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## Zeros of Certain Bessel Functions of Fractional Order

The following tables contain the zeros of  $J_\nu(x)$  for  $x \leq 25$ , where  $\nu = \pm 1/3, \pm 2/3, \pm 1/4, \pm 3/4$ . These zeros were obtained by inverse interpolation in a thirteen-place manuscript of these functions, computed by the NYMTP. The accuracy of the zeros to 10D is guaranteed, and the two additional places have a high probability of being correct.

$s$	$j_{+1/4,s}$	$j_{-1/4,s}$
1	2.78088 77239 95	2.00629 96717 90
2	5.90614 26988 43	5.12306 27427 46
3	9.04238 36635 83	8.25795 11756 42
4	12.18134 15289 55	11.39646 76969 87
5	15.32136 98260 12	14.53629 98843 38
6	18.46192 72456 89	17.67675 35868 47
7	21.60278 44489 13	20.81754 94222 32
8	24.74382 77961 28 (24.740)	23.95855 34952 86 (23.955)

$s$	$j_{+3/4,s}$	$j_{-3/4,s}$
1	3.49100 83741 08	1.05850 82594 04
2	6.65263 55231 22	4.28405 38127 24
3	9.80161 23591 40	7.44045 44040 05
4	12.94703 48891 39	10.58817 91486 60
5	16.09096 95281 99	13.73311 84505 74
6	19.23414 17604 82	16.87681 75138 75
7	22.37687 15748 16 (22.384)	20.01985 75839 86
8		23.16250 59340 75 (23.169)

  

$s$	$j_{+1/2,s}$	$j_{-1/2,s}$
1	2.90258 62484 17	1.86635 08588 74
2	6.03274 70572 66	4.98785 32314 35
3	9.17050 66694 64	8.12426 53819 40
4	12.31019 37716 45	11.26351 48254 28
5	15.45064 89678 17	14.40377 58801 36
6	18.59148 63361 81	17.54451 06557 21
7	21.73254 11617 47	20.68550 48061 24
8	24.87373 14228 06 (24.871)	23.82665 62470 57 (23.824)

  

$s$	$j_{+2/3,s}$	$j_{-2/3,s}$
1	3.37561 06526 94	1.24304 62596 19
2	6.53025 59365 13	4.42912 06776 99
3	9.67658 06352 38	7.57945 84465 30
4	12.82060 86784 66	10.72474 69244 99
5	15.96368 38809 06	13.86837 45833 31
6	19.10627 35045 92	17.01125 45001 33
7	22.24858 23933 60 (22.253)	20.15373 45371 51
8		23.29597 58670 60 (23.300)

For  $x > 25$ , ten or more decimal places in the zeros may be obtained from the well-known formula (five terms) for the roots of Bessel functions, given below; see, for example, G. N. WATSON, *Treatise on the Theory of Bessel Functions*, 1922 and 1944, p. 506; the sixth term was supplied by W. G. BICKLEY,<sup>1</sup> *Phil. Mag.*, s. 7, v. 34, 1943, p. 40:

$$j_{\nu,s} = \beta - \frac{\mu-1}{2^3\beta} - \frac{(\mu-1)(7\mu-31)}{3 \cdot 2^7 \cdot \beta^3} - \frac{(\mu-1)(83\mu^2-982\mu+3779)}{15 \cdot 2^{10} \cdot \beta^5} - \frac{(\mu-1)(6949\mu^3-15385\mu^2+1585743\mu-6277237)}{105 \cdot 2^{15} \cdot \beta^7} - \frac{(\mu-1)(70197\mu^4-24\,79316\mu^3+480\,10494\mu^2-5120\,62548\mu+20921\,63573)}{315 \cdot 2^{18} \cdot \beta^9} - \dots,$$

where  $\beta = (s + \frac{1}{2}\nu - \frac{1}{4})\pi$ ;  $\mu = 4\nu^2$ . The first five terms of the above expression will yield at least 10 decimals for roots greater than those given here. The values of  $\beta$  corresponding to the highest roots given here are noted in parenthesis; it is apparent that for  $x$  close to 25,  $\beta$  approximates the root to two decimals at least.

The author desires to express his appreciation of assistance rendered by several members of the NYMTP in checking these values.

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<sup>1</sup> It may be noted that in Bickley's article the fifth term of the formula has an erroneous number 6277327, for 6277237.