

So it seems that it would be possible, without excessive labour, to derive a double-entry table for $n = 0(.1)20$, $x = [0(.1)10; 7D]$, that would be interpolable in both n and x without going further than second differences at most. Since $\Lambda_n(x)$ is an even function of x , it would be unnecessary to tabulate for x negative (compare $J_n(x)$, for fractional real n and negative real x , complex). But difficulties arise for Λ_n when n is negative, since $\Lambda_n = \infty$ for n a negative integer. Even conversion to J_n is of little practical use here [$J_n(.1)$ fluctuates from zero to -38 and back, between $n = -1$ and -2]. The only practical solution would appear to be interpolation (if required) in the region n positive, followed by the use of recurrence formulae.

In equation (3), p. 46, the n in the denominator of the last fraction should be deleted so as to read $x^2/4(n+1)(n+2)$.

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182[L].—G. N. WATSON, *A Treatise on the Theory of Bessel Functions*. Second edition, Cambridge: at the University Press, New York: The Macmillan Co., 1944, viii, 804 p. 18.7×23.2 cm. \$15.00.

This offset print of the first edition of 1922 seems to have been in the press for more than three years since Watson's preface to the second edition is dated "March 31, 1941." It is as follows (apart from acknowledgments of assistance):

"To incorporate in this work the discoveries of the last twenty years would necessitate the rewriting of at least chapters XII–XIX; my interest in Bessel functions, however, has waned since 1922, and I am consequently not prepared to undertake such a task to the detriment of my other activities. In the preparation of this new edition I have therefore limited myself to the correction of minor errors and misprints and to the emendation of a few assertions (such as those about the unproven character of Bourget's hypothesis) which, though they may have been true in 1922, would have been definitely false had they been made in 1941."

Hence the numbers of pages in the first and second editions are the same. In the Bibliography, p. 753–788, the only change seems to be an addition of a title on p. 788. We note that equations (3), (4), p. 81 have been corrected. In *MTAC*, p. 307, we have already listed the tables in this work, p. 666–752. The errors which we there noted in T. I–II have now vanished but many others still remain; see *MTAC*, p. 296, and *MTE* 58, 60, where a beginning has been made in listing such errors. This reprint has filled a great need.

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183[M].—H. W. LINDEMANN, "Innenbackenbremsen," *Automobiltechnische Z.*, v. 46, Aug. 1943, p. 367–372; also English transl., "Internal shoe brakes," *Engineers' Digest* (Amer. ed.), v. 1, Sept. 1944, p. 558–563. 19.7×27.2 cm. English ed., v. 9, Aug., 1944, p. 230–235.

On p. 562 is a three-place table of eight integrals, from 0 to θ , of (a) $\sin t \cos t$; (b) $\sin^2 t$; (c) $\cos^2 t$; (d) $\cos t$; (e) $\sin t$; (f) $\sin^2 t \cos t$; (g) $\sin t \cos^2 t$; (h) $\sin^3 t$, for $\theta = 0(2^\circ)180^\circ$.

MATHEMATICAL TABLES—ERRATA

References have been made to Errata in RMT 169 (BARLOW, HAYASHI, WEIGEL, L. ZIMMERMANN), 170 (LEHMER), 172 (N.D.R. COMM.), 176 (DAVIS), 181 (NYMTP); N26 (AGETON).

55. J. M. BATES, "Zeros of a class of polynomials associated with Bateman's k -function," Iowa State College *J. Sci.*, v. 12, 1938, p. 474.

The first 5 zeros of $J_{1/2}[(2/3)x^{3/2}] + J_{-1/2}[(2/3)x^{3/2}]$ are here given as follows:

2.338107, 4.137258, 5.520555, 6.786701, 7.944136.

These values should be:

2.338107, 4.087949, 5.520560, 6.786708, 7.944134.

Bates states that he calculated his zeros from those for the function $J_{1/3}(x) + J_{-1/3}(x)$, given by WATSON, *Bessel Functions*, 1922 [and 1944], p. 751; but, as noted below, Watson's second zero is highly erroneous.

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56. A. R. FORSYTH, *Messenger Math.*, s. 2, v. 50, 1921, p. 129–149. Compare *MTAC*, p. 215, **IB**₆.

This paper gives exact values of certain sums extending over the roots of Bessel functions.

