58. J. C. ADAMS, "Useful formulae, connecting Legendre's coefficients, which are employed in the theory of terrestrial magnetism," *Scientific Papers* of Adams, v. 2, Cambridge, 1900, p. 243–296. On p. 268–281, Gaussian functions $G_n^m(\mu)$, $n = 0(1)10$, $m = 0(1)10$, $\mu = [0.00(0.05)1; 10D]$.

Authors

R. C. A.

RECENT MATHEMATICAL TABLES

98[S].—PROJECT FOR COMPUTATION OF MATHEMATICAL TABLES (A. N. Lowan, technical director), *Miscellaneous Physical Tables. Planck's Radiation Functions, and Electronic Functions.* Prepared by the Federal Works Agency, Works Projects Administration for the City of New York, conducted under the sponsorship of the National Bureau of Standards. New York, 1941, vii, 58 p. 20.2 × 26.3 cm. Reproduced by a photo offset process. Sold by the U. S. Bureau of Standards, Washington, D. C. The work was not distributed until 1943. $1.50; foreign price $1.75.

The tables of Radiation Functions are reprinted from Optical So. Amer., *Jn.*, Feb. 1940. A body of alsolute temperature T emits electromagnetic waves of all possible wave lengths λ. The radiated energy is however very unequally distributed among the waves of different length. The radiation and photon functions are defined as

$$R_\lambda = c_1\lambda^{-5}(e^{c_2/\lambda T} - 1)^{-1};\ R_{0-\lambda} = \int_0^\lambda R_\lambda d\lambda;\ N_\lambda = 2\pi c\lambda^{-4}(e^{c_2/\lambda T} - 1)^{-1};\ N_{0-\lambda} = \int_0^\lambda N_\lambda d\lambda.$$

In a region of wave lengths ranging from zero to λ the integrals $R_{0-\lambda}$ and $N_{0-\lambda}$ express, respectively, the rates of emission of energy and photons per units of area and time. The letters c, c_1, c_2 have special numerical values. It is chiefly in the values of these constants that there is a difference between the various tables previously published and listed on p. 12–13; c represents the velocity of light (cm./sec.). The values of c_1 and c_2 which were adopted after consultation with L. J. Briggs and H. T. Wensel of the National Bureau of Standards use numbers 3.732 and 1.436 while the recent tables of P. Moon[1] use 3.697 and 1.432 respectively. On p. 4 of the introduction a way of correcting for a change in c_1 and c_2 is explained. Remarks are made also on the number of significant figures in the tabulated entries and on the method of interpolation for λ. In making the computations results obtained by means of series were checked by evaluating the integrals by a method of numerical integration. In Table I we are given values of $R_{0-\lambda}/R_{0-\infty}$, $R_\lambda/R_{\lambda\max}$, $N_{0-\lambda}/N_{0-\infty}$, $N_\lambda/N_{\lambda\max}$, for $\lambda T = 0.050(0.001)\cdot1(0.005).4(0.01).6(.2)1(0.05)2$. In Table II are values of $R_{0-\lambda}$, R_λ, $N_{0-\lambda}$, N_λ, for λ (in microns) $= .5(0.01)1(0.05)4(0.1)6(0.2)10(0.5)20$. In Table III are given values of N_λ, to 4D, for $T = 1000°$K, 1500°K, 2000°K, 2500°K, 3000°K, 3500°K, 6000°K, and $\lambda = 0.25(0.05)1.6(0.2)3(1)10$. The last table is to be used for changes in c_1 and c_2, from 4D to 5D are given for $\lambda T = 0.025(.005).13(.01).2(.05).6(.1)1(.5)2$.

For an electron of velocity v, charge e in electromagnetic units and mass m_0, let $x = v/c$, where c is the velocity of light, let $G = (1 - x^2)^{-1/2}$. The need of tables of the function G has been felt for a long time. In his work on the distribution of electricity on a thin circular disc, George Green[2] gave a 3-place table for $x = 0(.2).8(.1)1$. A more extensive 3-place table for $x = 0(.01).4$ was prepared by M. Hamy[3] for use in the theory of perturbations. When the function occurred in the expressions for mass and energy in electromagnetic theory more extensive tables were needed. G. Fournier[4] gave from 5 to 6S for $x = 0(.01).9(.02)$-$.99(.001).995(.0005)1$ while E. N. Da C. Andrade[5] gave from 3 to 4S for $x = 0(.005)$-$.01(.01).05(.05).8(.01).99, .995, .998$.

The present tables were computed at the suggestion of A. E. Ruark. From 7 to 10D are given for the range $x = .005(.005)1(.001).9(.0005).96(.0002).99(.0001).995(.00005)$-$.998(.00002).999(.00001).99999(.000001).999999(.0000001).9999999(.00000001).99999999$ $(.000000001).999999996,.999999997(.0000000005).999999998(.0000000001).9999999990$. Corresponding to these values of x are given not only the values of G, but also of Gx, and $V = 10^{-8}e^{-1}T$ [where T the kinetic energy $= m_0c^2(G - 1)$, m_0 being the mass; Einstein's formula], and $H_\rho = (m_0/e)cxG$.

H. B.

[1] P. Moon, "A table of Planck's function from 3500 to 8000K," *Jn. Math. Phys.*, Mass. Inst. Tech., v. 16, 1938, p. 133–157.

