UNPUBLISHED MATHEMATICAL TABLES

We have also referred to unpublished mathematical tables of (a) SILBERSTEIN, in the second article of this issue; of (b) PETERS, in RMT **125**, and in N **13**; and of (c) PETERS and COMRIE in RMT **127**.

15[A].—SOLOMON ACHILLOVICH JOFFE (1868–) *Table of the first 100 factorials, and the method of its computation.* Ms., prepared in January 1921, in possession of the author; in 1921 photographic copies were placed in the libraries of the Edinburgh Mathematical Society, the University of Edinburgh, and the Royal Society of Edinburgh, and in 1943, in the Library of Brown University. Compare UMT **10**.

A table of the first 50 factorials is contained in a paper by FRANK ROBBINS, "Factorials and allied products with their logarithms," R. So. Edinb., *Trans.*, v. 52, part 1, 1917, p. 167–174. This factorial table was extended by the writer shortly after it was brought to his attention. Beginning with $n = 51$, the multiplications (and additions for verification) were made on the Electric Ensign Machine. Each value $n!$ was obtained from $(n - 1)!$ through multiplication by n, the multiplicand being broken up into 9-figure groups. The resulting product was verified by the congruence $n! \equiv 0 \pmod{10^6 - 1}$. E.g. for 68! $2480 + 035542 + 436830 + 599600 + 990418 + 569171 + 581047 + 399201 + 355367 + 672371 + 710738 + 018221 + 445712 + 183296 = 5999994$.

Each quinary value $n!$, for $n = 5k$, was computed a second time from $(n - 5)!$ through multiplication by $n^{(5)}$, the latter values having been obtained by successive addition from the table of $0^{(5)}$, $5^{(5)}$, $10^{(5)}$, $15^{(5)}$, $20^{(5)}$ and $25^{(5)}$. In multiplying $(n - 5)!$ by $n^{(5)}$, the latter was taken as the multiplicand and $(n - 5)!$ was separated into 6-figure groups for multipliers. The final value 100! was also verified by the congruence $100! \equiv -1$ (mod. 101).

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16[A, E, L].—*Miscellaneous Tables.* Mss. prepared by, and in possession of, the MATHEMATICAL TABLES PROJECT, 50 Church St., New York City. None of the items listed below is intended for publication.

1. Square roots of the integers up to 100 to about 18S.
2. The functions $x^n/n!$ and $x^n/n(n!)$ for $x = [0.00(0.01)2;\ 15D]$, and $n = 1, 2 \cdots p$, where p depends on x and is such that the functions in question are of the order of magnitude of 10^{-13}.
3. Values of $x^n/n!$ for $x = [1(1)10;\ 10S]$, and $n = 1(1)40$.
4. The first fourteen derivatives of the function $(1/\sqrt{2\pi})e^{-\frac{1}{2}x^2}$, for $x = [0(0.1)8.4;\ 20D]$.
5. The first fourteen derivatives of the function $(2/\sqrt{\pi})e^{-x^2}$ for $x = [0.0(0.1)8.0;\ 20D]$.
6. The first ten derivatives of the sine and cosine integrals for $x = [10(1)100;\ 12D]$.
7. The first eighteen derivatives with respect to r of the functions $J_0(z)$ and $J_1(z)$, where $z = re^{i\theta}$, for $\theta = 0°(5°)90°$ and $r = 0.0(0.1)10$. The derivatives are given to a number of places (varying from 8 to 15) adequate for the subsequent subtabulation of the functions to about 12D.
8. The first eighteen derivatives of the functions $x^{-\nu}J_\nu(x)$ and $x^{-\nu}I_\nu(x)$ for $x = 0.0(0.1)10$ and $\nu = \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}$. These derivatives are given to a number of places (varying from 8 to 15) adequate for the subsequent subtabulation of the functions to about 12D.
9. The first fifteen derivatives of $J_\nu(x)$ and $I_\nu(x)$ for $x = 10.0(0.1)25$. These derivatives are given to a number of places (varying from 8 to 15) adequate for the subsequent subtabulation of the functions to about 12D or 12S.

10. The first eighteen derivatives with respect to r of the functions

$$S_0(z) = Y_0(z) - (2/\pi)J_0(z)(\ln z + \gamma),$$
$$S_1(z) = Y_1(z) - (2/\pi)J_1(z)(\ln z + \gamma) + 2/\pi z,$$

where $z = re^{i\theta}$, for $\theta = 0°(5°)90°$ and $r = 0.0(0.1)10$. These derivatives are given to a number of places (varying from 8 to 15) adequate for subsequent subtabulation of the functions to 12D.