P. 145, for $\sum_{r=1}^{\infty} \mu_r^{-8} J_0^{-2}(\mu_r) = 73/(5 \cdot 27 \cdot 4096)$,

read $\sum_{r=1}^{\infty} \mu_r^{-8} J_0^{-2}(\mu_r) = 73/(5 \cdot 27 \cdot 4096)$;

p. 146, for $\sum_{r=1}^{\infty} \mu_r^{-8} J_0^{-3}(\mu_r) = -37/(3^3 \cdot 128)$, $\sum_{r=1}^{\infty} \mu_r^{-8} J_0^{-3}(\mu_r) = -379/(3 \cdot 5^3 \cdot 2048)$,

read $\sum_{r=1}^{\infty} \{\mu_r^{-4} + 8\mu_r^{-8}\} J_0^{-3}(\mu_r) = -37/(3 \cdot 128)$,

$$\sum_{r=1}^{\infty} \{\mu_r^{-8} + 8\mu_r^{-8}\} J_0^{-3}(\mu_r) = -379/(3 \cdot 5 \cdot 2048).$$

To the list on p. 139 we may add the following:

$$\sum_{r=1}^{\infty} \kappa_r^{-1} J_1^{-1}(\kappa_r) = \frac{1}{2},$$

though this is not obtainable by the author's method.

D. H. L.

57. K. KARAS, "Tabellen für Besselsche Funktionen erster und zweiter Art mit den Parametern $\nu = \pm 2/3, \pm 1/4, \pm 3/4$," *Z. f. angew. Math. u. Mech.*, v. 16, 1936, p. 249–251. It has been already noted (*MTAC*, p. 295) that NYMTP discovered that in half of these tables, those for $J_\nu(x)$, there are 108 errors, namely: 90 of a unit in the last decimal place, and 18 of two or more units. These errors are as follows:

$\nu = 2/3$					
<i>x</i>	<i>for</i>	<i>read</i>	<i>x</i>	<i>for</i>	<i>read</i>
.1	.15011	.15012	2.1	.52905	.52906
.2	.23723	.23722	4.2	-.28517	-.28518
.8	.54534	.54535	4.8	-.36057	-.36058
1.2	.62892	.62893	5.0	-.35712	-.35713
1.6	.62955	.62956	7.9	.27822	.27823
1.7	.61777	.61778			

$\nu = -2/3$					
<i>x</i>	<i>for</i>	<i>read</i>	<i>x</i>	<i>for</i>	<i>read</i>
.1	2.72981	2.72976	3.0	-.45760	-.45759
.2	1.68082	1.68084	8.5	-.21775	-.21774
.4	.96247	.96246	8.6	-.23186	-.23185
.5	.76835	.76834	8.9	-.25916	-.25915
.8	.37691	.37690	9.3	-.25895	-.25894
2.6	-.48311	-.48310	9.6	-.23233	-.23232

$\nu = 1/4$

x	for	read	x	for	read
.4	.7143	.7144	6.9	.2548	.2547
1.6	.5803	.5804	7.0	.2681	.2680
1.7	.5383	.5384	7.2	.2861	.2860
2.3	.2443	.2444	7.3	.2907	.2906
3.3	-.2189	-.2188	7.4	.2925	.2923
4.3	-.3836	-.3835	7.5	.2913	.2910
4.4	-.3789	-.3788	7.6	.2873	.2869
4.7	-.3436	-.3437	7.7	.2805	.2800
4.9	-.3046	-.3045	7.8	.2711	.2704
5.5	-.1345	-.1346	7.9	.2591	.2582
5.7	-.0684	-.0685	8.0	.2449	.2436

 $\nu = -1/4$

x	for	read	x	for	read
.2	1.4310	1.4319	7.1	.2742	.2743
.4	1.1560	1.1559	7.3	.2415	.2416
2.9	-.3668	-.3669	7.4	.2219	.2220
3.3	-.4227	-.4226	7.5	.2003	.2004
3.5	-.4243	-.4242	7.6	.1770	.1771
6.4	.3018	.3019	7.7	.1522	.1523
6.5	.3069	.3070	7.8	.1263	.1264
6.9	.2967	.2968	7.9	.0994	.0995
7.0	.2868	.2869	8.0	.0719	.0720

 $\nu = 3/4$

x	for	read	x	for	read
.8	.4986	.4987	5.7	-.2723	-.2722
1.2	.5988	.5989	6.1	-.1693	-.1694
1.3	.6115	.6116	6.7	.0145	.0146
1.5	.6224	.6225	6.8	.0447	.0449
1.9	.5888	.5889	6.9	.0741	.0743
3.1	.1719	.1718	7.0	.1024	.1025
3.2	.1274	.1273	7.3	.1778	.1779
3.3	.0830	.0829	7.4	.1990	.1991
3.7	-.0855	-.0856	7.5	.2180	.2182
4.0	-.1934	-.1935	7.6	.2347	.2348
4.1	-.2243	-.2244	7.7	.2488	.2489
4.2	-.2524	-.2525	7.8	.2602	.2604
4.7	-.3439	-.3440	7.9	.2689	.2692
5.2	-.3482	-.3481	8.0	.2749	.2752

