edition, improved, Edinburgh, 1878. A set of stereotype plates for this work was given to the R.S.E. with Sang's manuscripts. See J. HENDERSON, *Bibliotheca Tabularum Mathematicarum*, 1926, p. 135f; E. SANG, "Account of the new table of logarithms to 200 000," R. So. Edinburgh, *Trans.*, v. 26, 1872, p. 521–528. Also *Specimen Pages of a table of the Logarithm of all Numbers up to One Million . . . shortened to nine figures from original calculations to fifteen places of Decimals*, [1874?], 26 p. 4to, not in the List of his Writings. There is a copy in the Brown University Library. The first page is a copy of a memorial of the president and council of the R.S.E. dated 19 June 1874, to the Chancellor of Her Majesty's Exchequer, and the last 16 pages are devoted to communications from academies and scholars, dated 1872–73, commending the idea of publishing a 9-place table of logarithms of numbers to $10^6$, in three volumes; but the publication was never achieved.

Sang "received a grant of £100 per annum from [the] Government as a recognition of his valuable scientific work; and the associated Scottish life assurance offices, feeling that some substantial recompense was due to him for his logarithms and actuarial tables, at a meeting in 1878 resolved to recommend to the offices the payment of an annuity of £100 . . . for the remainder of his life, which was agreed to, and subscribed by the offices."

R. C. A.

## MECHANICAL AIDS TO COMPUTATION

**12[Z].**—L. J. COMRIE, "Mechanical computing," Appendix I, p. 462–473 of DAVID CLARK, *Plane and Geodetic Surveying*, v. 2, third ed. revised by JAMES GLENDENNING, London, Constable, 1943. 14 × 21.7 cm. Also as a pamphlet reprint.

Some computations possible on several calculating machines are described and there are illustrations of four of them, namely: (1) Brunsviga, (2) 10-key Facit, (3) Marchant Electric, (4) Twin Marchant. Two pages are devoted to a listing and discussion of published tables, and some other literature.

**13[Z].**—L. J. COMRIE, "Recent progress in scientific computing," *J. Sci. Instruments*, v. 21, Aug. 1944, p. 129–135, illustrated.

The substance of this paper was delivered 3 July 1943 at a joint meeting of the London and Home Counties' Branch of the Institute of Physics, and the London Mathematical Society. The headings in the paper are as follows: "Differential analysers," "The training of computers," "Numerical integration by hand," "Finite differences," "Direct and inverse interpolation," "Double-entry interpolation," "Punched-card machines," "Solution of simultaneous equations," "Mathematical tables," "Short bibliography."

## NOTES

**26.** AGETON'S METHOD.—Navigators and other persons who use Hydrographic Office, *Publication*, no. 211 (see *MTAC*, p. 80–81) regularly, will be interested in two notes recently published on Ageton's method of celestial navigation. The first is "The accuracy of Ageton's method in celestial navigation" by SAMUEL HERRICK, Astron. So. Pacific, *Publications*, v. 56, 1944, p. 149–155. Herrick points out that the warning appearing in H.O. 211 against the use of sights for which the value of $K$ is found to lie between the limits of 87° 30′ and 92° 30′ is not adequate. He shows that, when the tables are used in the conventional fashion and $K$ lies outside the forbidden range, the maximum error in the computed altitude, $h$, is six minutes of arc. He shows also that an error of a minute of arc in $h$ may occur even when $K$ is quite small. He points out that the "caution" mentioned above is unfortunately omitted from Ageton's *Manual of Celestial Navigation*, New York, Van Nostrand, 1942, T. II, and from three texts which reproduce the tables. Herrick concludes that if a maximum error of two minutes of arc in $h$ is acceptable, Ageton's method can be used without interpolation in the tables so long as $K$ lies outside the range, 82° to 98°.

The second note is "Increased accuracy with Ageton's method" by C. H. SMILEY, *Pop. Astr.*, v. 52, 1944, p. 379–383. Smiley points out that Ageton's method could be used in many of the problems of practical astronomy if its accuracy were increased slightly. He notes that seven-place logarithms of sines and cosines can be used in place of Ageton's $A$'s and $B$'s, otherwise keeping the computing form and rules as given by Ageton. He points out that the indeterminacy in the computed altitude, $h$, resulting from the value of $K$ being near 90° can often be avoided by interchanging the values of the declination and the latitude. There is a brief discussion of the probability that $K$ will lie near 90°.

