

A formula for δE , the part of the change in the retardation due to the variations in x' , y' and y , is¹

$$\delta E = E[F(v) \cdot v \Delta v + \eta \cdot d \ln H/dy]$$

where $v \Delta v = (x' \xi' + y' \eta')$, $H(y) = e^{-.0001028y}$. Since numerical integration must be used in determining the differential corrections to be applied at each point of the trajectory it is necessary to have tabulated values of $F(v)$.

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¹D. JACKSON, *The Method of Numerical Integration in Exterior Ballistics*, Washington, 1921, p. 24; this was a text-book prepared in the office of the Chief of Ordnance, 1919.

EDITORIAL NOTE: These tables were later superseded by smoother functions based on more recent firings. In Jackson's publication is an extended table of $H(y)$.

MATHEMATICAL TABLES—ERRATA

References have been made to Errata in RMT 319 (Duffield, Lefort, Vega), 332 (Rydbeck); N62 (Corey, Hardy & Rogosinski, Harvard, Zygmund, etc.).

88. H. T. DAVIS, *Tables of the Higher Mathematical Functions*, v. 2, Bloomington, Indiana, 1935, p. 29.

There are three serious errors on this page, in $\psi'(x)$ which Davis defines as $d^2 \ln \Gamma(x)/dx^2 = d^2 \ln(x-1)!/dx^2$, and in $\log \psi'(x)$.

For $\psi'(.05) = 401.552357\ 342115$, read 401.532357 342115;

For $\psi'(.11) = 84.077927\ 249967$, read 84.059535 747392;

For $\log \psi'(.11) = 1.92468\ 19966$, read 1.92458 69871.

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89. FMR, *Index*, 1946. See *MTAC*, v. 2, p. 13–18, 136.

A. Apart from a few definite errata noted below we have also indicated inconsistencies, indefinite statements, and a couple of notation changes, which the authors may desire to consider when preparing a new edition. The authors clearly recognized that slight blemishes of this kind existed, because of various elements entering into the preparation and publication of their work. Changes are in italics. See also MTE 90.

P. 23, 2.3 Higher *Positive* Integral Powers [2.5 Higher Negative . . .].

P. 25, 2.3 Higher *Positive* Integral.

P. 33, l. 7, $n = 440 \times 2^p$.

P. 35, 3.14 27 dec. Thoman.

P. 48, 4.18, l. 4, 4.021.

P. 51, 4.41, for 4.412, read 4.4121; for 4.413, read 4.4132.

P. 76, 4.9333, for 2^m , read 2^a .

P. 100, last l., Cauchy 1882, why not 1827? Also p. 101, 5.7115.

P. 111, last l., *de* Decker; p. 124 and 125, 5 d., *de* Lella.

P. 151, 9.24, l. 2, δ for d (longitude)? [see 9.23].

P. 192, 13.4 4 dec. for 10^4 , 10^5 , 10^6 , read $10^{(1)6}$?

P. 200, 7 dec. Brownlee 1923 (Russell, which one?).

P. 208, 14.92, for the heading "Tables of x ," read "Inverse tables relating to $B_z(p, q)$ "?

P. 251, 17.33 heading to make uniform with 17.35, read $G_0(x)$ and $G_1(x)$, then on next line $G_n(x) = -\frac{1}{2}\pi Y_n(x)$.

- P. 252, **17.341** $G_n(x)$: General Tables [to make uniform with **17.361**].
- P. 262, **17.7212** read $\log_{10} |J_1(j_{0,a})|$; also, p. 263, **17.7312**, read $\log_{10} |J_0(j_{1,a})|$.
- P. 265, **17.751**, for n Integral, read m Integral.
- P. 278, before **18.61** enter **18.6**. Expressions involving both $J_n(x)$ and $I_n(x)$.
- P. 326, **22.0**. Introduction, as heading and subheading, omitted.
- P. 368, after l. 5, **24.0** Introduction omitted; in **24.21**, l. 1, for requisite, read requisites.
- P. 412, after Lohmann insert reference to T. Lohnstein 1892 [see p. 429, Runge 1891].
- P. 435, Steinhauser 1865, l. 2, fünfzehnstelliger.
- P. 437, Terrill & Sweeny, no cross-reference from Sweeny.

