 259–364) for a total number of teeth, the corresponding gear pairs and decimal equivalents are given for every integer from 25 to 239; under 25, for example, are 12/13 = .923077 and 13/12 = 1.083333. Table IV (p. 365–403) is a gear factor table and consists of all numbers from 20 to 14400 that are the product of two factors neither of which exceeds 120. This table is used in finding gear combinations equivalent to a given numerator and denominator. For example, if a ratio = 697/1081, the table shows that (17×41)/(23×47) is an equivalent expressed in practical gear sizes.

Mr. Page's collection of gear-ratio tables is incomparably more extensive than anything previously published. The first person to conceive of such tables in connection with gear ratios was the French horologist, Achille Brocot, and the first publication in this connection was a paper presented by Brocot at a meeting of the Société des Horlogers at Paris, in June, 1860, "Calcul des rouages par approximation, nouvelle méthode," *Revue Chronométrique, Journal des Horlogers, Scientifique et Pratique*, v. 3, p. 186–194. He here refers to his work with tables which he had ready for publication. Their titles are transcribed from the *Catalogue Général des Livres Imprimés de la Bibliothèque Nationale:*

°*Calcul des Rouages par Approximation, Nouvelle Méthode,* Paris, l'auteur, 1862. Gr. in 8°, 97 p.

°*Table de Conversion en Décimale des Fractions Ordinaires, à l'usage du Calcul des Rouages par Approximation. Nouvelle Méthode.* Paris, P. Dupont, 1862. In 8°, 51 p.

No copies of these works are to be found in any of the principal libraries of America. But it would seem as if the gist of Brocot's work may have been contained in the following publication:

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well as other publications (1854–77) on spelling reform, stenography, etc. He was the chief reporter of the U. S. Senate debates 1848–54, and one of the reporters of the *Debates* of the Maryland Constitutional Convention of 1864. The 1869 edition of the *Tables* is in Library of Congress and the 1868 and 1873 editions in the New York Public Library (Astor Lib.). There is a sketch of Parkhurst in *Pop. Astr.*, v. 16, 1908, p. 231–239, and portrait plate.—Editor.

Berechnung der Räderübersetzungen. Herausgegeben von dem Verein „Hütte." Bearbeitet nach Calcul des Rouages par Approximation, Nouvelle Méthode par Achille Brocot, Horloger. Second ed., Berlin, Ernst & Korn, 1879. 68 p. 11.4×17.8 cm.

This little volume contains two tables, of which the first, occupying p. 15–54, gives the decimal equivalent to eleven places of decimals of the proper fractions 1/100 and 99/100 and the decimal equivalent of every proper fraction between these two whose denominator is not greater than 100. These decimal expressions are arranged in order of magnitude. There seems to be no doubt that this is the form of Brocot's original table. In the preface to the first German edition (Berlin, 1871, xvi, 52 p.), which I have not seen, it is stated that this table was wholly recalculated. The next edition of this table of 1879 was in the monograph

W. H. RASCHE, *Gear Train Design. A new method of Using Brocot's Table of Decimal Equivalents in Calculating the Numbers of Teeth in a Gear Train* (Virginia Polytechnic Institute, *Engineering Experiment Station Series, Bulletin,* no. 2, June 1926) 111 p. 15.4×23.8 cm. A second edition, with a revision of the introductory text only, appeared in this series as *Bulletin,* no. 14, March, 1933. In both editions, the table occupies pages 71–111. I am indebted to Professor Rasche for kindly supplying me not only with a copy of his *Bulletin* 14, but also with an incomplete copy of the German work of 1879.

After Brocot's table of 1862 no addition was made to it for 73 years. Then came

EARLE BUCKINGHAM, *Manual of Gear Design Section One. Eight Place Tables of Angular Functions in Degrees and Hundredths of a Degree and Tables of Involute Functions, Radians, Gear Ratios, and Factors of Numbers,* New York, Machinery, 148 Lafayette St., 1935. 21.5×26 cm. In this volume Professor Buckingham has reduced the Rasche-Brocot Table (p. 147–169) from eleven to eight places of decimals, throughout its range, but he has added a new eight-place table, corresponding to the increase in teeth of a gear from 100 up to 120. As already noted, Page extends this new table to 11 places of decimals. In none of these tables before Page and back to Brocot, was it possible readily to find the decimal equivalent of any particular fraction.

Many references might be given to other works where minor gear-ratio tables may be found. The following two samples will suffice:

BROWN & SHARPE MFG. CO., *Formulas in Gearing with Practical Suggestions.* Seventeenth ed., Providence, R. I., 1942, p. 223–245. 15.1×22.9 cm.

Machinery's Handbook for Machine Shop & Drawing Office . . . , Eleventh ed. New York, The Industrial Press, 1941, p. 671–675, 1080–1103. 11.5×17.9 cm.

But the mathematical consideration of expressing proper fractions decimally was developed many years earlier, by Henry Goodwyn in the following four publications of 1816–1823:

1. [*The First Centenary of a Series of Concise and Useful Tables of all the Complete Decimal Quotients which can arise from dividing a Unit, or any Whole Number less than each Divisor, by all Integers from 1 to 1024,* London,] 1816, xiv, 18 p., 20×25.3 cm.

