of a unit in the last decimal) in HAYASHI, *Siebenstellige . . . Tafeln* (1926) give ample proof (a) that he did not check by differencing and (b) that he used a building-up process for intermediate values. Curiously enough, his end figures are fairly reliable. A run of errors in an early volume of DAVIS[1] showed (a) that he had made independent subtabulations in each interval, (b) that he had relied on repetition—the poorest possible check, and (c) that he had not checked by differencing. Other errors showed (d) that he had neglected second differences when interpolating 10-figure logarithms, and (e) that he had taken 10-figure logarithms of rounded-off quantities containing only five or six significant figures. But to his credit be it said that he was an apt "pupil" and can be trusted not to fall into any of these traps again!

Mrs. GIFFORD's end figures,[2] especially in the tangents, show the neglect of higher order differences; at one point there is a perfect wave in each 10″ interval, with an amplitude of 3 units. The observation that her sines near 90° were often in error by 99, 100 or 101 units led to a confession (in the true Sherlock Holmes style) that she "pre-fabricated" the first six decimals, and later added the seventh and eighth, with the not unnatural result that the sixth is often one out!

DUFFIELD's claim to have computed his logarithms to 12 decimals, increasing the tenth when the last two were 50 or more, is immediately shown to be false by the fact that his end-figure errors are (with a few exceptions, which can be accounted for) the same as those of Vega![3]

The fact that BENSON had copied from BRANDENBURG was revealed by his end-figure errors. He, too, was forced into a confession (*MTAC*, v. 1, 1943, p. 9) that shows he had not been honest either in his compilation, or in his preface. IVES, who also wrote a deceitful preface, provided at least a part of the clue to his plagiarisms by his errors.[4]

I have seen a 5-figure navigational table which contained just five per cent of end-figure errors, because it had been prepared from a six-figure table, but rounding off all 5's in the *same* direction.

L. J. C.

EDITORIAL NOTES: In *MTAC*, v. 1, p. 144 and 58, accuracy of half a unit in the last decimal place by PETERS, and PETERS & STEIN has been noted; and also on p. 145 accuracy less than .502 in the last decimal place. See further "Cayley and tabulation," p. 98. In the quotation of a passage from Glaisher's pen "Lefort's errata" are those referred to in *MTAC*, v. 2, p. 164–165. The well-known 7D table of CHARLES BABBAGE is *Table of the Logarithms of the Natural Numbers from 1 to 108 000*, London 1827, and various later editions.

[1] H. T. DAVIS, *Tables of the Higher Mathematical Functions*, v. 1. Bloomington, Ind., 1933.

[2] E. GIFFORD, see *MTAC*, v. 1, p. 11, 24f, 64f.

[3] See *MTAC*, v. 2, p. 164.

[4] H. C. IVES, see *MTAC*, v. 1, p. 9f.

QUERIES

21. PORTRAITS AND BIOGRAPHIES OF BRITISH MATHEMATICAL TABLE MAKERS.—Where may portraits be seen, or copies possibly be procured, of any of the following individuals: PETER BARLOW (1776–1862), HENRY BRIGGS (1561–1630), OLIVER BYRNE (publs. of 1838–77), ALLAN JOSEPH CHAMPNEYS CUNNINGHAM (1842–1928), JAMES DODSON (d. 1757), RICHARD

FARLEY (publs. of 1840–56), HERSCHELL E. FILIPOWSKI (publs. of 1849–57), EMMA GIFFORD (1861–1936), PETER GRAY (1807?–1887), EDWARD LINDSAY INCE (1891–1941), HENRY SHERWIN (publs. of 1705–1741), ROBERT SHORTREDE (1800–1868), JOHN SPEIDELL (publ. of 1619), MICHAEL TAYLOR (1756–1789)? None of them are listed in Ball's Collection of portraits (E. M. HORSBURGH, *Modern Instruments and Methods of Calculation*, London, 1914). Karl Pearson tells us (*Logarithmetica Britannica*, part IX, 1924) that he vainly sought a portrait of Briggs. The *Dict. Nat. Biog.* contains sketches of Briggs and Gray, and there is biographical material about Cunningham in (a) Poggendorff, (b) London Math. Soc., *Jn.*, v. 3, 1928 (by A. E. Western), and about Ince in (a) *Who's Who 1941*, (b) London Mathem. Soc., *Jn.*, v. 16, 1941 (by E. T. Whittaker), (c) Edinb. Math. Soc., *Proc.*, s. 2, v. 6, 1941 (by A. W. Young), (d) *Nature*, v. 148, 1941 (by A. C. Aiken). Where may one find biographical data concerning the other persons listed?

R. C. A.

QUERIES—REPLIES

28. TABLES OF TAN$^{-1}(m/n)$ (Q**14**, v. 1, p. 431; QR**18**, v. 1, p. 460; **20**, v. 2, p. 62; **24**, p. 147).—In *MTAC*, v. 2, p. 63, footnote 2, J. C. P. MILLER suggested the following problem: Determine all positive integers N which are fundamental in the sense that there is no relation of the form

$$\tan^{-1} N = \sum_i \lambda_i \tan^{-1} n_i \qquad (1)$$

(where the λ_i are integers and the n_i are positive integers *less than* N).

This problem can be completely solved using methods of elementary number theory (C. F. GAUSS, *Werke*, v. 2, 1863, and 1876, p. 477 and 523, and the papers by C. STØRMER and others quoted in *MTAC*, v. 2, p. 28). It can be shown that N is fundamental if and only if $N^2 + 1$ has a prime factor which is not a factor of a number $n^2 + 1$ with $n < N$. An effective construction for the relation of the form (1) in the case when N is not fundamental can be given; a table giving all such relations with N satisfying $N^2 + 1 < 100\,000$ has been prepared. The positive integers which are fundamental form a minimal "integral" basis for the set $\{\tan^{-1} n\}$ and therefore also for the set[1] $\{\tan^{-1}(m/n)\}$. The connection between this "integral" basis and the "rational" basis which was apparently known to GAUSS[2] can be made clear by the following remark. Corresponding to a prime $p = 2$ or $4n + 1$, Gauss has $\tan^{-1}(a/b)$ where a and b are determined uniquely by the conditions $a \geqslant b > 0$ and $a^2 + b^2 = p$ while we have $\tan^{-1} N$ where N is the least positive integer such that $N^2 + 1$ is a multiple of p. A table has been prepared, covering all such primes $p < 500$, expressing $\tan^{-1}(a/b)$ in terms of $\tan^{-1} N$ and $\tan^{-1} m$ with $m < N$. These results will be submitted to the London Math. So., *Jn.*

The reduction of $\tan^{-1}$ 1 40333 78718 was attempted in order to test the reduction algorithm. This is reducible since the prime factors of 1 40333 78718^2 + 1 all occur as factors of $n^2 + 1$ with $n <$ 1 40333 78718. Use of the factorisation given by Gauss (*Werke*, v. 2, p. 481) led to a contradiction which was found to be due to an extra factor 13 in the decomposition