11. Modulus and argument of $J_0(z)$ and $J_1(z)$, where $z = re^{i\theta}$, for $\theta = 0°(5°)90°$ and $r = [0.0(0.1)10; 10D$ or $10S]$.

A. N. LOWAN

17[B].—H. T. DAVIS, *Tables of powers*. Mss. in possession of the author. Compare Q 5.

I. Tables of reciprocals, values of $1/n$, for $n = [100(1)1000; 17D]$, together with differences. The following work giving results to 7D is well known: W. H. OAKES, *Table of the Reciprocals of Numbers from* 1 *to* 100,000 . . ., London, 1865; there is also the more recent work, *Barlow's Tables* . . . , fourth ed. by L. J. COMRIE, London, 1941 (RMT **82**) which gives the reciprocals of all integers up to 12500, to 7S, with differences; "Reciprocals of primes less than 10,000," in units of the tenth and eleventh decimal, are given with differences in B.A.A.S., *Mathematical Tables*, v. 5, *Factor Table*, London, 1935, p. 284–291. II. Tables of reciprocals of squares and cubes, values of $1/n^2$ and $1/n^3$, for $n = [100(1)1000;$ 16S–18S]. There are tables for $n = [50(1)200; 10D–13D]$ in K. PEARSON, *Tables for Statisticians and Biometricians*, part 2, London, 1931, p. 244–245. III. Tables of reciprocal fourth and fifth powers, values of $1/n^4$ and $1/n^5$, for $n = [100(1)1000; 18S]$. The values of $1/n^p$, to 32D, for $n = 2(1)100$, $p = 1(1)16$, are given in J. T. PETERS, *Zehnstellige Logarithmentafel*, v. 1, Berlin, 1922, *Anhang*, by Peters and Stein, p. 36–57. IV. Tables of the functions $x^m(1 - x^2)^n$ for $x = [0.00(0.01)1.00; 14D]$; $m = 0$, 1, and 2; $n = 1/2$, 3/2, 5/2, 7/2, and 9/2. These tables were used in computing the values of the integrals of the Legendre associated functions. In his *Tables of* $(1 - r^2)^{\frac{1}{2}}$ *and* $1 - r^2$ *for use in Partial Correlation and Trigonometry* (Baltimore, Md., 1922), J. R. MINER gives the values of $(1 - r^2)^{\frac{1}{2}}$ for

$$r = [0.0001(0.0001).9999; \text{mostly to 6D}].$$

H. T. D. and R. C. A.

18[L].—*Tables of the Associated Legendre Functions and their First Derivatives*, Mss. prepared by, and in possession of, the MATHEMATICAL TABLES PROJECT, 50 Church St., New York City.

These are six-place tables of the functions $P_n{}^m(x)$ and $Q_n{}^m(x)$ and their first derivatives for real and purely imaginary arguments, integral and half-integral values of n, and integral values of m. Details of the tabulation are as follows:

Tables of $P_n{}^m(\cos\theta)$ and $dP_n{}^m(\cos\theta)/d\theta$ for $\theta = 0°(1°)90°$; $n = 0(1)10$; $m = 0(1)4$; $m \leq n$.

Tables of $P_n{}^m(x)$, $dP_n{}^m(x)/dx$, $Q_n{}^m(x)$, $dQ_n{}^m(x)/dx$, for $x = 1.0(0.1)10$, $n = 0(1)10$; $m = 0(1)4$; $m \leq n$.

Tables of $i^{-n}P_n{}^m(ix)$, $i^{-n}dP_n{}^m(ix)/dx$, $Q_n{}^m(ix)$, and $dQ_n{}^m(ix)/dx$, for $x = 1.0(0.1)10$; $n = 0(1)10$; 10; $m = 0(1)4$; $m \leq n$.

Tables of $P^m_{n+\frac{1}{2}}(x)$, $dP^m_{n+\frac{1}{2}}(x)/dx$, $Q^m_{n+\frac{1}{2}}(x)$ and $dQ^m_{n+\frac{1}{2}}(x)/dx$, for $x = 1.0(0.1)10$; $n = 0(1)4$; $m = 0(1)4$ (even where m is greater than n).

