28. Henry Goodwyn. *A Tabular Series* . . . (see *MTAC*, p. 22).—It is perhaps not generally realized that only a part of this table was ever published (one fifth). The complete title is: *A/Tabular Series/of/Decimal Quotients/for all the/Proper Vulgar Fractions/of which/when in their lowest terms/Neither the Numerator nor the Demoninator/is greater than*/1000/London/Printed by W. Marchant, Ingram Court, Fenchurch Street/Published by J. M. Richardson, 23 Cornhill/opposite the Royal Exchange/1823. 15×23.5 cm. 5 p. (introduction) + 107 p. (tables) + 5 unpaged leaves.

In his preface Goodwyn writes that he is now publishing Part I of the complete tabular series, consisting of all decimals beginning with .0. Part II is to contain all beginning with .1, and so on to part V containing all decimals beginning with .4. He explains that it is unnecessary to print the remaining decimals from .5 to 1. since they can readily be found as the arithmetical complements of those already tabulated (because in effect $n/d = 1-(d-n)/d$). Thus . . . "the number of entire quotients will amount altogether to 152096, and there being an equal number of complementary (or easily calculated but not printed ones) the total will exhibit the vulgar fractions corresponding to 304192 decimal quotients." He goes on to say that he is now printing Part I, containing all decimals beginning with 0.0, of the *Tabular Series* derived from the values given in his *Table of Circles*, . . . "but whether that table [i.e. of circles] shall be printed, or the present work completed, will depend altogether on the reception which this result of the Computer's labour may meet with from the public."

Then follow the tables from $1/1000 = .001$ to $99/991 = .09989909$, after which is printed "END OF PART I." Then follow 5 unpaged leaves with 2^n, $n = 0(1)19$; 5^n, $n = 0(1)8$; 2^n5^m, $n = 0(1)6$, $m = 0(1)6$.

It appears therefore that Part I of the *Tabular Series* was first printed, and then (also in 1823) the *Table of Circles*. It is to be presumed that, like most computers of tables, he found that sales did not cover the cost of printing, and no more of the *Tabular Series* was ever printed.

In Glaisher's articles on Goodwyn (see *MTAC*, p. 23) the incomplete state is noted. I imagine that Glaisher had a copy, but it is not in the "Glaisher Collection" in the Cambridge University Library, possibly because the Library already possesses two copies. As Goodwyn's daughter presented her father's 'complete works' in 1831 (see *MTAC*, p. 100) and no subsequent parts accompanied the gift, it may safely be assumed that they do not exist.

C. R. Cosens

QUERIES

11. Cube Roots.—We have recently needed a table of cube roots of numbers $N = .1(.0001).2$, to 6 or 7D. Is such a table available in print?

R. F. Boyer

Dow Chemical Co.,
Midland, Michigan

12. Items in Interpolation History.—Who originated (a) The use of the 'throw-back' to take account of higher differences neglected in interpolation, and (b) The use of interpolation formulae involving only even central differences, not advancing differences? Both these methods are generally supposed to have come into use much more recently than 1826, but they are illustrated by Legendre in volume 2 of his *Traité des Fonctions Elliptiques*, Paris, 1826, paragraphs 672, 673, 676, as the following extracts show:

672, p. 28. "Je remarquerai que lorsque les différences quatrièmes $\delta^4 A$ sont assez grandes pour que les différences suivantes $\delta^5 A$ aient quelqu'influence dans les interpolations, il conviendra de prendre $\delta^4 A - \frac{7}{10}\delta^5 A$ au lieu de $\delta^4 A$." He states that, x being the fraction of tabular interval required in the interpolate (he is here using forward differences) the relevant terms are $\frac{x(x-1)(x-2)(x-3)}{1.2.3.4}\left(\delta^4 A + \frac{x-4}{5}\delta^5 A\right)$; then—"comme $\delta^5 A$ est censé très petit par rapport à $\delta^4 A$, si l'on donne à x une valeur moyenne $\frac{1}{2}$, le terme $\frac{x-4}{5}\delta^5 A$ se reduira à $-\frac{7}{10}\delta^5 A$."

673, p. 29. Similarly a sixth difference can be "thrown back," as we should now say, to the preceding fifth, by taking $\delta^5 A - \frac{3}{4}\delta^6 A$ in place of $\delta^5 A$, $-\frac{3}{4}$ being, as before, the mean value of $\frac{x-5}{6}$ obtained for $x = \frac{1}{2}$.

676, p. 31. "Pour avoir le milieu entre deux termes consécutifs A, AI d'une suite dont les différences deviennent progressivement plus petites qu'une quantité donnée, il est bon d'avoir recours aux termes qui précèdent et qui suivent les deux termes proposés."

Legendre is interpolating into the middle of an interval, and in modern notation the formula he gives is the following:

$$f_{\frac{1}{2}} = \frac{f_0 + f_1}{2} - \frac{1}{2}\,\frac{(\Delta_0^2 + \Delta_1^2)}{8} + \frac{1.3}{2.4}\,\frac{\Delta_0^4 + \Delta_1^4}{32} - \frac{1.3.5}{2.4.6}\,\frac{\Delta_0^6 + \Delta_1^6}{128} + \cdots$$

precisely the Everett formula for interpolation at 0.5 tabular interval.

C. R. COSENS

QUERIES—REPLIES

12. LOG LOG TABLES (Q 4, p. 131; QR 9, p. 336).—In answer to this question it would seem to be in order to draw attention to Slide Rules with log log scales, which permit ready solution of complicated problems in raising of powers, and extracting a root besides obtaining values of natural logarithms, and hyperbolic functions. For example, to evaluate

$2.31^{5.67}$; $(.371)^{1/15.8}$; $2.87^x = 73.7$; $x^{.378} = .582$; $e^x = .748$; ln 31, $e^{\pm 3}$

and various hyperbolic functions of 3.

The Keuffel & Esser Co., Hoboken, N. J., has manufactured (1) The log log duplex trig slide rule with trigonometric scales to represent degrees and minutes; (2) The log log duplex decitrig slide rule, with trigonometric scales to represent degrees and decimals of a degree; (3) The log log duplex vector rule. In Jan. 1943 these rules, 10 in. size, were respectively listed at \$12.50, \$12.50, \$13.50; and the 20 in. size at \$27.00, \$27.00, and \$31.50. Reproductions of the scales of these rules are given in *K & E, Slide Rules and Calculating Instruments*, New York, 1941, p. 313f–313i; and also in figs. 5–7 of the last of the following works:

C. N. PICKWORTH, *The Slide Rule: A Practical Manual*, fourteenth ed.,