19[L].—*Table of the Bessel Functions $K_0(x)$ and $K_1(x)$ for Small Arguments.* Ms. prepared by, and in possession of the MATHEMATICAL TABLES PROJECT, 50 Church St., New York City.

The B. A. A. S. published volume of tables of *Bessel Functions* contains a table of $K_0(x)$ and $K_1(x)$ for $x = 0.00(0.01)5$. Eight significant figures are given and second differences are tabulated for $x > 0.3$. For $x < 0.3$ the auxiliary functions $E_0(x)$, $E_1(x)$, $F_0(x)$ and $F_1(x)$ are also tabulated for purpose of interpolation; the last mentioned functions being defined from the formulae

$$K_0(x) = E_0(x) + F_0(x) \log x, \qquad K_1(x) = E_1(x)/x + F_1(x) \log x.$$

Because of the frequent use of these functions in various branches of research, the Project has subtabulated the functions above mentioned in the range from 0 to 1. The ranges and intervals for the various functions are as follows: For $K_0(x)$ and $K_1(x)$, $x = 0.0000(0.0001)$-$0.0300(0.001)1.000$. For $E_0(x)$, $E_1(x)$, $F_0(x)$ and $F_1(x)$, $x = 0.000(0.001)0.030$. All entries are given to seven significant figures with first and second advancing differences.

A. N. LOWAN

20.[L].—*Bessel Functions of the Second Kind for Complex Arguments.* Mss. prepared by, and in possession of, the MATHEMATICAL TABLES PROJECT, 50 Church St., New York City. Film copy at Brown University.

These are ten-place tables of Bessel functions of the second kind, $Y_0(z)$ and $Y_1(z)$, for complex arguments $z = re^{i\theta}$, where $r = 0.00(0.01)10$, $\theta = 0°(5°)90°$.

Bessel functions of orders 0 and 1 of the first and second kinds are encountered in the general solution of boundary value problems arising in the theory of potential, heat conduction, and wave motion, when the domain is a cylinder or a cylindrical annulus. They occur in particular in the problem of propagation of electromagnetic waves, the theory of the skin effect for poorly conducting wires, the oscillatory motion of a sphere in a viscous medium, the vibration of a heavy chain in a resisting medium, the determination of the frictional and gliding coefficients of a fluid in rotary vibrational motion and in many other boundary value problems. They have further theoretical importance in connection with the problem of conformal mapping and the evaluation of contour integrals involving Bessel functions.

No tables of Bessel functions of the second kind for complex arguments are to be found in the literature except for arguments along the 45° ray, which are known as ker and kei functions. These functions (or functions closely related to them) have been tabulated by H. G. SAVIDGE, *Phil. Mag.*, s. 6, v. 19, 1910 for $x = [1(1)30; 4S]$; by F. TÖLKE, *Besselsche und Hankelsche Zylinderfunktionen nullter bis dritter Ordnung vom Argument $r\sqrt{i}$*, Stuttgart, 1936, for $r = [0.00(0.01)21.00; 4S]$; by H. B. DWIGHT, *Tables of Integrals and Other Mathematical Data*, 1934, for $x = [0.0(0.1)10; 7D–9D]$; by N. W. MCLACHLAN and A. L. MEYERS, *Phil. Mag.*, s. 7, v. 18, 1934, in polar form, for $z = [0.0(0.1)10; 5D$ to $8D]$; by K. HAYASHI, *Fünfstellige Funktionentafeln*, Berlin, 1930, p. 111–118. Shorter tables are given by N. W. MCLACHLAN, *Bessel Functions for Engineers*, Oxford, 1934; by E. JAHNKE and F. EMDE, *Tables of Functions with Formulae and Curves*, New York, 1943; and by B.A.A.S., *Report*, 1916.

A. N. LOWAN

MECHANICAL AIDS TO COMPUTATION

8[Z].—S. L. BROWN and L. L. WHEELER, "Use of the mechanical multi-harmonigraph for graphing types of functions and for solution of pairs of non-linear simultaneous equations," *Review of Scientific Instruments*, v. 13, 1942, p. 493–495.

This note indicates the application of the Brown harmonic analyzer (described in *MTAC*, p. 128–129) to the drawing of "complex Lissajous figures" and their application. Such figures are curves whose parametric equations express x and y as trigonometric polynomials in θ, such as

$$x = \cos 13\theta + \cos 15\theta, \qquad y = -\sin 13\theta + \sin 15\theta,$$

which is clearly a rose of 28 petals. Nine drawings, some of them quite complicated, are reproduced in this article.

If $f(x, y)$ and $g(x, y)$ are polynomials, the machine can be used to solve the system

$$f(x, y) = h, \qquad g(x, y) = k,$$

for x and y by setting

(A) $$x = r\cos\theta, \qquad y = r\sin\theta$$

and thereby transforming f and g into a pair of functions of r and θ, which, for a fixed (trial) value of r, become trigonometric polynomials. If the corresponding complex Lissajous figure passes through the point (h, k) for a certain value of θ, this value, and the value selected for r, determine x and y by (A). If not, a new trial value of r is interpolated.