There are many places in Part I where initials seem necessary, exactly to identify individuals in Part II. Such, for example, are the following: *K. Pearson* (p. 21, 24), *C. F. Gauss* (p. 22), *P. L. H. Davis* (p. 144), *W. P. Elderton* (p. 200), *J. B. Russell* (p. 228), *G. W. Hill* and *G. N. Watson* (p. 316), *J. G. Schmidt* (p. 317), *E. A. Milne* and *J. C. P. Miller* (p. 332), *G. N. Watson* (p. 343), *C. J. Hill* (p. 344), *Ch. Jordan* ? (p. 352), *K. Pearson* (p. 355), *R. A. Fisher* (p. 364). But the name and year, given in each of these places but one, are uniquely determinate.

There are a number of cases of names mentioned in Part I which are not represented in the Bibliography, at least in connection with items in question, for example: *Wingquist* (p. 22), *Glaisher* (p. 59), *Wrench* (p. 85), *Stirling* (p. 105), *Atwood* (p. 178), *E. Wright* (p. 185), *Isaacson & Salzer* (p. 202), *Ikehara* (p. 207), *Norton*, *W. E. Deming* and *L. S. Deming* (p. 209), *Jeffreys* (p. 212), *Julia Bell* (p. 219), *B. A. Gould* (p. 221), *Blumer* (p. 239), *Sarmousakis* (p. 242), *Stoneley* (p. 338), *F. E. Allan* (p. 364), *Carse & Urquhart* (p. 367), *Buys-Ballot* (p. 369), *Darwin* and *Doodson* (p. 370), *Darwin-Börger* (p. 371).

S. A. JOFFE

B. Herewith are memoranda dealing with matters not of outstanding importance.

- P. 83, 427. "Richter 1855," Richter died in 1854, and his 500D value of π first published in October 1854 (see *MTAC*, v. 2, p. 144), was reprinted in *Archiv* 1855. Hence "Richter 1854" is desirable here.
- P. 112. The Duffield 1897 table is to 100 009, not 100 000.
- P. 144. One wonders at the omission in **8.4** of a reference to Legendre's table of $\log \tan(45^\circ + \frac{1}{2}x)$, *Traité des Fonctions Elliptiques*, v. 2, 1826, p. 256–259.
- P. 377. According to the form of entry the authors declare that they have seen a copy of Bertrand's *Calcul des Probabilités*, dated 1888. The Brown University copy, dated 1889, and the *Catalogue* of the Bibliothèque Nationale give no hint of an 1888 edition.
- P. 384. Where so many much less worthwhile items are listed one is surprised not to see a reference to OLIVER BYRNE, *Tables of Dual Logarithms*, London, Bell & Daldy, 1867.
- P. 387. One might readily infer that the first English edition of Crelle's Calculating Tables was in 1908; the statement in *FMR Index* is "O. Seeliger's new edition first appeared . . . in English . . . in 1908," which is, of course, perfectly correct. The first English edition, however, was of Bremiker's revision of Crelle, London, Nutt, 1897. But much earlier there appeared *Tables for Facilitating the Operations of Multiplication, Division, and Evolution. Abridged from Dr. A. L. Crelle's Rechentafeln* by J. A. NORRIS, Washington, Govt. Printing Office, 1885. 20 p. Quarto format. [Text, p. 5–8; Tables p. 9–20.]
- P. 390. The Desvallées, H. R. entry should be under Rocques-Desvallées, H.; see *Annuaire pour l'an 1919 publié par Le Bureau des Longitudes*, p. C4, and *Catalogue Général de la Librairie Française*, v. 29, p. 821.
- P. 390. Why not refer to the 1934 reprint by Stechert, New York, of Dickson's *History of the Theory of Numbers*, v. 1, 1919?
- P. 391. It would seem better to have expanded the title for Dupuis 1862 to become *Tables de Logarithmes à Sept Décimales d'après Callet, Véga, Bremiker, etc.*, so as to indicate their lack of originality.
- P. 392. Surely it were especially desirable to give as a second reference for Encke 1832, his *Ges. math. u. astron. Abhandlungen*, Berlin, v. 3, 1889, p. 71–78.

