Dr. Wrench independently discovered another error in his work. Thus 12 digits have to be changed in the previously published value of π in the interval 721D–808D. The correct sequence is as follows:

$\pi =$	.....	.....	.....	.....	86403	44181	59813	62977	47713	09960
	51870	72113	49999	99837	29780	49951	05973	17328	16096	31859
	50244	594(55)								

The corrected sequence in $\tan^{-1} \frac{1}{5}$, 721D–808D is as follows:

.....	.....	.....	.....	40468	13622	41107	68081	00362	13759
92595	89071	66648	51452	55706	79954	88100	43132	95466	83892
29036	883(09)								

There is no change to be made in Mr. Smith's previously published values of $\tan^{-1} \frac{1}{239}$ to 811D or in Mr. Ferguson's (a) $\tan^{-1} \frac{1}{4}$, (b) $\tan^{-1} \frac{1}{20}$, (c) $\tan^{-1} \frac{1}{1985}$, each to 710D. We add, however, Mr. Ferguson's values of (a), (b), and (c) 711D–808D.

(a)	.....	.....	59314	07269	83047	72505	80573	53263	40402	52936
	68088	15266	48595	21663	12393	77666	38170	19070	41862	22868
	53718	412(24)								
(b)	.....	.....	66995	11975	27660	30938	09302	51266	37651	02972
	47760	03233	34506	66330	41815	96327	61997	38784	39560	96639
	31126	113(14)								
(c)	.....	.....	35385	21705	44797	37589	88930	29687	78069	65707
	60943	18995	57207	68639	53447	83160	99985	33336	38876	42719
	95279	798(75)								

D. F. Ferguson & John W. Wrench, Jr.
Univ. of Manchester, England
4711 Davenport St., N. W., Washington 16, D. C.

RECENT MATHEMATICAL TABLES

450[A, B, C, D].—Donald V. Mitchell, (a) *Six-Place Tables for Precision Computing to accompany* [b] *Streamlined Methods of Computing with Slide Rule and Mathematical Tables* (a) 1945, 47 p., stiff paper cover \$1.00; [b] revised and greatly enlarged ed., 1947, 80 p., stiff paper cover \$1.00, Seattle, Washington, Craftsman Press. 13.5 × 20.5 cm. Procurable from the author, 12345 Sand Point Way, Seattle 55, Wash.

The tables are: Log N, $N = 1000$–9999, with Δ; the six natural trigonometric functions and their logarithms for each tenth of a degree, with c.d.; decimal equivalents of common fractions; N^2, N^3, $N^{1/2}$[4D], $N^{1/3}$[4D], $N = 1(1)1000$. The "Slide Rule Topics" in [b] occupy p. 37–58.

451[A, D].—Friedrich Schulze, *Hilfstafeln für die Schrägmessung mit 5 m-Latten und mit dem 20 m. und dem 10 m-Stahlband.* Liebenwerda, Verlag von R. Reiss, 1941 (?) 23 p. 14 × 16 cm. Limp cover. 50 pfennige.

T. 1, arctan $(h/5)$, $h = .1(.01)2.09$, to the nearest 0′.1.
T. 2–3, $Z = (25 - h^2)^{1/2} - 5000$, and $0.2Z$, $h = .1(.01)2.09$, h in m. and Z in mm.
T. 4–5, $r = 5000 - 25/(25 + h^2)^{1/2}$ and $0.2r$, $h = .1(.01)2.09$.
T. 6–7, $Z = 20(\sec \alpha - 1)$ and $0.05Z$, $\alpha = 0(0°.1)24°.9$.
T. 8–9, $r = 20(1 - \cos \alpha)$ and $0.05r$, $\alpha = 0(0°.1)24°.9$.

T. 10–11, $Z = 10(\sec\alpha - 1)$ and $0.1Z$, $\alpha = [24°(0°.1)48°.9$; mostly 3S].
T. 12–13, $r = 10(1 - \cos\alpha)$ and $0.1r$, $\alpha = [24°(0°.1)48°.9$; mostly 3S].
T. 14, $r = Z - Z^2/(5000 + Z)$, Z in mm. $= 0(2)408$.
T. 15, $r = Z - Z^2/(2000 + Z)$, Z in cm. $= 0(1)209$.
T. 16, $r = Z - Z^2/(1000 + Z)$, Z in cm. $= 80(1)349$.

Errata: T. 1, $h = 0.42$, *for* 49.0, *read* 48.1; T. 10, $\alpha = 48°.7$, *for* 575, *read* 515.