2. *The First Centenary of a Series of Concise and Useful Tables of all the Complete Decimal Quotients, which can arise from dividing a Unit, or any Whole Number less than each Divisor, by all integers from 1 to 1024. To which is now added A Tabular Series of Complete Decimal Quotients for all the Proper Vulgar Fractions, of which, when in their lowest terms, neither the Numerator, nor the Denominator, is greater than 100: with the Equivalent Vulgar Fractions Prefixed.* London, 1818, xiv, 18, viii, 32 p. 20×25.3 cm. No. 1, without title page, and called by Goodwyn a "specimen" is identical with the first part of no. 2 and all of these pages were printed in 1816. The title page and independently paged second part were added in 1818.

3. *A Tabular Series of Decimal Quotients for all the Proper Vulgar Fractions, of which, when in their lowest Terms, neither the Numerator nor the Denominator is greater than 1000,* London, 1823, v, 153 p. 20×25.3 cm.

4. o *A Table of the Circles arising from the Division of a Unit, or any other Whole Number, by all the Integers from 1 to 1024, being All the pure Decimal Quotients that can arise from this Source.* London, 1823, v, 118 p. To express 1/1021 as a repeating decimal requires a "circle" of 1020 digits.

These publications are excessively rare. There is a copy of no. 1 in the Yale University Library; of no. 2, and a film of no. 3, at Brown University; of no. 3 at the John Crerar Library in Chicago. There is no copy of no. 4 in the principal libraries of America. Nos. 3 and 4 were published anonymously. All four of these tables are described by J. W. L. Glaisher (1) in his *Report of the Committee on Mathematical Tables*, Br. Assoc. Adv. Sci., *Report*, 1873; (2) in his "On circulating decimals with special reference to Henry Goodwyn's *Table of Circles* and *Tabular Series of Decimal Quotients* (London, 1818–1823)," Cambr. Phil. So., *Proc.*, v. 3, p. 185–206.

And, finally, we may give a reference to C. F. Gauss's "Tafel zur Verwandlung gemeiner Brüche mit Nennern aus dem ersten Tausend in Decimalbrüche," in his *Werke*, v. 2, Göttingen, second ed., 1876, p. 411–434. 22.5×27.6 cm. When this table was prepared does not seem to be known; it was found among Gauss's papers after his death in 1855.

For such discussions by Gauss, Goodwyn, and others, see D. H. Lehmer, *Guide to Tables in the Theory of Numbers*, Washington, D. C., National Research Council, 1941.

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88[A, F, P].—*Formulas in Gearing with Practical Suggestions.* Seventeenth ed., Providence, R. I., Brown and Sharpe Mfg. Co., 1942. 266 p. 15.1×22.9 cm.

Brown and Sharpe has long been one of the greatest concerns in this country for manufacturing machines, tools, and instruments involving high precision in workmanship. Indeed, during past years their products have been in international demand. Among various publications of this firm are two anonymous volumes which have passed through many editions. *Practical Treatise on Gearing*, by Oscar James Beale, appeared first in 1886 as a volume of about 130 p.; the twenty-fourth ed. of 1942 contains 244 p. The second volume, by Charles C. Stutz, and the one before us for review, was first published in 1892 as a volume of 69 p. The chief additions and changes were made in the fourth, tenth, and twelfth editions of 1905, 1929, and 1936. The current demand for the work is indicated by the fact that there have been four editions, each of 1000 copies, since the twelfth in 1936. The sixteenth and seventeenth editions both appeared in 1942.

The first quarter of the book is mainly occupied by descriptive matter and formulas, while the remaining pages are filled with tables. The descriptive text deals with such topics as systems of gearing, definitions and classifications applied to gearing and pitch of gears, spur gearing, bevel gears, worm and worm wheel, spiral and screw gearing, epicyclic gearing, gearing of lathes for screw cutting, sprockets, and strength of gears. Among the tables most of the pages (123–245) are taken up with tables used by mathematicians, namely: five-place table of natural sines, cosines, tangents, and cotangents for every minute of the quadrant (reproduced by "courtesy of The International Correspondence Schools, Scranton, Pa."); table of prime numbers and factors 1 to 10,200; table of prime numbers and lowest factors 10,000 to 100,000; gear ratios and their decimal equivalents ("Brocot's table" from Machinery's *Handbook*, various ratios from 1/60 =0.0167 to 59/60=0.9833); six-place logarithms for all gear ratios from 100/24 to 24/100 arranged in numerical order; table of ratios of two gears with their decimal equivalents (100/24 =4.1667 to 24/100=.2400). We have discussed gear ratios at length in RMT **87**.

The tables of prime numbers and factors are almost wholly taken from Edward Hinkley, *Tables of Prime Numbers and Prime Factors of the Composite Numbers from* 1 *to* 100,000, Baltimore, 1853. Many errors in this volume are listed by L. J. Comrie, p. xi–xii of Br. Ass. Adv. Sci., *Mathematical Tables*, v. 5: *Factor Table*, London, 1935. In 1935 Mr. Comrie drew the attention of the Brown & Sharpe Mfg. Co. to errors in its factorization tables, in the eleventh edition (1933), mainly copied from Hinkley, of the following 30 numbers: 2198, 2798, 3632, 4086, 4396, 5506, 6998, 7011, 7160, 7264, 8172, 8285, 8792, 8815, 8844, 8901, 9224, 9542, 9543, 9696, 9788, 9810, 10134, 11747, 16107, 17633, 56323, 58301, 65959, 93617. Corrections of all of these errors have been made in the present edition; indeed, except for 8172, they had been already made in the twelfth edition (1936).

R. C. A.