[2] G. Green, "Mathematical investigations concerning the laws of the equilibrium of fluids analogous to the electric fluid with other similar researches," Cambridge Phil. So., *Trans.*, v. 5, 1833, p. 1–63; table on p. 62.

[3] M. Hamy, "Sur le développement approché de la fonction perturbatrice dans le cas des inégalités d'ordre élevé," *Jn. de Math.*, s. 4, v. 10, 1894, p. 391–472, s. 5, v. 2, 1896, p. 381–439; table on p. 466.

[4] G. Fournier, "Tables relatives à l'électron," *Jn. de Physique*, s. 6, v. 6, 1925, p. 23–32.

[5] E. N. da C. Andrade, *The structure of the atom*, London, Bell, 1924, p. 300; third ed., 1927, p. 720.

99[J, K, L].—CATHERINE M. THOMPSON, "Tables of percentage points of the incomplete beta-function," *Biometrika*, v. 32, 1941, p. 168–181. 19.3 × 27.3 cm.

The incomplete beta-function is here defined as

$$I_x(p, q) = \frac{\Gamma(p + q)}{\Gamma(p)\Gamma(q)} \int_0^x x^{p-1}(1 - x)^{q-1}dx.$$

The percentage points of this function are the values of x which satisfy the integral equation

$$I_x(p, q) = P$$

for given values of P, p, and q.

A table of x is given for each of seven values of P: 0.50, 0.25, 0.10, 0.05, 0.025, 0.01, and 0.005. For each P, $\nu_1 = 2q = 1(1)$ 10, 12, 15, 20, 24, 30, 40, 60, 120, ∞; $\nu_2 = 2p = 1(1)$ 30, 40, 60, 120, ∞; x to 5S (in most parts of the table this is equivalent to 5D).

In a prefatory note (p. 151–153), E. S. Pearson points out some uses of the Table in statistical problems. In particular, if S_1, S_2 are two independent sums of squares, distributed as $\chi^2\sigma^2$ with ν_1 and ν_2 degrees of freedom respectively, the table gives the percentage points of

$$x = S_2/(S_1 + S_2).$$

The multiple correlation coefficient R^2 follows the beta-function distribution when the population is normal with zero correlation. The beta-function distribution also serves frequently as an approximation to the distribution of a random variable x which is constrained to lie between 0 and 1.

Partial sums of a binomial expansion are expressible in terms of an incomplete beta-function. From a result due to Laplace, *Théorie Analytique des Probabilités*, 2nd ed., Paris, 1814, p. 151, the sum of the first s terms of the binomial $(q' + p')^n$, where $q' + p' = 1$, is found to be

$$I_q'(n + 1 - s, s).$$

Thus the present tables, if read at $\nu_1 = 2s$, $\nu_2 = 2(n + 1 - s)$, provide the value of q' for which the sum has the given value P. This result may be used to construct fiducial limits for the probability p' of occurrence of an event which has been observed to happen $(s - 1)$ times in $(n - 1)$ trials.

The first table of this type was constructed by R. A. Fisher, *Statistical Methods for Research Workers*, Edinburgh, Oliver and Boyd, 1925. For $P = 0.05$, n_1, $n_2 = 1(1)6$, 8, 12, 24, ∞, Fisher's table gave the values of z to 4D, where $z = (1/2) \ln F$, $F = [p(1 - x)]/qx$. In the 2nd ed. (1928) of Fisher's work a table for $P = 0.01$ is also given; and in the 8th ed. (1941), a further table for $P = 0.001$, is credited to C. G. Colcord and L. S. Deming (1936). In Fisher's notation n_1, n_2 replace ν_1, ν_2 respectively. These tables were subsequently extended to $P = 0.20$ (H. W. Norton, Iowa State College Master's thesis, 1937). These tables, for $P = 0.20$, 0.05, 0.01, 0.001, $n_1 = 1(1)6$, 8, 12, 24, ∞; $n_2 = 1(1)30$, 40, 60, 120, ∞; z to 4D, appear in: R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research*, Edinburgh, Oliver and Boyd, 1938.

From Fisher's table, P. C. Mahalanobis, *Indian Jn. Agric. Sci.*, v. 2, 1932, p. 678–693, constructed tables of $z' = (1/2) \log F$; $y = F^{1/2}$. In these tables, $P = 0.05$, 0.01; $n_1 = 1(1)6$, 8, 12, 24, ∞; $n_2 = 1(1)30$, 60, ∞; z' to 4D; F to 3D (throughout most of the table); y to 3D (throughout most of the table). Where no interpolation is required, the table of F (in statistical terminology the *variance ratio*) is usually the most convenient for tests of significance associated with the analysis of variance. A further table of F, for $P = 0.20$, 0.05, 0.01, 0.001; $n_1 = 1(1)6$, 8, 12, 24, ∞; $n_2 = 1(1)30$, 40, 60, 120, ∞; F to 2D in most parts of the table, is given by Fisher and Yates, *loc. cit.* Miss Thompson's tables do not include the values $P = 0.20$, 0.001; in this respect Fisher and Yates's tables and the present tables supplement each other.

The article by Miss Thompson also contains a description of the methods of computation by L. J. Comrie and H. O. Hartley (p. 154–161), who advised on this phase of the work, and an account (p. 161–167) by H. O. Hartley of suitable methods of interpolation for users of the tables.

W. G. Cochran

100[H, J, K].—L. J. Comrie and H. O. Hartley, "Table of Lagrangian coefficients for harmonic interpolation in certain tables of percentage points," *Biometrika*, v. 32, 1941, p. 183–186. 19.3 × 27.3 cm.