 $\nu = -3/4$

x	for	read	x	for	read
.7	.3277	.3234	3.9	-.1509	-.1508
.8	.2194	.2193	4.1	-.0719	-.0718
1.9	-.4211	-.4212	5.3	.2941	.2942
2.1	-.4672	-.4673	5.7	.3306	.3305
2.5	-.5008	-.5009	6.3	.2889	.2890
3.1	-.4204	-.4205	6.5	.2526	.2527
3.2	-.3943	-.3945	7.1	.0998	.0999
3.3	-.3656	-.3657	7.9	-.1257	-.1258
3.5	-.3007	-.3006			

See RMT 72, *MTAC*, p. 8.

58. E. C. J. v. LOMMEL, Bayer. Akad. d. Wissen., *math. natw. Abt., Abh.*, v. 15, 1886, p. 648; also G. N. WATSON, *A treatise on the Theory of Bessel Functions*, 1922 and 1944, p. 744; see RMT 182 and *MTAC*, p. 296. Tables of $S(2x/\pi)^{\frac{1}{2}}$ and $C(2x/\pi)^{\frac{1}{2}}$:

$S(2x/\pi)^{\frac{1}{2}}$			$C(2x/\pi)^{\frac{1}{2}}$		
x	for	read	x	for	read
0.2	.023788	(Watson .023720 correct)	11	.380390	.380392
1.5	.415348	.415483	11.5	.395149	.395152
6.5	.347099	.347100	15.0	.569335	.569336
8.0	.512010	.512009	15.5	.524009	.524010
11.0	.504784	.504786	17.5	.406589	.406590
11.5	.447809	.447810	20.5	.587849	.587848
12.0	.405810	.405811	21.0	.573842	.573841
13.5	.432489	.432488	21.5	.542266	.542265
15.5	.598183	.598184	25.5	.526896	.526897
20.5	.504875	.504874	34.0	.537026	.537027
21.0	.545885	.545884	34.5	.504881	.504882
24.0	.467029	.467028	38.5	.545560	.545559
28.5	.573060	.573059	47.5	.479313	.479311
34.0	.557490	.557489	48.5	.443930	.443929
36.5	.476871	.476872	The 14 errors for $x < 20$ were noted by comparison with the tables of J. R. AIREY, in B.A.A.S., <i>Report</i> , 1926, p. 274-275.		
39.5	.513690	.513689	J. W. WRENCH, JR.		
40.5	.558799	.558800			
44.5	.447720	.447721			

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59. NYMTP, *Tables of Circular and Hyperbolic Sines and Cosines*, 1939. See RMT 89, p. 45f.

The integral part in the value of $\sinh x$ corresponding to the argument 1.8190, on p. 364, should be 3 instead of 2.

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60. G. N. WATSON, *Bessel Functions . . .*, p. 751. See RMT 182, p. 364.

s_n , zeros of $J_{-1/2}(x) + J_{1/2}(x)$
 s_2 , for 5.6101956, read 5.5101956.

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UNPUBLISHED MATHEMATICAL TABLES

Reference has been made to an unpublished table in RMT 168 (TERRILL & SWEENY).

29[C].—GEORGE HASTINGS HEPPEL (1793-1845), *Table of Logarithms*, mss. and stereotyped plates possibly in possession of the Institute of Actuaries.

The principal tables in these mss. consist of the logarithms, to 12D, of 100 000 to 118 600, stereotyped in quarto; of 131 000 to 136 636; and of 200 000 to 210 000, to 15D as far as 202 024, and to 12D thereafter. Mr. Heppel was at various times an accountant and an actuary. Having decided shortly before he died that he had sufficient material to publish he got as far as having stereotyped plates made for the range indicated above. The accuracy of the tables has not been tested. The author's son has told us that the logarithms were constructed on the basis of Hutton's logarithms of numbers to 20 places. Mainly through the influence of A. DEMORGAN the tables and plates were purchased and placed in the hands of CHARLES JELLCOE, later president of the Institute of Actuaries. It would seem as if all that is known of the tables and their author is given in an article by Jellicoe, "Heppel's logarithms," *Institute of Actuaries, J.*, v. 10, 1862, p. 82-84.

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