452[C, D].—OTTO MÜLLER & MICHELE RAJNA, *Tavole di Logaritmi con Cinque Decimali. Ventinovesima edizione riveduta per cura di* LUIGI GABBA. (*Manuali Hoepli*). Milan, Hoepli, 1940. xxxii, ii, 203 p. 10.2 × 15 cm.

The tables in this volume are as follows: **T. 1,** p. 1–33, log N, $N = 1(1)9999$ with Δ. From 1000 to 9990 the corresponding number of degrees, minutes and seconds are given, 16′40″ to 2°46′30″. Values of S and T are also given 1000″(100″)9900″. **T. 2,** p. 35–125, log sin, log tan, log cot, log cos at interval 1′ with Δ and P. P. **T. 3,** p. 139–187, Addition and Subtraction logarithms, A, $-D = 0(.001)2(.01)4(.1)5$, $I\Delta$, *fPPd*; S, $-D = 0(.00001).051(.0001).4(.001)1.8(.01)4(.1)5$, *PPd*. On p. 33–34 are various constants, mainly functions of π and M; on p. 126–127, lengths of arc of unit circle for each degree, minute, and second; on p. 128 dimensions of the earth; on p. 129–137, 4D tables of the natural trigonometric functions sin, tan, cot, cos, at interval 10′; on p.138, value of gravity at different latitudes and elevations; on p. 188–191, conversion tables of degrees, minutes and seconds to time; on p. 192–194, of degrees and minutes to seconds; on p. 195, physical constants; and on p. 196–203, N^2, $N = 0(1)1000$.

This work was originally compiled by Müller, and first published by Hoepli at Milan in 1883; second ed. 1886, 143 p.; third ed. 1891, 142 p. The table of addition and subtraction logarithms was added "per cura Michele Rajna" (1854–1920), astronomer at the Royal Observatory in Milan, in the fourth ed. 1895 (xxxvi, 185 p.). The fifth ed. 1897, sixth ed., 1900 (xxxvi, 191 p.), seventh ed. 1903, eighth ed. 1905, ninth ed. 1906, tenth ed. 1908, eleventh ed. 1911, and the twelfth and thirteenth eds., 1915, and the fifteenth ed., 1922, were issued in similar form, and with the same number of pages. But with the twentieth revised ed., in 1926, "per cura LUIGI GABBA," also astronomer at the Milan Observatory, Müller & Rajna are listed as joint authors. In this edition, the twenty-fifth, 1932, the twenty-sixth, 1935, and in the present twenty-ninth ed., the number of pages is the same except for the leaf of 17 errata in **T. 2,** added in the last edition; but the first three of these were corrected. Four of these errors (p. 106, 114, 124, 125) were also in the twenty-fifth edition, but none of those four occurred in the fifteenth edition.

The following errata of the present volume were noted: p. 63, cos 13°55′, *for* 9.98906, *read* 9.98706; p. 100, cot 32°5′, *for* 0.20381, *read* 0.20281; cot 32°6′, *for* 0.20203, *read* 0.20253; cot 32°20′, *for* 0.19960, *read* 0.19860; p. 105, cos 34°54′, *for* 9.91386, *read* 9.91389; p. 192, col. 4, last l., *for* 210 000, *read* 216 000.

R. C. A. & S. A. J.

EDITORIAL NOTE: In the preparation of this review we were greatly assisted by H. E. MOSE, reference librarian of the John Crerar Library, Chicago. Müller is the author of two other tables, namely: *Tavole per la Determinazione del Tempo dietro le Altezze del Sole o di una Stella*. Milan, Hoepli, 1881, xxi, 34 p.; and *Hilfstafeln für praktische Messkunde nebst logarithmisch-trigonometrischen Tafeln*. Zürich, F. Schulthess, 1897, 144 p.

453[D].—ANT. PROKEŠ, *Tabured. Tabellen zur Reduktion von schräg gemessenen Entfernungen für zentesimale Kreisteilung.* Berlin-Grunewald, Wichmann, 1943, 53 p. 16.8 × 24.1 cm.

Let O be a point of observation of an object P whose projection on the horizontal plane is P'. If $OP = m$, $OP' = D$, angle $POP' = \alpha^g$, $D = m\cos\alpha = m - r$, where $r = m(1 - \cos\alpha)$. The table is of r, that is, multiples of versed sin α, for $m = 10(10)100$, $\alpha = [0(2^c)50^g; 3D]$, Δ. Random checking revealed an error in r, $\alpha = 49^g.26$, *for* 28.372, *read* 28.472. While there are numerous tables of versed sine for argument in degrees, minutes and seconds, and also tables with argument in radians and time, we recall only one other table with centesimal argument, namely the Chinese table by E. A. SLOSSE, to which we referred, *MTAC*, v. 1, p. 38: *Tables des Valeurs Naturelles des Expressions Trigonométriques. Division Centésimale* . . . 1923.