C. H. SMILEY

Brown University

**27.** EARLIEST TABLES OF $S$ AND $T$.—In N22 A. Fletcher refers to early tables of the auxiliary functions $S$ and $T$ for arguments 0°.01(0°.01)2°.99 given by John Newton in 1658 in his *Trigonometria Britanica*, and quotes a suggestion that these may be the first published tables. Tables embodying an equivalent idea, although only for arguments 0(0°.01)0°.5, were, however, given in the previous year by William Oughtred in the second part of his *Trigonometria*,[1] 1657, which has a separate title-page *Canones Sinuum, Tangentium, Secantium: et Logarithmorum pro Sinubus et Tangentibus*, London, 1657. On p. 235 Oughtred gives quantities which he labels "$S$" and "$A$" (being respectively negative and positive); these quantities, given in units of the seventh decimal, are respectively $S - S_0$ and $T - T_0$, in which $S_0$ and $T_0$ are the limiting values of $S$ and $T$ for zero argument. The end-figure is not completely reliable (although never more than a unit in error), so that it is not easy to decide whether (i) $S$ and $S_0$, $T$ and $T_0$ were individually rounded-off before subtraction in pairs, or whether (ii) the subtractions were first performed with more decimals and the results rounded-off. The former seems more likely since, if we assume this to have been done, Oughtred has 16 errors of $+1$ (i.e. errors in which his "$S$" is numerically too small and his "$T$" too large) and 7 of $-1$; with process (ii), he would have 5 errors of $+1$ and 23 of $-1$.

Process (i) seems to imply that Oughtred regarded $S$ and $T$ as the primary functions and performed the subtraction simply as a device for giving small numbers to tabulate and interpolate; it is the correct process if the quantities are always to be used in conjunction with the same $S_0$ and $T_0$ (i.e. for a specified unit) since the resulting value of $S$ or $T$ is then subject to a slightly smaller total rounding-off error. It seems improbable that Oughtred envisaged any other unit than that of 0°.01 which he used, but it is worth while to note that his "$S$" and "$A$," if rounded-off by process (ii) above, would be independent of the unit used, except insofar as this determines the tabular arguments.

J. C. P. MILLER

[1] The writer has seen three copies of this work, not all in agreement in all details. One of these belonged to Isaac Newton, whose library was recently purchased by the Pilgrim Trust and presented to the Library of Trinity College, Cambridge.

**28.** Henry Goodwyn. *A Tabular Series* . . . (see *MTAC*, p. 22).—It is perhaps not generally realized that only a part of this table was ever published (one fifth). The complete title is: *A/Tabular Series/of/Decimal Quotients/for all the/Proper Vulgar Fractions/of which/when in their lowest terms/Neither the Numerator nor the Demoninator/is greater than*/1000/London/Printed by W. Marchant, Ingram Court, Fenchurch Street/Published by J. M. Richardson, 23 Cornhill/opposite the Royal Exchange/1823. 15 × 23.5 cm. 5 p. (introduction) + 107 p. (tables) + 5 unpaged leaves.

In his preface Goodwyn writes that he is now publishing Part I of the complete tabular series, consisting of all decimals beginning with .0. Part II is to contain all beginning with .1, and so on to part V containing all decimals beginning with .4. He explains that it is unnecessary to print the remaining decimals from .5 to 1. since they can readily be found as the arithmetical complements of those already tabulated (because in effect $n/d=1-(d-n)/d$). Thus . . . "the number of entire quotients will amount altogether to 152096, and there being an equal number of complementary (or easily calculated but not printed ones) the total will exhibit the vulgar fractions corresponding to 304192 decimal quotients." He goes on to say that he is now printing Part I, containing all decimals beginning with 0.0, of the *Tabular Series* derived from the values given in his *Table of Circles*, . . . "but whether that table [i.e. of circles] shall be printed, or the present work completed, will depend altogether on the reception which this result of the Computer's labour may meet with from the public."

Then follow the tables from 1/1000 = .001 to 99/991 = .09989909, after which is printed "END OF PART I." Then follow 5 unpaged leaves with $2^n$, $n = 0(1)19$; $5^n$, $n = 0(1)8$; $2^n5^m$, $n = 0(1)6$, $m = 0(1)6$.

It appears therefore that Part I of the *Tabular Series* was first printed, and then (also in 1823) the *Table of Circles*. It is to be presumed that, like most computers of tables, he found that sales did not cover the cost of printing, and no more of the *Tabular Series* was ever printed.

In Glaisher's articles on Goodwyn (see *MTAC*, p. 23) the incomplete state is noted. I imagine that Glaisher had a copy, but it is not in the "Glaisher Collection" in the Cambridge University Library, possibly because the Library already possesses two copies. As Goodwyn's daughter presented her father's 'complete works' in 1831 (see *MTAC*, p. 100) and no subsequent parts accompanied the gift, it may safely be assumed that they do not exist.

C. R. Cosens

## QUERIES

**11.** Cube Roots.—We have recently needed a table of cube roots of numbers $N = .1(.0001).2$, to 6 or 7D. Is such a table available in print?

R. F. Boyer

Dow Chemical Co.,
Midland, Michigan

**12.** Items in Interpolation History.—Who originated (a) The use of the 'throw-back' to take account of higher differences neglected in interpolation, and (b) The use of interpolation formulae involving only even central differences, not advancing differences? Both these methods are generally