In this table the Lagrange coefficients have been calculated by the standard formulas for interpolation by a polynomial of sixth degree. However, in determining Lagrange coefficients for harmonic interpolation, the coefficients are those for polynomials of the sixth degree in the reciprocal of the parameters used as the argument. The table of coefficients is so constructed that the reciprocals of the numbers in the following progression (or any submultiple progression) may be used as arguments of tabular values: 10, 12, 15, 20, 24, 30, 40, 60, 120, ∞. The Lagrange coefficients are given to five places of decimals and for the following values of the interpolate: 16(1)19 with 10, 12, 15, 20, 24, 30, 40 as the arguments of tabular values 21(1)23 with 12, 15, 20, 24, 30, 40, 60 as the arguments of tabular values, 25(1)29 with 15, 20, 24, 30, 40, 60, 120 as the arguments of tabular values 31(1)119 (excluding 40 and 60) with 20, 24, 30, 40, 60, 120, ∞ as the arguments of tabular values.

For smaller values of the interpolate (i.e., 11 to 19) ordinary Lagrange coefficients (to five places of decimals) are given for sixth degree polynomial interpolation since the parameter itself rather than its reciprocal is preferable as the argument in this range.

The table of Lagrange coefficients for harmonic interpolation are very useful for finding percentage points of the incomplete beta-function and Fisher's z distribution for new combinations of values of the arguments.

Illustrative examples are given, showing how to apply the tables to these two functions.

S. S. W.

101[J, L].—C. M. Thompson, "Table of percentage of the χ^2 distribution," *Biometrika*, v. 32, 1941, p. 187–191. 19.3 × 27.3 cm.

The probability distribution of χ^2 with ν degrees of freedom is

$$f(\chi^2) = \frac{(1/2)^{\nu/2}}{\Gamma(\frac{1}{2}\nu)}(\chi^2)^{\frac{1}{2}\nu-1}e^{-\frac{1}{2}\chi^2}$$

The percentage points of this distribution are the values of χ^2 which satisfy the equation

$$\int_{\chi^2}^{\infty} f(\chi^2)d(\chi^2) = P$$

for given values of P, ν. In the present table $P = 0.995, 0.99, 0.975, 0.95, 0.9, 0.75, 0.5, 0.25, 0.1, 0.05, 0.025, 0.01, 0.005$; $\nu = 1(1)30(10)100$; χ^2 to 6S. The editor states that for $\nu > 50$ the sixth figure may be in error by one or two units. Interpolation formulae are suggested both for the body of the table and for values of ν outside the range of the table.

Apart from the normal frequency distribution, that of χ^2 is perhaps the most frequently encountered in modern statistical theory. Under certain assumptions about the nature of the population, the distribution of an estimated variance s^2, derived from ν degrees of freedom, is related to that of χ^2 by the equation $\nu s^2 = \chi^2\sigma^2$, where σ^2 is the population variance of which s^2 is an estimate. The percentage points of the χ^2 distribution are accordingly used to calculate fiducial limits for σ^2 from a given estimate s^2. The χ^2 distribution serves also as an approximation to many distributions whose exact forms are not yet known or have not been tabulated. In this connection the most important instances are the uses of χ^2 in K. Pearson's test of "goodness of fit" of an observed to a theoretical frequency distribution and in the tests of homogeneity of a binomial distribution, of a Poisson distribution and of a contingency table.

The χ^2 distribution is closely related to the partial sum of a Poisson frequency distribution. The sum of the first s terms of the latter, with mean m, satisfies the relation

$$e^{-m}\left\{1 + m + \frac{m^2}{2!} + \cdots + \frac{m^{s-1}}{(s-1)!}\right\} = \int_{2m}^{\infty} f(\chi^2)d(\chi^2)$$

where the number of degrees of freedom in χ^2 is $\nu = 2s$. It follows that from the percentage points of the χ^2 distribution the value of m can be found for which the sum of a given number of terms of the Poisson series has a given value.

The first table of the percentage points of χ^2 was published by R. A. Fisher, *Statistical Methods for Research Workers*, Edinburgh, 1925. A recent edition of this table, in R. A. Fisher and F. Yates, *Statistical Tables for Agricultural, Biological and Medical Research*, Edinburgh, 1938, gives χ^2 to 3D (except for some small values of n, where 3S are given) for $P = 0.99, 0.98, 0.95, 0.90, 0.80, 0.70, 0.50, 0.30, 0.20, 0.10, 0.05, 0.02, 0.01, 0.001$; $n = 1(1)30$. Except for $P = 0.001$, the same table is to be found in the eighth edition of Fisher's *Statistical Methods*, 1941. Fisher uses n in place of ν. It should be noted that Fisher's table and Miss Thompson's table cover somewhat different values of P.

W. G. Cochran

102[K].—M. Merrington and C. M. Thompson, "Tables of percentage points of the inverted beta (F) distribution," *Biometrika*, v. 33, 1943, 16 p. 19.3 × 27.3 cm. Compare RMT **99**.