R. C. A.

454[D, Q].—D. K. KULIKOV, "Novyĭ metod obrabotki nabliûdeniĭ par Tsingera" [New method for the treatment of observations of Zinger star-pairs], Akad. N., SSSR, Institut Teoreticheskoĭ Astronomii, Biûlleten', v. 4, no. 2(55), March, 1947, p. 77–86. 19.3 × 25.1 cm.

There is a 3D table on p. 82 of $\sigma_0(n) = \frac{1}{8}n^3 \sin^2 1^s$ for $n = 300(10)610(5)800$. There are 24 entries for the first 299^s, such as $n = 0 - 82^s$, $\sigma_0(n) = .000$; $n = 82^s - 119^s$, $\sigma_0(n) = .001°$; $n = 119^s–141^s$, $\sigma_0(n) = .002°$, $\cdots$; $n = 295^s–299^s$, $\sigma_0(n) = .023°$.

455[E, M].—ADRIAAN VAN WIJNGAARDEN, *Enige Toepassingen van Fourier-integralen op Elastische Problemen.* Diss. Delft Technische Hoogeschool. Delft, 1945. 125 p. 15.5 × 24.5 cm.

T. VII. $\int_0^\infty t^p \cosh^n \pi t dt/\sinh^m \pi t$, 31 exact values, $n = 0(1)3$, $m = 1(1)6$, $p = 1(1)6$.

T. VIII. $\sum_{k=N}^{\infty} (2k + 1)^{-n}$, $N = 0(1)10$, $n = [2(1)6; 5D]$.

T. IX. $\int_0^\infty t \sinh pt \coth qt dt/\sinh t$, $q = .1(.1)1$, $p = [0(.1).9$; mostly 4D].

T. X. $\sum_{k=N}^{\infty} k^{-n}$, $N = 1(1)10$, $n = [2(1)6; 5D]$.

T. XI. $\int_0^\infty t \sinh^2 pt \coth qt dt/\sinh^2 t$, $q = .1(.1)1.8$, $p = [0(.1).9$; mostly 4D].

T. XII. $\int_0^\infty t^2 \coth rt dt/\sinh t$, $r = [.1(.1)1; 4D]$.

T. XIII. $\int_0^\infty t^3 \coth rt dt/\sinh^2 t$, $r = [.1(.1)1; 4D]$.

456[F].—A. GLODEN, "Factorisation de Nombres de la forme $16x^8 + 1$," *Euclides*, Madrid, v. 7, 1947, p. 95. 16.6 × 24.1 cm.

This note gives 40 factorizations of the function $16x^8 + 1$ for values of x between 23 and 94 inclusive. Seven of these are incomplete, the missing prime factors being in excess of $5 \cdot 10^6$. Twelve of the complete factorizations had been given previously by the author (see *MTAC*, v. 2, p. 300) in a more extensive table of the function $x^4 + 1$. The present tables are based on the inversion of tables of solutions of the quartic congruence $y^4 + 1 \equiv 0$ (mod p), *MTAC*, v. 2, p. 71–72, 210–211, 252, 300.

D. H. L.

457[F].—ALBERT GLODEN, *Liste des formes linéaires des nombres dont le carré se termine dans le système décimal par une tranche donnée de 4 chiffres.* Luxembourg, author, rue Jean Jaurès 11, 1947. 15 leaves mimeographed. 21.1 × 29.9 cm.

This interesting table gives not only all four digit endings of squares but also, for each ending, the sort of numbers whose squares have that particular ending. Tables giving merely the endings, arranged in increasing sequence, have been given before by KULIK,[1] SCHADY,[2] and THÉBAULT.[3] There are precisely 1044 such endings of four digits. The numbers whose squares have these endings are described by linear forms $mn \pm a (n = 0, \pm 1, \pm 2 \cdots)$. Thus we have the entry

$$1801 \qquad 5000n \pm 349 \qquad 5000n \pm 901$$

indicating that numbers congruent to ± 349 and ± 901 modulo 5000 and no others have squares ending in 1801. Of the 1044 possible endings 500 have $m = 5000$, as above; 500 have $m = 2500$; 20 have $m = 1000$; 20 have $m = 500$; two have $m = 200$; and two have $m = 100$. The table is useful not only in recognizing non-squares by their endings but also in setting up excluding programs by means of quadratic congruences.

