	$S(2x/\pi)^{\frac{1}{2}}$			$C(2x/\pi)^{\frac{1}{2}}$	
x	*for*	*read*	x	*for*	*read*
0.2	.023788	(Watson .023720 correct)	11	.380390	.380392
1.5	.415348	.415483	11.5	.395149	.395152
6.5	.347099	.347100	15.0	.569335	.569336
8.0	.512010	.512009	15.5	.524009	.524010
11.0	.504784	.504786	17.5	.406589	.406590
11.5	.447809	.447810	20.5	.587849	.587848
12.0	.405810	.405811	21.0	.573842	.573841
13.5	.432489	.432488	21.5	.542266	.542265
15.5	.598183	.598184	25.5	.526896	.526897
20.5	.504875	.504874	34.0	.537026	.537027
21.0	.545885	.545884	34.5	.504881	.504882
24.0	.467029	.467028	38.5	.545560	.545559
28.5	.573060	.573059	47.5	.479313	.479311
34.0	.557490	.557489	48.5	.443930	.443929
36.5	.476871	.476872			
39.5	.513690	.513689			
40.5	.558799	.558800			
44.5	.447720	.447721			

The 14 errors for $x < 20$ were noted by comparison with the tables of J. R. Airey, in B.A.A.S., *Report*, 1926, p. 274–275.

J. W. Wrench, Jr.

4211 Second St., N. W.
Washington 11, D. C.

59. NYMTP, *Tables of Circular and Hyperbolic Sines and Cosines*, 1939. See RMT **89**, p. 45f.

The integral part in the value of sinh x corresponding to the argument 1.8190, on p. 364, should be 3 instead of 2.

A. N. Lowan

60. G. N. Watson, *Bessel Functions* . . . , p. 751. See RMT **182**, p. 364.

s_n, zeros of $J_{-1/3}(x) + J_{1/3}(x)$
s_2, *for* 5.6101956, *read* 5.5101956.

J. C. P. Miller

UNPUBLISHED MATHEMATICAL TABLES

Reference has been made to an unpublished table in RMT **168** (Terrill & Sweeny).

29[C].—George Hastings Heppel (1793–1845), *Table of Logarithms*, mss. and stereotyped plates possibly in possession of the Institute of Actuaries.

The principal tables in these mss. consist of the logarithms, to 12D, of 100 000 to 118 600, stereotyped in quarto; of 131 000 to 136 636; and of 200 000 to 210 000, to 15D as far as 202 024, and to 12D thereafter. Mr. Heppel was at various times an accountant and an actuary. Having decided shortly before he died that he had sufficient material to publish he got as far as having stereotyped plates made for the range indicated above. The accuracy of the tables has not been tested. The author's son has told us that the logarithms were constructed on the basis of Hutton's logarithms of numbers to 20 places. Mainly through the influence of A. DeMorgan the tables and plates were purchased and placed in the hands of Charles Jellicoe, later president of the Institute of Actuaries. It would seem as if all that is known of the tables and their author is given in an article by Jellicoe, "Heppel's logarithms," Institute of Actuaries, *J.*, v. 10, 1862, p. 82–84.

R. C. A.

30[C].—John Thomson (1782–1855), *Table of Twelve-Figure Logarithms of Numbers from* 1 *to* 120 000, mss. presented to the Royal Astronomical Society in 1873 by his sister Catherine Thomson. Mss. in the R.A.S. Library, though not listed in its printed *Catalogue* of 1886 or 1925.

All that is known about these tables is contained in an "Account" drawn up at the request of the Council of the R.A.S. by J. W. L. Glaisher, and published in R.A.S., *Mo. Notices*, v. 34, 1874, p. 447–475. A few extracts from the information thus supplied are now given.

The tables in about a dozen volumes include a complete 12-place table of the logarithms of numbers from unity to 120 005 (besides the logarithms of 744 higher numbers at irregular intervals, the highest being log 123 187) to 12D; first and second differences for numbers from 29 561 to 120 001; and logarithms of numbers from unity to 4 000 to 15D. Another volume containing second differences for numbers from 10 000 to 29 560 has probably been lost. Here are wholly original tables.

To determine the accuracy of the mss. Glaisher read parts of them with existing tables giving more than 12 figures. One such table is Briggs, *Arithmetica Logarithmica*, London, 1924 with 14-place logarithms of numbers from 1 to 20 000 and from 90 000 to 101 000. It was found that most of the ms. errors were in end-figures. By comparison with a number of other tables Glaisher assembled many interesting results.

Little is known of Thomson except that he was born at Strachur, Argyleshire, and was the second in a family of eleven children of a farmer. He was educated at Strachur till he was twelve years of age, when he went to Greenock, and was placed under the care of a teacher of mathematics and navigation, from whom he imbibed a taste for mathematical studies. For many years he occupied the position of clerk to a firm in Greenock, but subsequently he commenced business as an accountant.