- P. 394. The second editions of GAUSS, *Werke*, v. 2, 1876, and v. 3, 1876, are not noted. D. H. LEHMER, *Guide to Tables in the Theory of Numbers*, p. 100, points out that v. 2, 1876, for example, is not merely a reprint, since it contains material not in v. 2, 1863.
- P. 396. The authors inquire whether the tiny booklet by "Serge de Glasenapp," *Tables de Logarithmes . . .*, Paris, 1934, 127 p., 8.8×12.9 cm., is by Serge P. Glazenap, author of *Matematicheskie i Astronomicheskie Tablitsy*, Leningrad, 1932, iv, 241 p. 17.3×25 cm. The answer is yes.
- P. 401. The second edition of Heger's *Fünfstellige logarithmische u. goniom. Tafeln*, is dated 1913, not 1914. In the title should be "Hülftafeln" not "Hilftafeln."
- P. 407. The authors apparently state that they have seen Kennelly's *Chart Atlas*, third ed. revised and enlarged, dated "1926"; the Brown Univ. copy is dated 1924.
- P. 408. Of Kepler's *Chilias Logarithmorum 1624* there was also a so-called 1639 edition; but except for the first two leaves the editions are identical.
- P. 410. Under L. "Leau" is "? *Tables des Parties Proportionnelles*, Paris, Gauthier-Villars." This 10-page pamphlet by Léau (not Leau), see *Catalogue Général de la Librairie Française*, v. 29, p. 571, was published in 1921. Poggenдорff lists this title and uses the form "Leau." "*Proportionnelles*" is misspelled by FMR.
- P. 416. The entry under A. Meyer (a work not seen by the authors) is misleading. Anton Meyer, professor of mathematics at the Univ. of Liège, died in 1857. On the request of his widow his ms. *Théorie Analytique des Probabilités* was edited by his former student F. J. P. FOLIE, and published at Brussels in 1874. This was translated into German by EMANUEL CZUBER and published at Leipzig in 1879 under the title *Vorlesungen über Wahrscheinlichkeitsrechnung*.
- P. 420. For Newcomb 1882, the authors list only the first edition of *Logarithmic and Other Mathematical Tables*. The dates of many other reprints up to 1921 are given in my Bibliography of Simon Newcomb, *Nat. Acad. Sci., Memoirs*, v. 17, 1924, p. 55. In 1921 this was the "best seller" of all of Newcomb's works on the market.
- P. 420. The title of Newton's work is *Trigonometria Britannica* (not *Britannica*).
- P. 427. The French edition, 1887 of F. W. Rex, *Fünfstellige Logarithmen-Tafeln* is listed. It might have been noted that the author's initials there become F. G.
- P. 431. After the title of Schorr 1916 should be: *Publ. A-F*, Hamburg, Lucas Gräfe.
- P. 431. Of Schrön 1860, there was an Italian edition in 1867 and a French edition in 1891.
- P. 433. The entry Sherman 1933, should be under J. Sherman & L. Brockway; their little table is described elsewhere in this issue, N 62.
- P. 434. The authors appear to state that they have seen a second edition of Spence's *Essay* dated 1820. As we have already noted, *MTAC*, v. 1, p. 458-459, the copy, which we consulted, of *Mathematical Essays* containing this, is dated 1819, not 1820.
- P. 439. Since the authors have Chebyshev not Tschebyshev, for "Tschuprow" should there not be at least a cross-reference from Chuprov, even though the title-page has the form of name given?
- P. 440. It seems somewhat remarkable that the authors do not list in the Bibliography the important publication of MAXIMILIAN VON LEBER, *Tabularum ad Faciliorem et Breuiorem, in Georgii Vegae "Thesauri Logarithmorum" magnis Canonibus, Interpolationis Computationem utilium, Trias*. Vienna, 1897, 51 p., including the listing of thousands of Vega errors.
- P. 443. Under Hoëne Wroński 1827 (see *MTAC*, v. 2, p. 18) it is stated that there was a Russian edition published at St. Petersburg in 1845. For this date the authors probably took as their excellent authority *Encycl. d. Sci. Math.*, I. 4. 2, 1908, p. 306. On the other hand, the very careful historian, S. DICKSTEIN, in his *Hoene Wroński*, Cracow, 1896, p. 328, gives the date of publication as 1844.
- P. 444. The authors usually make a point of indicating the date of the first edition of each work listed, but have not done this in the case of L. ZIMMERMANN, *Vollständige Tafeln der Quadrate aller Zahlen*. This is the complete title of the work appearing at Liebenwerda, Verlag des technischen Versandgeschäfts R. Reiss, 1898.

C. By courtesy of Mr. D. H. SADLER, Superintendent of the Nautical Almanac Office, the authors have now seen J. W. CAMPBELL, *Numerical Tables of Hyperbolic and Other Functions*, Boston, etc., Houghton Mifflin, 1929. This useful little volume, which we vainly tried both to buy and to borrow during the war, is doubtless known to many of our American readers.