D. H. L.

[1] J. P. KULIK, *Tafeln der Quadrat- und Kubik-Zahlen aller natürlichen Zahlen bis Hundert Tausend, nebst ihrer Anwendung auf die Zerlegung grosser Zahlen in ihre Factoren.* Leipzig, 1848.

[2] SCHADY, "Tafeln für die dekadischen Endformen der Quadratzahlen," *Jn. f. d. reine u. angew. Math.*, v. 84, 1878, p. 85–88.

[3] V. THÉBAULT, "Sur les carrés parfaits," *Mathesis*, v. 48, Oct. 1934 Suppl., 22 p. See also *MTAC*, v. 2, p. 72.

458[F].—H. GUPTA, "A table of values of $N_3(t)$," Nat. Inst. Sciences, India, Calcutta, *Proc.*, v. 13, 1947, p. 35–63.

Let $n_2(k)$ and $n_3(k)$ stand for the number of solutions in non-negative integers of the respective equations

$$x^2 + y^2 = k, \qquad x^2 + y^2 + z^2 = k,$$

and let their sums be denoted by

$$N_2(t) = \sum_{k=0}^{t} n_2(k), \qquad N_3(t) = \sum_{k=0}^{t} n_3(k).$$

The main table of this paper gives (p. 39–63) $n_3(t)$ for $t \leq 10^4$. The values of the argument t are classified modulo 8 into seven columns (since $n_3(8m + 7) = 0$ this class of values is omitted). In the right margin the values of $N_3(t)$ for $t = 8m + 7$ are accumulated. Other values of this function may be found quickly by adding or subtracting the appropriate values of $n_3(k)$. This table was built up from a similar manuscript table of the function $n_2(k)$ by means of the formula

$$n_3(t) = \sum_{m=0}^{\infty} n_2(t - m^2).$$

It has been checked by means of the class number relation

$$n_3(8i + 3) = \sum h'[(32i + 12)/d^2],$$

where the sum extends over the square divisors d^2 of $8i + 3$ and where $h'(D)$ denotes the number of properly primitive classes of binary quadratic forms $ax^2 + bxy + cy^2$ of negative determinant $-D = b^2 - 4ac$. The necessary values of h' were taken from the author's table[1] (part I). Conversely, the present table may be used to find isolated values of the class number function.

There is also a small table (p. 37–38) connecting the results of the main table with the number of lattice points inside circles and spheres. Let $C(r)$ and $S(r)$ denote respectively the number of points with integer coordinates inside or on the circle and sphere of radius r (with center at the origin). If points on these loci are given the same weight as interior points then

$$C(r) = 4N_2(r^2) - 4r - 3, \qquad S(r) = 8N_3(r^2) - 12N_2(r^2) + 6r + 5.$$

The table gives for $r = 1(1)100$ the values of

$$N_2(r^2), \qquad C(r), \qquad N_3(r^2), \qquad S(r),$$

and also, for comparison with $S(r)$, the nearest integer V to the volume of the sphere of radius r. At $r = 100$ we find

$$S(100) = 41\,87857, \qquad V(100) = 41\,88790 = 1.000223S(100).$$

D. H. L.

[1] H. GUPTA, "On the class-numbers of binary quadratic forms," Tucuman, Argentina, Universidad, *Revista*, S. A., *Matem. y Fisica Teorica*, v. 3, 1942, p. 283–299. See *MTAC*, v. 1, p. 180–182.

459[F].—D. B. LAHIRI, "On Ramanujan's function $\tau(n)$ and the divisor function $\sigma_k(n)$.—I," Calcutta Math. Soc., *Bull.* v. 38, Dec. 1946, p. 193–206. 18.7 × 24.2 cm.

In this first part of the paper is found tabular material relating to the function $\sigma_k(n)$ only. This function is the sum of the k-th powers of the divisors of n. The main results of the paper are incorporated in a table giving 89 congruence relations, with respect to various moduli, between the functions $\sigma_k(n)$. One of these, for example, is

$$11\sigma_9(n) \equiv 10(3n - 2)\sigma_7(n) + \sigma(n) \pmod{480}.$$

These are derived from tables of relations between the power series

$$\Phi_{r,s}(x) = \sum_{n=1}^{\infty} n^r\sigma_{s-r}(n)x^n$$

which are extensions of similar tables of RAMANUJAN.[1] Parts II and III are to contain results on Ramanujan's function $\tau(n)$.