Comparison of the contents of these manuscripts with the tables referred to in H. T. Davis's review of Mursi's table of $P_n{}^m(x)$ (RMT **118**) reveals the following facts: (a) Tallqvist's tables of $P_n{}^m(\cos\theta)$ cover a greater range in n and give more decimals than ours. (b) The function $dP_n{}^m(\cos\theta)/d\theta$ is not given by Tallqvist; (c) Mursi's tables of $P_n{}^m(x)$ for x less than unity, are completely complementary to our tables for real x. (d) There is some slight overlapping between our values of $dP_n{}^m(\cos\theta)/d\theta$ and those in the Tallqvist manuscript.

A. N. LOWAN

19[L].—*Table of the Bessel Functions $K_0(x)$ and $K_1(x)$ for Small Arguments.* Ms. prepared by, and in possession of the MATHEMATICAL TABLES PROJECT, 50 Church St., New York City.

The B. A. A. S. published volume of tables of *Bessel Functions* contains a table of $K_0(x)$ and $K_1(x)$ for $x = 0.00(0.01)5$. Eight significant figures are given and second differences are tabulated for $x > 0.3$. For $x < 0.3$ the auxiliary functions $E_0(x)$, $E_1(x)$, $F_0(x)$ and $F_1(x)$ are also tabulated for purpose of interpolation; the last mentioned functions being defined from the formulae

$$K_0(x) = E_0(x) + F_0(x) \log x, \qquad K_1(x) = E_1(x)/x + F_1(x) \log x.$$

Because of the frequent use of these functions in various branches of research, the Project has subtabulated the functions above mentioned in the range from 0 to 1. The ranges and intervals for the various functions are as follows: For $K_0(x)$ and $K_1(x)$, $x = 0.0000(0.0001)$-$0.0300(0.001)1.000$. For $E_0(x)$, $E_1(x)$, $F_0(x)$ and $F_1(x)$, $x = 0.000(0.001)0.030$. All entries are given to seven significant figures with first and second advancing differences.

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20.[L].—*Bessel Functions of the Second Kind for Complex Arguments.* Mss. prepared by, and in possession of, the MATHEMATICAL TABLES PROJECT, 50 Church St., New York City. Film copy at Brown University.

These are ten-place tables of Bessel functions of the second kind, $Y_0(z)$ and $Y_1(z)$, for complex arguments $z = re^{i\theta}$, where $r = 0.00(0.01)10$, $\theta = 0°(5°)90°$.

Bessel functions of orders 0 and 1 of the first and second kinds are encountered in the general solution of boundary value problems arising in the theory of potential, heat conduction, and wave motion, when the domain is a cylinder or a cylindrical annulus. They occur in particular in the problem of propagation of electromagnetic waves, the theory of the skin effect for poorly conducting wires, the oscillatory motion of a sphere in a viscous medium, the vibration of a heavy chain in a resisting medium, the determination of the frictional and gliding coefficients of a fluid in rotary vibrational motion and in many other boundary value problems. They have further theoretical importance in connection with the problem of conformal mapping and the evaluation of contour integrals involving Bessel functions.

No tables of Bessel functions of the second kind for complex arguments are to be found in the literature except for arguments along the 45° ray, which are known as ker and kei functions. These functions (or functions closely related to them) have been tabulated by H. G. SAVIDGE, *Phil. Mag.*, s. 6, v. 19, 1910 for $x = [1(1)30; 4S]$; by F. TÖLKE, *Besselsche und Hankelsche Zylinderfunktionen nullter bis dritter Ordnung vom Argument $r\sqrt{i}$*, Stuttgart, 1936, for $r = [0.00(0.01)21.00; 4S]$; by H. B. DWIGHT, *Tables of Integrals and Other Mathematical Data*, 1934, for $x = [0.0(0.1)10; 7D$–$9D]$; by N. W. MCLACHLAN and A. L. MEYERS, *Phil. Mag.*, s. 7, v. 18, 1934, in polar form, for $z = [0.0(0.1)10; 5D$ to $8D]$; by K. HAYASHI, *Fünfstellige Funktionentafeln*, Berlin, 1930, p. 111–118. Shorter tables are given by N. W. MCLACHLAN, *Bessel Functions for Engineers*, Oxford, 1934; by E. JAHNKE and F. EMDE, *Tables of Functions with Formulae and Curves*, New York, 1943; and by B.A.A.S., *Report*, 1916.

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MECHANICAL AIDS TO COMPUTATION

8[Z].—S. L. BROWN and L. L. WHEELER, "Use of the mechanical multi-harmonigraph for graphing types of functions and for solution of pairs of non-linear simultaneous equations," *Review of Scientific Instruments*, v. 13, 1942, p. 493–495.