By making the transformation

$$x = \frac{1}{1+u} \tag{1}$$

on the incomplete beta distribution

$$f(x) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)}x^{p-1}(1-x)^{q-1}, \qquad (0 < x < 1) \tag{2}$$

we obtain the so-called inverted beta distribution

$$g(u) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} u^{q-1}(1+u)^{-p-q}, \qquad (0 < u < \infty). \tag{3}$$

If χ_1^2 and χ_2^2 are independently distributed according to χ^2-laws with ν_1 and ν_2 degrees of freedom respectively, the χ^2-law with ν degrees of freedom respectively being defined by

$$\frac{1}{2}\frac{(\frac{1}{2}\chi^2)^{(\nu/2)-1}}{\Gamma(\frac{1}{2}\nu)} e^{-(1/2)\chi^2} d(\chi^2), \tag{4}$$

then $\chi_1^2/\chi_2^2 = u$ is distributed according to $g(u)$ with $q = \frac{1}{2}\nu_1$, $p = \frac{1}{2}\nu_2$. But $\chi_1^2/\chi_2^2 = (\nu_1/\nu_2)F$, where F is the Snedecor ratio which is the most convenient form for the analysis of variance test criterion. Hence the distribution law of F, say $h(F)dF$, may be found by making the transformation $u = (\nu_1/\nu_2)F$ in (3) with $q = (1/2)\nu_1$, and $p = (1/2)\nu_2$. Let

$$I_F(\nu_1, \nu_2) = \int_F^\infty h(F)dF.$$

Values of F are tabulated to five significant figures, for $I_F(\nu_1, \nu_2) = 0.50, 0.25, 0.10, 0.05, 0.025, 0.01, 0.005$ and for $\nu_1 = 1(1)10, 12, 15, 24, 30, 40, 60, 120$, $\nu_2 = 1(1)30, 40, 60, 120, \infty$. Harmonic interpolation is therefore possible for both ν_1 and ν_2.

S. S. W.

103[Q, U].—J. Y. DREISONSTOK, *Navigation Tables for Mariners and Aviators*, sixth ed., 1942, iv, 109 p. 14.9 × 23.2 cm. (H. O. no. 208.) For sale by the Hydrographic Office and by the Superintendent of Documents, Washington, D. C. $1.20; foreign price, postage extra.

These tables were originally designed by Dreisonstok, a Lieut. Commander, U.S.N.; the sixth edition was modified and revised by E. B. COLLINS, a Senior Scientist, U. S. Hydrographic Office. They have been popular and widely used; for a time they were standard equipment in the British Royal Air Force. Except for a number of large errors, their accuracy is entirely adequate for surface navigation (altitudes are accurate to about four-tenths of a minute of arc when the tables are used without interpolation, one-tenth with) and more than adequate for aerial navigation. Compare MTE **11**.

To simplify the discussion of these and other navigation tables, the following paragraph on notation is presented. The astronomical triangle is the triangle on the infinite celestial sphere with vertices at the observer's zenith, Z, the celestial object under consideration, S, and the visible celestial pole, P. In the usual notation of navigators, $\angle SPZ = t =$ local hour angle; $\angle PZS = Z =$ azimuth; side $PZ = 90° - L$ where L is the observer's latitude; side $PS = 90° - d$ where d is the declination of the celestial object; side $ZS = 90° - h$ where h is the altitude of the celestial object.

The fundamental problem in celestial navigation as commonly practised today is this: Given t, d, and L; to find h and Z. The computed altitude, h, is compared with an observed altitude, properly corrected. Using the computed azimuth, Z, and the difference between the observed and computed altitudes, a "line of position" is drawn; this is used as the locus of possible positions of the observer.

Dreisonstok drops a perpendicular from Z on the great circle through P and S, meeting it at E. This perpendicular divides the astronomical triangle into two right triangles, the polar triangle containing P and the stellar triangle containing S.

Tables I and IA are double-entry tables in which are tabulated four quantities associated with the polar triangle, b, A, C, and Z', with arguments L and t, each given for integral degrees, t 0° to 360°. $b =$ side PE (degrees, minutes and tenths); $A = 10^5 \log \sec a$ (to nearest unit); $C = 10^3 \log \csc a$ (to nearest unit) where $a =$ side ZE; and $Z' = PZE$ (degrees and tenths). Table I contains values for latitudes (L) 0° to 65° North or South; Table IA is for latitudes 66° to 90°. It may be noted that Table IA was first computed by E. B. Collins for the Byrd Antarctic expedition in 1929; this table is included in the third and sixth editions, not in the others.

Table II is essentially a table of logarithms of cosecants and cotangents for every minute of arc, 0° to 180°.

$B = 10^5 \log \csc (b + d)$; $D = 10^3 \log \cot (b + d)$, each given to the nearest integer.

h and Z are obtained by the formulas:

$$10^5 \log \csc h = A + B, \qquad 10^3 \log \tan Z'' = C + D, \qquad Z = Z' + Z''.$$

Dreisonstok's method is known as an "assumed position" (or A.P.) method since the altitude and azimuth are computed for a position with integral values (in degrees) of t and L. This position may in the limit be as much as 42 nautical miles from the dead-reckoning position. In most cases in practise, this is satisfactory. In general, this method, like other A.P. methods, is not very satisfactory when the observed altitude is near 90°; the azimuth is poorly determined in these cases.

There are two ways in which the form of the tables could be improved. Tables I and IA might be united in a single table with the same form throughout. Secondly, all the values corresponding to a single latitude might be brought together on a single page; at present values corresponding to a single hour-angle are brought together on two pages which are widely separated in the volume. A reduction in page-turning would result from these changes since hour-angle varies more rapidly and more widely than does latitude. In a letter, the Chief Hydrographer has stated that the values in Table I and IA have been re-computed using seven-place logarithms. It is to be hoped that when the volume is again set up in type, the tabular values may be correct to the last place given and that the two suggested changes in form may be made.