D. H. L.

[1] S. RAMANUJAN, Camb. Phil. Soc., *Trans.*, v. 22, 1916, p. 159–184; *Coll. Papers*, Cambridge, 1927, p. 136–162.

460[F].—KURT MAHLER, "Lattice points in two-dimensional star domains (III)," London Math. Soc., *Proc.*, s. 2, v. 49, p. 168–183, 1946. 17.3 × 25.2 cm.

This paper, presented to the Society in 1942, contains two small tables (p. 178, 181) giving data on lattice points in a domain bounded by two concentric ellipses each of area π. These results are illustrations of the author's general method[1] and are too special to merit a detailed explanation here.

D. H. L.

[1] K. MAHLER, "Lattice points in two-dimensional star domains (I)," London Math. Soc., *Proc.*, s. 2, v. 49, p. 128–157, 1946.

461[F].—HARRY C. ROBERT, JR., "Prime-factor table, numbers $6n \pm 1$, base XII," *Duodecimal Bulletin*, v. 3, no. 2, June 1947, p. (14–17, i.e.) 16–19. 13.7 × 21 cm.

This table gives the factorization into primes, or indicates the primality, of all integers prime to 6 and less than 5184 ($= 3 \cdot 12^3$). The arrangement is such that numbers in the same column but in adjacent lines differ by 144. Thus the table has 48 columns headed by the numbers prime to 6 and less than 144. These headings are simply the last two digits of numbers $6n \pm 1$ when written to the base 12. All numbers are written duodecimally.

D. H. L.

462[F].—ARNOLD WALFISZ, "On the additive theory of numbers. X.," Akad. N., SSSR, Gruzinskiĭ Filial, Matem. Inst., *Trudy*, Tiflis, v. 11, 1942, p. 173–186.

This paper dealing with the universality of the two cubic forms $ax_1^3 + x_2^3 + \cdots + x_n^3$, $a = 1, 20$, contains two tables which serve to show that all positive integers n for which $5745 \leq n \leq 11000$ or $12000 \leq n \leq 62000$, except for 5818 and 8042, are sums of not more than 6 positive cubes. Such results for much more extensive ranges have been obtained by von Sterneck[1] and Dickson.[2] The point of the present paper is to show that more extensive tabular evidence is unnecessary.

D. H. L.

[1] R. D. VON STERNECK, "Über die kleinste Anzahl Kuben, aus welchen jede Zahl bis 40 000 zusammengesetzt werden kann," Akad. d. Wiss., Vienna, math-natw. Kl., *Sitzb.*, v. 112, section 2a, 1903, p. 1627–1666.

[2] L. E. DICKSON, Manuscripts in the University of Chicago Library: (a) Table of the minimum number of cubes required to represent each integer from 40 000 to 270 000; (b) Table of sums of four cubes from 270 000 to 560 000.

463[G, K].—(**a**) M. ZIAUD-DIN, "Tables of symmetric functions for statistical purposes," Nat. Acad. Sci., India, *Proc.*, v. 10, 1940, p. 53–60. (**b**) S. M. KERAWALA, "Table of monomial symmetric functions of weight 9," *ibid.*, v. 11, 1941, p. 51–55. (**c**) S. M. KERAWALA and A. R. HANAFI, "The table of symmetric functions of weight 10," *ibid.*, v. 11, p. 56–63. (**d**) S. M. KERAWALA and A. R. HANAFI, "Table of monomial symmetric functions of weight 11," *ibid.*, v. 12, 1942, p. 81–96.

All four tables give the coefficients in the linear representation of the monomial symmetric functions of weight m in terms of products of the sums of like powers. The values of m considered in (a) are 7, 8, and 9. The weights of the other tables are as indicated in their titles. As an example of a relation of weight 7 we find in Table 1 of (a) that

$$2 \sum \alpha_1^5\alpha_2\alpha_3 = S_1^2S_5 - 2S_1S_6 - S_2S_5 + 2S_7. \qquad S_i = \sum \alpha^i.$$

The reason for using the sums S_i as basis instead of the more usual[1] elementary symmetric functions $\sum \alpha_1\alpha_2 \cdots \alpha_i$ is because the S's occur as moments in statistical problems. There is another good reason not connected with statistics: isolated values of the S's are more easily computed than other symmetric functions. The three tables in (a) are so poorly printed that it is difficult to tell which entry lies at the intersection of a given row and column. Also the arrangement of the symmetric functions is not lexicographical so that the tables have an unkempt, jumbled appearance instead of the usual attractive triangular layout. The tables for $m = 1(1)8$ were published in 1938 by SUKHATME.[2] The new table of (a), for $m = 9$, contains a number of errors which are corrected by the republishing of the table in (b). Here, and in (c) and (d), the arrangement and printing are good. There are 5 errors in (c) which are noted in the introduction of (d). The reviewer has checked the table in (d) and finds only one error: In the expansion of

$$720 \sum \alpha_1^3\alpha_2^2\alpha_3\alpha_4\alpha_5\alpha_6\alpha_7\alpha_8$$

the coefficient of $S_1^3S_2S_6$ should be -240, not -24.