31[C, D, Q].—*Arithmetic, Logarithmic, Trigonometric, and Astronomical Tables*, computed, 1848, 1869–89, by Edward Sang, and his daughters Jane Nicol Sang, Flora Chalmers Sang, and presented in 1907 to the Royal Society of Edinburgh, in custody for the British Nation.

These manuscripts, now at the R.S.E., consist of 47 volumes which were originally accompanied by transfer duplicates of 33 of the volumes; these latter volumes were deposited in the Library of the University of Edinburgh. Of the 47 volumes, 26 (1–6, 12–19, 28–30, 39–47) are the work of E.S., 5 (7, 32, 33, 36, 37) of J.N.S., and 16 (8–11, 20–27, 31, 34, 35, 38) of F.C.S. The chief tables in these volumes are as follows:

The first 10 000 Prime Numbers (up to 104 729) with their logarithms to 28D. Computed 1848, 1865–75.

Logarithms of Numbers 100 000–370 000, to 15D, Δ^2. Computed 1869–1873? Also the last 10 figures of the logarithms of numbers from 1 00000 0000 to 1 00000 9999, and from 1 00000 0000 to 99999 0000. Computed 1884.

Sines for each centesimal 5 minutes of the quadrant, to 33D, Δ^2. Computed 1877. See *MTAC*, p. 36f.

Log Sines for every .0001 radian up to .25, and log [arc/sine], to 7D, with Δ. Computed 1880–85.

Sines and cosines for each centesimal minute in the quadrant, to 15D, Δ^2. Computed 1881.

Log Sines for each centesimal minute in the quadrant, to 15D, 0 to 50^g, Δ; 50^g to 100^g, Δ^3. Log Tangents, for each centesimal minute, to 15D, 0 to 50^g, Δ. Computed 1885–88; revised 1889.

Sines for each centesimal minute in the quadrant, to 8D, Δ^2. Computed 1879.

The tables so far listed, along with their computations, are in v. 1–44. Then follow:

Tables of Circular Segments and Mean Anomalies. Writing the equation for Kepler's problem of finding the mean anomaly (m) of a planet's orbit from the eccentric anomaly (u)

in the form

$$m = u - e \sin u = u - \cos\theta \sin u$$
$$= \tfrac{1}{2}\{(u + \theta) - \sin(u + \theta)\} + \tfrac{1}{2}\{(u - \theta) - \sin(u - \theta)\},$$

it appears that if u and θ are known, m may be derived from a table which gives the quantity $\frac{1}{2}(x - \sin x)$ with argument x. This quantity is the area of the segment of angle x^r in a circle of unit radius. Sang first calculated a table of circular segments for each centesimal minute in the 400^g, to 8D, Δ^2. The results are given in terms of the unit one grade of area, the sector of a circle in which the arc subtends one grade; hence the whole circle contains 400^g or units of area.

Thus were derived the corresponding mean anomalies, to 8D, by a double reference, for each arc $u = 0(1^g)\ 200^g$, and each degree of arc of ellipticity, $e = 0(1^g)\ 99^g$.

Such matters are discussed, and some printed tables to 4D are given by Sang, in his "Nouveau calcul des mouvements elliptiques," Accad. Sci., Turin, *Mem.*, v. 32, 1880, p. 187–199, 305–307. The table of segments is given p. 192–193; and the tables of anomalies for $e = \cos 10^g, \cos 50^g, \cos 90^g$, p. 194–199.

And in v. 47 "the Anomalies are given only to the nearest second, but the differences for a change of 1^g of position, and the variations for a change of 1^g in ellipticity, are filled in; and thus, of the three—the eccentricity, the position, the anomaly—any one may be determined from the others." The previous table to 8D is here cut to 4D, with Δ.

In this same volume are also a Table of all Rational Right-angled Triangles having the hypotenuse less than 1000, arranged according to the magnitude of the hypotenuse; and a Table of Other Triangles whose sides ($\leqslant 1000$), and areas, are integers.

Greater details concerning these manuscripts, and ideas developed in them, may be found in

1. "Dr. Edward Sang's logarithmic, trigonometrical, and astronomical tables," R. So. Edinburgh, *Proc.*, v. 28, 1908, p. 183–196. Reprinted, with a portrait photo of Sang, in E. M. Horsburgh, *Modern Instruments and Methods of Calculation*, London, 1914, p. 38–47.

2. C. G. Knott, "Edward Sang and his logarithmic calculations," *Napier Tercentenary Memorial Volume*, London, 1915, p. 261–268; see also R. A. Sampson, p. 236–237.

3. E. Sang, "On the need for decimal subdivisions in astronomy and navigation, and on tables requisite therefor," R. So. Edinburgh, *Proc.*, v. 12, 1884, p. 533–544.

4. E. Sang, "On the construction of the canon of logarithmic sines," *idem*, p. 601–619; see also v. 9, 1878, p. 343–352.

5. E. Sang, "Notice of fundamental tables in trigonometry and astronomy, arranged according to the decimal division of the quadrant," R. So. Edinburgh, *Proc.*, v. 16, 1889, p. 249–256.