The tables may be used for the determination of great circle course and distance and for star identification. See further RMT **106**.

Brown University C. H. Smiley

104[Q, U].—A. A. Ageton, *Dead Reckoning Altitude and Azimuth Table*, third ed., Washington, D. C., 1940, v, 49 p. 14.5 × 23.4 cm. (H.O. no. 211.) For sale by the Hydrographic Office and by the Superintendent of Documents, Washington, D. C. \$.90; foreign price, postage extra.

This is a table neither of solutions nor of partial solutions of the astronomical triangle; instead it is essentially a table of logarithms of secants and cosecants with which the altitude and azimuth of a celestial body may rapidly and accurately be computed from the local hour angle, declination and latitude. The chief advantages of the table lie in the elegant simplicity of the working rules devised by Ageton and in the fact that the computations are made for the dead reckoning position. Because of the latter fact, Ageton's method is known as a D.R.P. method and, for a celestial object near the zenith, it is better than an A.P. method.

In the notation of RMT **103**, Ageton drops a perpendicular from S on the great circle through P and Z, meeting it at X. This perpendicular divides the astronomical triangle into two right triangles, the zenith triangle and the polar triangle. In the polar triangle, the side PX is called $90° - K$ and the side SX is denoted by R. In the zenith triangle, $K \sim L$ is used to denote $|K - L|$.

$A(x)$ and $B(x)$ are defined as follows: $A(x) = 10^5 \log \csc x$; $B(x) = 10^5 \log \sec x$. These functions are given to the nearest integer for each half-minute of arc from 0° to 180°, except that when the value of a function is less than 240, one decimal is given. Because of the tabulation of the functions for every half-minute, interpolation is not required for ordinary navigation. By the introduction of the factor of 10^5, the complication of a decimal point is eliminated. The adoption of the quadrantal form for the table simplifies the matter of entering it for a function without making the volume bulky. The B's are printed in a heavy-face type, the A's in a light-face type, thereby reducing the possibility of error. As is true of all Hydrographic Office tables, the volume is well printed on good paper.

The basic formulas are:

$$A(t) + B(d) = A(R); \qquad A(d) - B(R) = A(K);$$
$$B(R) + B(K \sim L) = A(h); \qquad A(R) - B(h) = A(Z).$$

There are only three places in the computation where judgment is required, namely when K, h, and Z are being taken from the table. Ageton has prepared simple rules to cover these points and two of the rules are printed at the top of every double-page through the table.

It may be pointed out that Ageton's basic formulas and precepts will work equally well if $A(x)$ is defined as log sin x and $B(x)$ as log cos x. Thus an ordinary five-place logarithm table can be used when a copy of H. O. 211 is not available, or a seven-place table can be used when greater accuracy is desired.

L. J. Comrie has commented (*Hughes' Tables for Sea and Air Navigation*, London, 1938, p. xxxi) that Ageton's table "has been prepared from a six-figure table, with no attempt to ascertain whether a five in the sixth decimal should be rounded up or down; consequently the last printed figure is in error in one out of twenty entries, although admittedly these errors are of no real consequence."

As for indeterminacy, Ageton has printed on the page facing the title page, the warning that when the local hour angle is near 90°, an error of one or two (nautical) miles may occur in computing an altitude. In every method where the altitude is determined by a single trigonometric function, there will be a region of indeterminacy; the one occurring here is no more troublesome in practise than most of them.

This table may also be used for the computation of the initial course and distance along a great circle between two terrestrial points, for star identification, and for locating points on a great circle track. In astronomical work, it may be used to determine the orientation of the crescent moon with reference to the horizon, the distance between two celestial objects whose positions are known, and to transform one set of spherical coördinates into another, and for other similar problems. See further RMT **106**.

C. H. Smiley

105[Q, U].—*Tables of Computed Altitude and Azimuth, Latitudes,* 0° *to* 9°, *inclusive*, v. 1, Washington, D. C., 1941, 3, xi, 2–253 p. 22.6 × 29.1 cm. This is the first of eight uniform v. of H.O. 214, each v. devoted to 10° of latitude, v. 8 for 70°–79°. V. 2, 4, 5, 6, 7 were each published in 1940, and each contained 3, xi, 2–265 p.; v. 8, 1941, 3, xiii, 2–265 p. For sale by the Hydrographic Office and by the Superintendent of Documents, Washington, D. C. $2.25 per v.; foreign price, postage extra.

These tables present complete solutions of the astronomical triangle. Their principal advantages are the rapidity and simplicity of use and the accuracy of results; their disadvantages are their weight and bulk. Since each volume includes both cases, declination same name as latitude and contrary name, the eight cover all latitudes between 80°S and 80°N. Volume 8 has a special section of two pages devoted to polar navigation. The material for volume 9 has been prepared but, because of the limited use to which such a table for latitudes 80°–90° would be put, and because of the expense, it is not planned to publish it.

Volume 4 was the first one to be completed; it appeared in 1936. Copies of it were sent to many navigators and their criticisms and suggestions were invited. The other volumes were then prepared with the assistance of the Philadelphia group of the Works Progress Administration.