D. H. L.

[1] See for example F. FAÀ di BRUNO, *Théorie des Formes Binaires*, Turin, 1876, for weights ≤ 11, on the three folding plates; second ed 1883. German ed., Leipzig, 1881.

[2] P. V. SUKHATME, "On bipartitional functions,' R. Soc. London, *Phil. Trans.*, v. 273A, 1939, p. 399.

464[L].—NBSCL, *Table of the Bessel Functions $J_0(z)$ and $J_1(z)$ for Complex Arguments.* Second ed., New York, Columbia Univ. Press, 1947. xliv, 403 p. 20 × 26.5 cm. $7.50.

That an edition of 800 copies of this valuable work should have been exhausted in the past four years demonstrated clearly how great was the need for a Table of this kind. For a review of the first edition by Professor PHILIP M. MORSE, see *MTAC*, v. 1, p. 187–189. In the new edition, boosted in price by fifty percent, and in red rather than buff colored jacket, the tabular pages are unchanged, but the incorrect contour charts on p. xv, xvii of the first edition (see *MTAC*, v. 1, p. 326) are now corrected. Several minor changes have been made in the Introduction, p. xxi, xxx–xxxi, and the former Supplementary Note 2, "Expression for differences in terms of derivatives" has been dropped. One title (67) has been added (p. xliv) to the Bibliography (namely, *Guide to Tables of Bessel Functions*, *MTAC*, v. 1, p. 205–308).

When the Bibliography (66 titles, a number of them non-tabular) of the first edition was published in 1943 it served a very useful purpose, and slips and infelicities might have been, to a certain extent, condoned by workers in the field. But when in 1947 the NBS brings out a new edition repeating all the old untidiness, inaccuracies, and gross errors, such careless editing under such high auspices causes no little astonishment.

One may be precise in indicating clearly to what one thus refers. On p. xix and xl, there are incorrect references to HAYASHI's, instead of DINNIK's short tables of $J_0(z)$ and $J_1(z)$. The description of the contents of the BADELLINO entry in (8) is wholly wrong (the correct statement was given in *MTAC*, v. 1, 1944, p. 281). The description of the SMITH entry in (58) contains several errors: MICHELL, not "Mitchell," functions are in question; the definitions of the third and fourth of these functions are incorrect. It is stated in 1947 that items (8) Badellino, (48) MEISSEL, and (60) "Toelke" (the name is TÖLKE), had not been seen, yet in 1944 members of the NBSCL staff gave for publication material indicating errors in Tölke (*MTAC*, v. 1, p. 306); since 1944 any alert editor could have in America readily consulted the Badellino and Meissel items. The titles in (6) ANDING and (38) Hayashi need repair. In (9) BAASMTC, line 9, the entry should be "$A_i(x)$, $B_i(x)$ used to calculate $J_i(x)$, $Y_i(x)$ to 8D." If the statement in (31) Dwight "with tables reprinted from (14) and (17)" be interpreted as only tables previously published, exactly the same as the statement in (28) DWIGHT, it is incorrect. In the (45) MASCART & JOUBERT entry it should have been indicated that the second edition of the work was being considered and the reference to "KELVIN'S" table ought to have been to MACLEAN'S (see *MTAC*, v. 1, p. 297). Meissel's table (47) was published in 1889, not 1888. Even the five page numbers xL–xLiv are not corrected.

But more remains to be set forth. Whatever reason may in 1943 have led to the insertion of (64) BRASEY, (65) TOPPING & LUDLAM, (66) BICKLEY & NAYLER out of alphabetical order, and 12 entries anonymously under the heading B.A.A.S. when all of the authors are well known (AIREY, ALGER, LODGE, SAVIDGE, WEBSTER), there was no way in 1947 for the meticulous editor to avoid putting all of these items with the others in one alphabet. He would also naturally in items of his list have noted new editions and given references to important published lists of errata; this would, for example, have included the 1945 edition of JAHNKE & EMDE, where elaborate corrections were made in at least one Bessel Function table. The insertion in the Bibliography of a selected list of important tables published during 1943–1947, would obviously have rounded out the revision, where of course all possibilities for improvement have not been here suggested.