We find that the following is the only major correction required by the *Index*: Art. 10.41 (p. 170), for 0(.0001).025, and 0(.01)2.5 read 0(.0005).025, and 0(.05)2.5. A minor correction in Art. 7.12 (p. 123) is that Campbell's table of $\tan x$ gives 5 figures (instead of 4 decimals) for $x = 1.472(.001)1.670$, i.e. round $x = \frac{1}{2}\pi$. There are also trivial corrections of the nature that, for example, tables ending at $x = 2.999$ and 7.99 have been described as ending at 3 and 8 respectively. Campbell gives no differences, but gives proportional parts on a folding sheet at the end of the book.

If the major (and possibly the minor) correction is made, users of the *Index* may like to delete the asterisk in the J. W. Campbell item in Part II (p. 384).

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90. FRANCE, Service Géographique de l'Armée, *Tables des Logarithmes à huit Décimales*, Paris, 1891.

J. HENDERSON, *Bibliotheca Tabularum Mathematicarum*, 1926, states, p. 143, "It will be of interest to remark that this is the first 8-place table since 1658. Error: Log 28917 should be one unit less in the last place." FMR, INDEX, p. 112, states one unit more. In the table $\log 28917 = 4.46115\ 324$. Since $28917 = 3^5 \cdot 7 \cdot 17$ it was easy to find, by means of Grimpen's table, that $\log 28917 = 4.46115\ 32349\ 908$ is correct to 13D. Hence Henderson, and not FMR, is correct. This is the only known error in this table.

S. A. JOFFE

91. A. M. LEGENDRE, *Tafeln der Elliptischen Normalintegrale erster und zweiter Gattung*, hrsg. von FRITZ EMDE, Stuttgart, 1931. See MTE 86, v. 2, p. 136 f.

In the previous survey we omitted to take account of three errors, noted by HEUMAN l.c., p. 144, occurring in the 1816 table of Legendre, but correct in Legendre's 1826 table. These are in $E(\phi, \theta)$ as follows:

Page	ϕ	θ	For	Read
385(50)	23°	51°	0.33502...	0.39502...
409(74)	14°	82°	0.241959...	0.341969...
413(78)	2°	86°	0.03489 9351	0.03489 9531

Heuman lists errors in $\log F(24^\circ.9)$, $\log F(30^\circ.9)$, $\log E(34^\circ.4)$, $\log E(86^\circ.0)$, in Legendre 1826, and therefore in Pearson. Only one of these errors occurs in Legendre 1816, namely in $\log E(34^\circ.4)$, but this table was not reprinted by Emde. In Legendre 1826 there are also errors in $F(73^\circ, 5^\circ)$, $E(54^\circ, 15^\circ)$, $F(35^\circ, 30^\circ)$, $E(45^\circ, 35^\circ)$. The erratum, *MTAC*, v. 2, p. 137, l. 11, for $F(35^\circ, 30^\circ)$ is to be deleted. Thus, there are at least 33 serious errors in the Emde and Potin Legendre tables, and at least 42 serious errors in the Pearson Legendre table. It may be noted that three of Legendre's errors previously listed were included by Heuman in a Supplementary Errata sheet published at Stockholm in 1941. Finally, on p. 137 (l.c.), for 1.17204, read 3.17204; and for 1.17024 read 3.17024; and in l. 4, for Seventeen, read Sixteen.

R. C. A.

92. NYMTP, "Table of $f_n(x) = n!J_n(x)/(x/2)^n$," *J. Math. Phys.*, v. 23, 1944. See *MTAC*, v. 1, p. 363.

P. 50, $n = 4$, $x = 8.5$, for .01527 8693, read .01527 8963. P. 52, $n = 6$, $x = 9.4$, *82, for 193518, read 193528. P. 59, $n = 19$, $x = 1$ for .98757 4123, read .98757 4124.

NYMTP.

93. U. S. HYDROGRAPHIC OFFICE, *Publication* no. 214, v. 4, 1940, *Tables of Computed Altitude and Azimuth, Latitudes 30° to 39°, inclusive*. Compare *MTAC*, v. 1, p. 81 f.