Within the section devoted to a particular degree of latitude, altitude (to 0′.1) and azimuth (to 0°.1) are tabulated for every degree of local hour angle, 0° to 180° or down to altitude 5°, and for selected values of the declination. The values of declination include every half-degree from 0° to 29° inclusive; these allow for the convenient determination of altitude and azimuth for the sun, moon and planets as well as stars of declination numerically less than 29°.5. Beyond 29°, the following 37 declinations are listed: 30°, 32°, 34°, 34°.5, 35°.5, 36°, 37°, 38°.5,40°, 42°, 42°.5, 43°, 45°, 46°, 47°, 48°.5, 49°.5, 50°.5, 51°.5, 52°.5, 54°, 54°.5, 55°, 56°, 56°.5, 57°, 57°.5, 59°, 59°.5, 60°, 60°.5, 62°, 62°.5, 63°, 69°, 69°.5, 74°.5. These values were chosen for the convenient reduction of observations of a selected list of stars, although the list is not given in the volumes.

For each latitude, a star identification table of two pages is given, tabulating declination and hour angle in integral degrees with arguments altitude 4°(4°)88°, and azimuth 0°(4°)180°.

In addition to altitude and azimuth for each entry, there are given the changes in altitude per minute of arc of declination, and per minute of arc of hour angle. With the aid of a multiplication table on the inside of the back cover and the page facing it, one can easily interpolate the altitude for the precise declination and hour angle and for an integral latitude. Another table is given for correcting the altitude for small changes in latitude; thus one may determine the altitude of a body for the dead reckoning position. There are no facilities for determining the azimuth for this position and it is recommended that one not attempt to use the tables for the dead reckoning position when the altitude is greater than 80°.

If one assumes that the tabular altitude and each of the three corrections are in error by 0′.1 and all of the errors of the same sign, the maximum error of an altitude for a dead reckoning position would be 0′.4. Generally one would expect the error to be 0′.2 or less.

In the Description of the Tables (p. iv), it is stated that "the altitudes have been computed to an accuracy of one tenth of a minute of arc by seven-place logarithms." [1] The reviewer found six errors, each of two tenths of a minute of arc, in a very brief check of a small part of volume 4; the amount of material checked was too small to serve as a basis for an estimate of the number and magnitude of the errors in the entire set of tables. In response to a query, the Chief Hydrographer has written that the tables were checked, but he gave no information as to the method or the thoroughness of the check. See further RMT **106**.

C. H. Smiley

[1] In computing the nine volumes of H. O. 214, the Hydrographic Office used 100 copies of Peters' *Siebenstellige Logarithmentafel der trigonometrischen Funktionen für jede Bogensekunde des Quadranten*, Leipzig, 1911. After the completion of the work copies of the tables were distributed to various Government departments.—Editor.

106[Q, U].—*Astronomical Navigation Tables, Latitudes* 65°–69°, *North and South, Volume P*, Washington, D. C., U. S. Government Printing Office, 1941, 233 p. 16.5 × 24.6 cm. This is the fourteenth and last v. of H.O.218, each one covering five degrees of latitude, v. A, 0°–4°, no volume lettered I or O. Not available for public distribution or sale.

These tables are "reproduced by photo-lithographic process for emergency use from British Air Publication 1618," of the same name. The reproduction was "undertaken to meet the emergency requirements of U. S. Naval Aviation and the U. S. Army Air Corps in cooperating with the British Air forces." The set is useful only between 70° South and 70° North latitude. Persons who have used these tables as well as other modern tables agree that they are the fastest tables available for navigational purposes at the present time. Altitudes obtained from them are said to be correct to the nearest minute of arc, an accuracy entirely adequate for aerial use for which they are intended, although not as close as many navigation tables now in common use.

In the front of each volume, about 89 pages are devoted to 22 bright navigation stars. Altitude and azimuth of each star are tabulated to the nearest minute and the nearest degree respectively with arguments latitude (the five degrees covered by the volume) and hour angle (0° to 180° or down to 10° altitude) given to integral degrees. For each entry, a value is given (called *t* but *not* the local hour angle) which serves as an argument in entering a table printed on the inside of the front cover, to correct tabulated altitudes for any year to 2000 A.D. inclusive. No correction is needed in most cases before 1944.

In the latter part of each volume are tables in which for an integral value of declination (in degrees, 0° to 28° inclusive) altitude and azimuth are tabulated to the nearest minute and the nearest degree respectively with arguments latitude (again for the five degrees covered by the volume) and local hour angle (0° to 180° or down to 10° altitude) given to integral degrees. For each entry in the tables, a value of d is given, the change in altitude in minutes of arc corresponding to a change of one degree in declination; a multiplication table on the inside of the back cover and on the page facing it is offered for purposes of interpolation.

This second half of each volume allows one to reduce observations of the Sun, Moon, planets and 13 additional bright stars whose declinations are numerically less than 28° and for which data are available in the Air Almanac. In both sections of the volume, data for north latitudes are printed on one pair of pages, for south latitudes on the following pair of pages. At the top of each page is a clear indication in heavy, black type of the hemisphere to which the data apply.

One of the features which makes for speed in the use of these tables is that the tabulated altitudes are already corrected for refraction under normal conditions at an elevation of 5000 feet above sea level; thus an interpolated altitude is directly comparable with an altitude measured by bubble octant from a plane flying at 5000 feet. A small table of additional corrections due to refraction is given for elevations other than 5000.

The bulk of the fourteen volumes, their weight and the fact that they are useful only between 70° North and 70° South are their chief disadvantages. The rapidity and simplicity of their use recommends them for the relatively crowded quarters of a plane under the perhaps unfavorable conditions of light, cold, low oxygen content of air, etc.