This will suffice by way of illustration of strictures made above. We have gone into the matter at some length in the hope, that as the NBS brings out new volumes, and later new editions, of its mathematical tables, much greater care may be taken in their preparation so that they may be immediately recognizable as finely edited up-to-date contributions to scholarship. The tables themselves have long been of this character. Our plea is that everything else in such volumes may ever be on an equally high plane of achievement.

R. C. A.

465[L].—NBSCL, *Tables of Spherical Bessel Functions*, volume 2. New York, Columbia University Press, 1947, xx, 328 p., 20 × 26.4 cm. $7.50. V. 1 reviewed by Prof. W. G. BICKLEY, *MTAC*, v. 2, p. 308–309.

The table of $F = (\frac{1}{2}\pi/x)^{\frac{1}{2}}J_\nu(x)$ is continued for $\pm 2\nu = 29(2)43$, for $x = 0(.01)10(.1)25$, and $\pm 2\nu = 45(2)61$, for $x = 10(.1)25$. The entries are given with δ^2 or modified δ^2, and sometimes δ^4, to 8S, 9S, or 10S for $x \leq 10$, and to 7S for $x > 10$, except for a few entries very close to the zeros of the function. For $2|\nu| = 43$ interpolation for F is laborious when $x < 5$ and the interval in x is .01. This difficulty increases with increasing $|\nu|$, so that at $2|\nu| = 61$, very limited accuracy could be obtained by a four-point interpolation formula when $x < 10$. For this reason the functions F have not been tabulated for $x < 10$, $2|\nu| > 43$. However, the closely related functions $\Lambda_\nu(x)$ are tabulated. $F = \pi^{\frac{1}{2}}[2^{\nu+\frac{1}{2}}\Gamma(\nu+1)]^{-1}x^{\nu-\frac{1}{2}}\Lambda_\nu(x)$, or $\Lambda_\nu(x) = 2^\nu\Gamma(\nu+1)J_\nu(x)/x^\nu$. Λ_ν is tabulated, with δ^2 (sometimes modified) and δ^4 (partly), for $x = 0(.1)10$, $2\nu = [1(1)41(2)61; 9D]$, and $x = 10(.1)25$, $2\nu = [1(2)61; 7S$ mostly$]$; also for negative values of ν in regions where F does not difference well, $-2\nu = 29(2)33$, $x = [0(.1)9.5(.05)10(.1)25; 7S$ mostly$]$; $-2\nu = 35(2)61$, $x = [0(.1)25; 7S$ mostly$]$. $\Lambda_n(x)$ is smoother than F and interpolation for $\Lambda_\nu(x)$ in the ν direction is good over a large portion of the $x - \nu$ plane when ν is positive. For these reasons available values of $\Lambda_\nu(x)$ have been included for integral[1] as well as half-integral values of ν, for ν positive and x less than 10. $\Lambda_n(x)$ becomes infinite with $\Gamma(\nu+1)$ for negative integral values of ν; hence interpolation in the ν direction is not feasible when ν is negative and less than -1. For this reason, $\Lambda_n(x)$ is published, for negative ν, only for half-integral values of $\nu < -29/2$, mainly for purposes of interpolation in the x direction in regions where interpolation in the functions F is difficult. There are tables of interpolating coefficients $E_2 = p(1-p^2)/6$, $F_2 = q(1-q^2)/6$, $p + q = 1$, $E_4 = p(1-p^2)(4-p^2)/5!$, $F_4 = E_4(q)$, $G_4 = [p(1-p^2)(4-p^2)/120] - [0.18393p(1-p^2)/6]$, $H_4 = G_4(q)$ at interval .001 in p. Also, tables of $C_\nu = 2^n n!/(2n+1)!$, $2\nu = 2n+1$ $= [1(2)61; 9D]$; and of $C_\nu = (-1)^n(2n)!/2^n n!$, $2\nu = -2n - 1 = [-61(2) -1; 9D]$.

The tables of zeros of $J_\nu(x)$ and of $J_\nu'(x)$ with corresponding values of $J_\nu'(j_{\nu,s})$, and $J_\nu(j'_{\nu,s})$. The values are given 6–10D, for $\pm 2\nu = 1(2)13$, and $s = 1(1)5$ or 6 or 7 or 8; for $\pm 2\nu = 15(2)39$, $s = 1(1)5$ or 4, or 3 or 2, or merely 1. Most of the entries are correct to within a unit in the last tabulated place; those given to at least eleven significant figures are correct to within two units in the last tabulated place.