D. H. L.

9[Z].—S. L. Brown and L. L. Wheeler, "The use of a mechanical synthesizer to solve trigonometric and certain types of transcendental equations, and for the double summations involved in Patterson contours," *Jn. Applied Phys.*, v. 14, 1943, p. 30–36.

The methods and results of applying the Brown analyzer (described in MTAC, p. 128–129) to the problem of solving such equations as

$$x + \sin x = 3$$
$$\log(2x^3 + 5x^2 + 10x + 15) = 5/x$$
$$3\tan^2 x + 2\cos^2 x + \sin x + \csc x = 5$$

are discussed in detail in this paper. In each case a trigonometric polynomial is obtained by some device and the machine used to graph a curve from which the desired roots are read off. In the case of the first equation the left member is expanded in powers of x and then x is replaced by $2.5\cos\theta$. Powers of the cosine are then reduced to trigonometric polynomials.

Two beautiful graphs are given of Legendre's $P_n(\cos\theta)$ ($n = 1, \cdots, 11$), and certain associated functions $P_n^{(m)}(\cos\theta)$. Unfortunately no scales are indicated. Finally there is a discussion of the double Fourier series

$$A(x, y) = \sum_{k=0}^{n}\sum_{h=0}^{m} a_{hk}\cos 2\pi hx \cos 2\pi ky$$

used in the Patterson method for determining interatomic distances in crystals. Here the machine is set to draw a series of $n + 1$ curves

$$C_{kx} = \sum_{h=0}^{m} a_{hk}\cos 2\pi hx. \quad (k = 0, \cdots, n).$$

Then, reading 5 values of C_{kx} on each curve (for $x = 0(.02)1$), the machine is set to draw the 5 curves

$$A(x, y) = \sum_{k=0}^{n} C_{kx}\cos 2\pi ky.$$

These curves can then be used to prepare a contour map of the function $A(x, y)$.

D. H. L.

10[Z].—R. L. DIETZOLD, "The isograph a mechanical root finder," and R. O. MERCNER, "The mechanism of the isograph," *Bell Laboratories Record*, v. 16, 1937, p. 130–134 and p. 135–140.

These short articles describe the theory, practice and construction of a large instrument designed in 1928 by T. C. Fry primarily for the solution of algebraic equations of degrees ≤ 10. The theory behind the machine is briefly set forth in a review of the S. L. Brown harmonic analyzer, to which review the reader is referred (*MTAC*, p. 128–129). The mechanical methods of generating and adding the ten pairs of motions $a_n r^n \cos n\theta$, $a_n r^n \sin n\theta$ and the drawing of this sum are practically the same as those of the Brown analyzer. There is one difference however. In the latter machine the two motions of any one frequency may be set with any preassigned phase difference, while in the Isograph this difference is inexorably 90°. This means that the use of the Isograph is restricted to equations with real coefficients. An interesting feature of the Isograph is a set of ten cylindrical slide rules driven with frequencies from 1 to 10, for computing the values of $a_n r^n$ ($n = 1, \cdots, 10$) which are the ten amplitudes to be set on the cross heads. These amplitudes must not exceed an inch and a half (as compared with over 6 inches in the Brown analyzer) so that frequent changes in scale must be required for accommodating rather different values of r.

The photographs and diagrams illustrating these articles have been put on 12 slides, and a film has been made. Since these may be borrowed from the Bell Laboratories (New York), together with mimeographed material about the machine, there is the basis for an interesting lecture about the Isograph for mathematics classes and club meetings.

D. H. L.

NOTES

11. ALFRED LODGE.—In referring to sketches of two tablemakers in *Who was Who 1929–1940*, London, 1941 (N 1) a third one was overlooked, namely: of ALFRED LODGE (1854–1937), brother of Sir Oliver Lodge (1851–1940), long principal of the University of Birmingham, and of Sir Richard Lodge (1855–1936), professor of history at the University of Edinburgh. Alfred Lodge was Fereday fellow of St. John's College, Oxford, 1876–91, professor of pure mathematics at the Royal Indian Engineering College, Cooper's Hill, 1884–1904, and assistant master (mathematics) of Charterhouse from 1904 until his retirement in 1919, at the age of 65. He was president of the Mathematical Association, 1897–98, and scores of his communications appeared in the *Mathematical Gazette*. For some years he was secretary and recorder of Section A of the B.A.A.S. He was a member of its Committee on Calculation of Mathematical Tables for nearly fifty years; except for Airey, more of his tables were published than those of any other member (see *MTAC*, p. 75). He was a joint author of the Committee's *Factor Table* (1935), of which three independent copies were made; compare N **13**. Since Lodge alone had been on the Committee from the day on which Bessel functions were first discussed up to 1937, when the Committee published its first volume of tables of *Bessel Functions*, the volume was dedicated to him. It appeared, however, a few days after his death. See also *The Times*, London, 6 Dec. 1937, p. 14; and C. O. TUCKEY, "Alfred Lodge," *Math. Gazette*, v. 22, 1938, p. 3–4.

12. INTERPOLATION.—If there is suspicion of the accuracy of an entry in a table with equal argument-intervals, the suspicious entry u_0 may be checked in terms of the six adjacent entries u_{-3}, u_{-2}, u_{-1} and u_1, u_2, u_3, by the