In order to decide whether the Japanese made use of H.O. No. 214 in the computation of the altitudes and azimuths given in their *Celestial Air Navigation Table*¹ (a problem which will not be considered in the present discussion), it was desirable to have a complete list of all errors in the altitudes in a specific section of the former tables. It was decided that the tabular material for the bright star Deneb as observed between 30°–39° North latitude would serve as a good basis for comparison. Hence the altitudes corresponding to declination 45°, declination same name as latitude, were computed for all integral degrees of local hour angles down to the horizon and for integral latitudes 30°–39°, with seven-place natural values of trigonometric functions. These altitudes were compared with those given (down to 5° altitude) in H.O. 214. The results of this comparison are given below:

<i>L</i>	Errors	Errors >0.1	Tabular Values
30°	14	0	115
31°	58	16	117
32°	42	1	118
33°	57	2	120
34°	41	1	121
35°	60	12	123
36°	40	2	125
37°	36	2	126
38°	58	2	128
39°	45	4	130
Totals	451	42	1223

More than a third of the values are in error by at least one unit in the last place given and approximately 3.5 per cent are in error by two or more units. As will be seen below, the largest errors found were three of 0.4 each. Though the sample examined is a small one, it is believed that it is fairly representative of the accuracy to be expected of H.O. 214.

The uneven distribution of errors among the various latitudes would seem to indicate that in the preparation of H.O. 214, different latitudes were assigned to different computers. For example, the work on latitude 30° is vastly superior to that on latitudes 31° and 35°.

Below is given a list of the 42 errata of two or more units in the last place:

<i>d = 45°</i>					
<i>L = 31°</i>			<i>L = 35°</i>		
<i>i</i>	<i>h</i> should be	error in 0.1		<i>h</i> should be	error in 0.1
5°	75°27.8	2	11°	76°56.9	2
9°	74°19.8	2	12°	76°26.8	3
10°	73°57.8	2	13°	75°55.3	2
14°	72°13.8	2	14°	75°22.5	2
15°	71°44.4	4	17°	73°38.2	2
16°	71°13.8	2	18°	73°01.8	3
17°	70°42.1	2	25°	68°31.1	2

t	h should be	error in 0'.1	t	h should be	error in 0'.1
19°	69°35'.9	2	27°	67°10'.3	2
21°	68°26'.4	2	29°	65°48'.5	2
22°	67°50'.6	2	31°	64°25'.9	2
24°	66°37'.2	2	35°	61°38'.9	2
25°	65°59'.7	2	52°	49°39'.4	2
29°	63°25'.1	2			
31°	62°05'.7	2			
36°	58°42'.5	2		$L = 36^\circ$	
39°	56°38'.3	2	11°	77°44'.1	2
	$L = 32^\circ$		12°	77°12'.6	2
				$L = 37^\circ$	
15°	72°32'.5	2	11°	78°29'.4	2
	$L = 33^\circ$		12°	77°56'.4	2
				$L = 38^\circ$	
12°	74°50'.3	2	20°	73°31'.5	2
17°	72°13'.2	2	27°	68°43'.2	2
	$L = 34^\circ$				
15°	74°04'.9	2		$L = 39^\circ$	
			5°	82°56'.8	3
			6°	82°31'.8	4
			7°	82°03'.9	4
			28°	68°27'.9	2

I wish to acknowledge the valuable assistance of Miss EVELYN LINDSAY and Miss NANCY ARNOLD in the work of computing and checking the values used.

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¹ See *MTAC*, v. 2, 1946, p. 44.

UNPUBLISHED MATHEMATICAL TABLES

The list of some tables prepared by The Radio Corporation of America, published below, suggests that many other mathematical tables must have been prepared during recent years at various research centers. The Editors would heartily welcome Reports on such tables. Other Unpublished Tables are referred to in *RMT* 320 (Strömngren). See also N61.

50[F].—ROBERT JAMES PORTER (1882-): *Tables giving the complete classification of primitive binary quadratic forms for negative determinants from $-D = 2$ to $-D = 1000$* . Ms. calculated during 1945 and the first quarter of 1946, the property of the author, residing at 266 Pickering Road, Hull, England.

The Ms. is in loose-leaf form, 298 pp. $8 \times 10\frac{1}{4}$ inches. The tables are in long-hand, in pencil, and consist principally of the main table, 259 pp., arranged in six parallel columns, the first containing the determinant number with its prime factors, in symbolic form; the second, the positive forms belonging to each determinant, arranged in ascending order of magnitude of the middle term; the third, the number of genera; the fourth, all the forms