For some facts we are also indebted to Henderson (see *MTAC*, p. 2).

There is a biography of Edward Sang (1805–1890) by D. B. Peebles in R. So. Edinburgh, *Proc.*, v. 21, 1897, p. xvii–xxxii; this includes a list of his writings. Born in Kirkcaldy, Scotland, he was a pupil of Edward Irving (Scottish church divine and friend of Thomas Carlyle) until 1818, when he went to Edinburgh and entered the University. With the exception of two years (1841–1843) spent in Manchester New College as professor of mechanical science, and of twelve years (1843–1854) spent in Turkey, where he assisted in establishing schools of civil engineering, and in laying out railroads, he lived and worked in Edinburgh as a private teacher of mathematics. For many years he was secretary of the Royal Scottish Society of Arts. In 1883 the University of Edinburgh conferred on him the degree of LL.D., and in 1884 he was elected an honorary member of the Franklin Institute in Philadelphia. His first published work was a large folio volume, *Life Assurance and Annuity Tables* with *a copious collection of rules and examples*, Edinburgh, 1841, second v. 1859. For interesting comments on this work, see A. De Morgan, *English Cyclopædia*, ed. C. Knight, *Arts and Science* section, v. 7, London, 1861, col. 1013.

He was the first one to publish a logarithmic table beyond 108 000, *A New Table of Seven-Place Logarithms of all numbers from* 20 000 *to* 200 000, . . . *prepared under the auspices of the managers of the Associated Life Insurance Offices in Scotland*, London, 1871, second edition, improved, Edinburgh, 1878. A set of stereotype plates for this work was given to the R.S.E. with Sang's manuscripts. See J. HENDERSON, *Bibliotheca Tabularum Mathematicarum*, 1926, p. 135f; E. SANG, "Account of the new table of logarithms to 200 000," R. So. Edinburgh, *Trans.*, v. 26, 1872, p. 521–528. Also *Specimen Pages of a table of the Logarithm of all Numbers up to One Million . . . shortened to nine figures from original calculations to fifteen places of Decimals*, [1874?], 26 p. 4to, not in the List of his Writings. There is a copy in the Brown University Library. The first page is a copy of a memorial of the president and council of the R.S.E. dated 19 June 1874, to the Chancellor of Her Majesty's Exchequer, and the last 16 pages are devoted to communications from academies and scholars, dated 1872–73, commending the idea of publishing a 9-place table of logarithms of numbers to 10^6, in three volumes; but the publication was never achieved.

Sang "received a grant of £100 per annum from [the] Government as a recognition of his valuable scientific work; and the associated Scottish life assurance offices, feeling that some substantial recompense was due to him for his logarithms and actuarial tables, at a meeting in 1878 resolved to recommend to the offices the payment of an annuity of £100 . . . for the remainder of his life, which was agreed to, and subscribed by the offices."

R. C. A.

MECHANICAL AIDS TO COMPUTATION

12[Z].—L. J. COMRIE, "Mechanical computing," Appendix I, p. 462–473 of DAVID CLARK, *Plane and Geodetic Surveying*, v. 2, third ed. revised by JAMES GLENDENNING, London, Constable, 1943. 14 × 21.7 cm. Also as a pamphlet reprint.

Some computations possible on several calculating machines are described and there are illustrations of four of them, namely: (1) Brunsviga, (2) 10-key Facit, (3) Marchant Electric, (4) Twin Marchant. Two pages are devoted to a listing and discussion of published tables, and some other literature.

13[Z].—L. J. COMRIE, "Recent progress in scientific computing," *J. Sci. Instruments*, v. 21, Aug. 1944, p. 129–135, illustrated.

The substance of this paper was delivered 3 July 1943 at a joint meeting of the London and Home Counties' Branch of the Institute of Physics, and the London Mathematical Society. The headings in the paper are as follows: "Differential analysers," "The training of computers," "Numerical integration by hand," "Finite differences," "Direct and inverse interpolation," "Double-entry interpolation," "Punched-card machines," "Solution of simultaneous equations," "Mathematical tables," "Short bibliography."

NOTES

26. AGETON'S METHOD.—Navigators and other persons who use Hydrographic Office, *Publication*, no. 211 (see *MTAC*, p. 80–81) regularly, will be interested in two notes recently published on Ageton's method of celestial navigation. The first is "The accuracy of Ageton's method in celestial navigation" by SAMUEL HERRICK, Astron. So. Pacific, *Publications*, v. 56, 1944, p. 149–155. Herrick points out that the warning appearing in H.O. 211 against the use of sights for which the value of K is found to lie between the limits of 87° 30′ and 92° 30′ is not adequate. He shows that, when the tables are used in the conventional fashion and K lies outside the forbidden range, the maximum error in the computed altitude, h, is six minutes of arc. He shows also that an error of a minute of arc in h may occur even when K is quite small. He points out that the "caution" mentioned above is unfortunately omitted from Ageton's *Manual of Celestial Navigation*, New York,