The excellent design of the tables indicates that experienced table-makers were consulted. This shows in the use of critical tables, the placing of the various tables, choice of type, and in other ways.

For purposes of comparison, the following table is offered:

	vol. (cc.)	weight (lbs.)	no. entries	no. nos. taken out	no. addits. and subtrs.	time reqd. (secs.)	cost	useful in latitudes	min. alt.
H. O. 208	317	0.8	4	8	4	75	$1.20	90°S to 90°N	−90°
H. O. 211	211	0.5	7	9	5	90	0.90	90°S to 90°N	−90°
H. O. 214	9752	15	3	4	1	50	18.00	80°S to 80°N	+5°
H. O. 218	11900	19	3	4	1	40	—	70°S to 70°N	+10°

No great emphasis is to be placed on the estimates of time required for the use of the tables in the standard way; they represent only the times required by a few experts working under the most favorable conditions. The number of entries is taken as the number of different pages which must be consulted and this is increased by one if a volume has to be chosen among several. The important fact is that none of the methods requires as much time as does the observation of a celestial body.

C. H. Smiley

The U. S. Hydrographic Office made a large contribution to the accuracy of the contents of H. O. no. 218, since it supplied the British, for comparison with their own computations, with all values for both altitude and azimuth to tenths, from the equator to the poles. —Editor.

107[C, D].—Coast and Geodetic Survey, *Table of S and T Values*, [Washington, D. C.], U. S. Govt. Printing Office, 1934, 45 p. 20 × 26.1 cm. Off-set printing of type script. Not available for public distribution.

Absolutely nothing but the tables are printed in this volume; $S = \log \sin x - \log x$, and $T = \log \tan x - \log x$. Here, as well as in the volumes of Peters, and others, the authors use column headings S and T, when $S + 10$, and $T + 10$ would be exact. This is a table of $S + 10$ and $T + 10$, 0°–4°30′, for every 10″, to 10D, with differences. Each page is devoted to 6′. These tabular values are based on the second as unit. There is no suggestion as to how this table was prepared but it may have been partly an abridgement of Andoyer's table to 14D, with differences, 0°–3°, p. 581–599 of

(1s). H. Andoyer, *Nouvelles Tables Trigonométriques Fondamentales* (*Logarithmes*), Paris, 1911.

A similar table to 8D, for each second, 0°–5°, 85°–90°, is also to be found in

(2s). J. Bauschinger and J. Peters, *Logarithmic-Trigonometrical Tables with Eight Decimal Places*, v. 2, Leipzig, 1911, p. 2–151. Another like table, to 7D, for each 10″, 0°–2°46′30″, is given in

(3s). G. Vega, *Logarithmic Tables of Numbers and Trigonometrical Functions, translated from the fortieth edition of Dr. Bremiker's thoroughly revised and enlarged edition* by W. L. Fischer, 72nd ed., Berlin, 1890, p. 2–185. In

(4d). J. T. Peters, *Hilfstafeln zur zehnstelligen Logarithmentafel*, Berlin, 1919, p. 25–26, are the values of $S + 10$, and $T + 10$, to 10D, for each .001°, 0°.000–2°.100. In

(5m). *Logarithmic and Trigonometric Tables*, ed. E. R. Hedrick, rev. ed., New York, Macmillan, 1935, p. 45, there are very brief tables for $S + 10$ and $T + 10$ (correctly marked) with the unit in minutes. There are other small tables of S and T, 0°–4.890°, with units in degrees, minutes, and radians, in

(6r). L. M. Milne-Thompson and L. J. Comrie, *Standard Four-Figure Mathematical Tables, Edition A*, London, MacMillan, 1931, p. 4–5.

Such are examples of four types of tables according as the units are seconds, degrees, minutes, or radians. For an angle of 2° the respective values of S are (1s). $\bar{6}.68557\ 48668\ 2354$; (4d). $\bar{2}.24178\ 91682$; (5m). $\bar{4}.46390$; (6r). $\bar{1}.9999$. Some further forms of S and T tables, in other units, may be mentioned.

(7c.s.). France, Service Géographique de l'Armée, *Tables des Logarithmes à huit Décimales des Nombres entiers et de* 1 *à* 120 000 *et des Sinus et Tangentes de dix Secondes d'Arc dans le Système de la Division Centésimale du Quadrant*. Paris, 1891. S and T are given for every centesimal minute up to 5^g, with centesimal seconds as units. $S = \bar{6}.19603\ 169$ (2° = approx. 22222″).

(8mi.). France, Service Geog. de l'Armée, *Tables de Logarithmes à cinq Décimales pour les Nombres de* 1 *à* 12 000 *et pour les Lignes Trigonométriques dans le Système de la Division de la Circonférence en* 64 000 *parties égales* (*dixième du millième de l'artillerie*), Paris, 1916. On p. [229] there are tables of S and T for each 4 mils from 0 to 600, mil as unit. $S = \bar{5}.99191$ (2° = approx. 356 mils).

(9mic.). J. de Mendizábel Tamborrel, *Tables des Logarithmes à huit Décimales des Nombres de* 1 *à* 125 000 *et des Fonctions Goniométriques sinus, tangente, cosinus et cotangente de centimiligone en centimiligone, et de microgone en microgone pour les* 25 000 *premiers microgones et avec sept décimales pour tous les autres microgones*, Paris, 1891, S and T, p. 1–59, for the first 12500 microgones, to 8D, unit microgone. $S = \bar{6}.79809\ 169$ (2° = approx. 5555 mic.).

(10ci.). J. C. F. Rey-Pailhade, "Table des logarithmes à quatres décimales de toutes les lignes trigonométriques dans la division décimale du cercle entier," Soc. Géog. de Toulouse, *Bull.*, v. 19, 1900. S and T table, p. 99, to 4D for $x = 0.0(0.1)2.5$ cirs. $S = \bar{2}.7981$ (2° = approx. .5555 cirs).

(11t.s.). N. Herz, *Siebenstellige Logarithmen der trigonometrischen Functionen für jede Zeitsecunde, zum astronomischen Gebrauche herausgegeben*, Leipzig, 1885. S and T are given for each 1^s, to 7D, from 0^h0^m to 0^h20^m, the unit being a second. $S = \bar{5}.861\ 5779$ ($2° = 8^m = 480^s$).

Tables of S and T, defined as the negative of the ordinary S and T, are found in

(12d.). C. Bremiker, *Logarithmisch-Trigonometrische Tafeln mit fünf Decimalstellen*, third stereotyped ed., edited by A. Kallius, Berlin, 1880, p. 180, where the table, to 6D, for every tenth of a degree, is from 0° to 4°, the degree as a unit. For 2°, S (Bremiker) = 1.758211.

S and *T* tables, with the sexagesimal second as unit, appeared already in the first stereotyped edition of

(13s). F. CALLET, *Tables Portatives de Logarithmes*, Paris, 1795. In his *Report . . . on Mathematical Tables*, London, 1873, J. W. L. Glaisher states (p. 54), "Tables of *S* and *T* are frequently called, after their inventor, Delambre's tables." In a letter of C. M. Merrifield written to Glaisher in 1873, listing matters he wishes to bring to his friend's attention, he notes "the so-called Delambre's tables of log $(\sin x/x)$ and log $(x/\tan x)$ really John Newton in 1658." I have examined Newton's *Trigonometria Britanica* (*sic*), of 1658, but as yet I have found no printed *S* or *T* tables before 1795. Delambre's dates are 1749–1822. We have already referred to the manuscript *S* and *T* tables of the *Tables du Cadastre* (*MTAC*, p. 34) possibly dating from 1792 or 1793.

S and *T* "are required for passing from log arc to log sin and log tan, and are of particular value in geodetic calculations, where long operations have sometimes to be performed with small arcs which are usually expressed in seconds, while four or five places of the second have to be retained" (3s).

R. C. A.

MATHEMATICAL TABLES—ERRATA

8. France, Service Géographique de l'Armée, *Tables des Logarithmes à huit Décimales des Nombres entiers de* 1 *a* 120000 *et des Sinus et Tangentes de dix Secondes en dix secondes d'Arc dans le Système de la Division Centésimale du Quadrant.* Paris, 1891. Compare *MTAC*, p. 36.

In the differences and proportional parts which correspond to Log cos $4^g75'$ to $5^g00'$,

	for					read			
	49	**48**	**47**	**46**		**51**	**52**	**53**	**54**
1	4.9	4.8	4.7	4.6	1	5.1	5.2	5.3	5.4
2	9.8	9.6	9.4	9.2	2	10.2	10.4	10.6	10.8
3	14.7	14.4	14.1	13.8	3	15.3	15.6	15.9	16.2
4	19.6	19.2	18.8	18.4	4	20.4	20.8	21.2	21.6
5	24.5	24.0	23.5	23.0	5	25.5	26.0	26.5	27.0
6	29.4	28.8	28.2	27.6	6	30.6	31.2	31.8	32.4
7	34.3	33.6	32.9	32.2	7	35.7	36.4	37.1	37.8
8	39.2	38.4	37.6	36.8	8	40.8	41.6	42.4	43.2
9	44.1	43.2	42.3	41.4	9	45.9	46.8	47.7	48.6

	for	read
Log Sin $4^g65'40''$	$\bar{2}.86355936$	$\bar{2}.86355935$
Log Tan 4 65 40	$\bar{2}.86472090$	$\bar{2}.86472089$
Log Cot 4 65 40	1.13527910	1.13527911
Log Cot 34 53 60	0.21981237	0.21981257
Log Cos 41 28 80	$\bar{1}.90143668$	$\bar{1}.90143666$

J. DE MENDIZÁBEL TAMBOREL, Sociedad Cientifica "Antonio Alzate," Mexico, *Revista*, v. 5, p. 9–10, 1891.

9. Authors of frequently used works in the field of Statistics display some carelessness in the preparation of tables they publish. Here are a few illustrations (an asterisk * denotes an exact result):

R. A. FISHER and F. YATES, *Statistical Tables for Biological, Agricultural and Medical Research*, Edinburgh, 1938. P. 33, $n_1 = n_2 = 2$, *for* 99.01, *read* 99.00*; and $n_1 = 2$, $n_2 = 3$, *for* 30.81, *read* 30.82. The same mistakes occur in

G. W. SNEDECOR, *Statistical Methods applied to Experiments in Agriculture and Biology*, Ames, Iowa, Collegiate Press, third ed., 1940, p. 184. On this same page (through $n_2 = 13$) are at least 53 other last figure errors of 1 to 3 units, which suggest that there may be 200 errors on the 4 pages of this table of 5% and 1% points for the *F* distribution. Five of these