Extracts from text

[1] Tables of $\Lambda_\nu(x)$ for integral values of ν were published by NBSCL in "Tables of $f_n(x)\cdots$", *Jn. Math. Phys.*, v. 23, 1944, p. 45–60. See *MTAC*, v. 1, p. 363–364.

EDITORIAL NOTE: That the Columbia University Press should have printed the gilt title and volume number of this volume more than an inch higher than the corresponding title and number of volume one is inexcusable.

466[L].—ANDERS REIZ, "On the numerical solution of certain types of integral equations," *Arkiv för Matem., Astr. och Fysik*, v. 29A, no. 29, 1943, 21 p. 13.8 × 21.7 cm.

On pages 6 and 12 are two 7D tables, of zeros (x_i), corresponding CHRISTOFFEL numbers (p_i), and $\alpha_i = \pi^{\frac{1}{2}}p_ie^{x_i^2}$, for T. 1, HERMITE polynomials[1] $H_n(x)$, $n = 2(1)9$; T. 5, LAGUERRE polynomials, $L_n(x)$, $n = 2(1)5$. $H_n(x) = (-1)^n e^{x^2}d^n e^{-x^2}/dx^n$ is here defined as by Hermite, and not as by E. R. SMITH and others (see *MTAC*, v. 1, p. 152–153). In T. 5, the values of zeros of L_5 and the corresponding p_i were previously given by N. S. KOSHLÎAKOV in 1933 (see *MTAC*, v. 1, p. 361, and MTE **121**). For the later much more complete table for zeros of $L_n(x)$ to 8D, and of Christoffel numbers to 8S, $n = 1(1)10$, in SRE/ACS 82, see *MTAC*, v. 2, p. 31. From this table it appears that in Reiz's T. 5, for $n = 5$, the final digits in the second, fourth, and fifth zeros should be respectively 1, 0, 8, instead of 0, 2, 7.

R. C. A.

[1] REIZ refers to A. BERGER, "Sur l'évaluation approchée des intégrales définies simples," K. Vetenskaps Societeten i Upsala, *Nova Acta*, p. 3, v. 16, no. 4, 1893, and states (p. 6) that "Berger has given numerical values for the x's and p's, for $n = 2, 3, 4$," on p. 50 of his paper. Since there are no such values on this page, presumably those given in formulae on p. 52 were meant. So far as I know at present Reiz's table is the first one of $H_n(x)$, in decimal form, which has appeared.

467[L, S, T].—G. W. KING, *Punched-Card Methods in Analyzing Infra-Red Spectra*. **a.** *Progress Report*, April 1–May 31, 1947, issued June, 1947. iii, 44 p. + 3 folding plates. **b.** *Progress Report*, June 1–July 31, 1947, issued August, 1947. iii, 14 p. + 2 folding plates. Each 24 × 29 cm. Cambridge, Mass., Arthur D. Little, Inc.

These are two consecutive bimonthly progress reports on work being done under Office of Naval Research contract. The text is produced from typescript and the distribution list is extremely limited, but some of the work (including, it seems and is to be hoped, the main mathematical table) has been submitted to the *Journal of Chemical Physics*. The two reports will be referred to below as the June and August reports. The mathematical theory is contained in the June report, but the August report gives an improved and enlarged version of Table I and an additional Table III.

The work is concerned with the analysis of infra-red band spectra and microwave absorption lines of triatomic molecules of the type of H_2S and its heavier forms HDS and D_2S; consideration is being given to the cases in which ordinary S is replaced by O or Se, or a less common isotope of S or O. These molecules, not being linear, have no axis about which they are *three-dimensionally* symmetrical, and although they are not completely rigid it has been found of value to know the rotational energy levels of the asymmetric rotor (freely rotating asymmetric rigid body). It is not necessary to work with three general values a, b, c for the reciprocal principal moments of inertia, since it has been shown by RAY,[1] as a result in matrix mechanics, that the energy is easily derived from a "reduced energy" $E(\kappa)$, where, assuming $a \geq b \geq c$, κ is a parameter of asymmetry defined by $(2b - a - c)/(a - c)$, so that $-1 \leq \kappa \leq 1$; oblate and prolate symmetry correspond to $\kappa = 1$ and $\kappa = -1$ respectively, and $\kappa = 0$ has been called the "most asymmetrical" case. The quantity $E(\kappa)$ can be formally considered as the energy of a hypothetical rotor with reciprocal moments $1, \kappa, -1$.

The values of $E(\kappa)$ have already been computed by the New (Matrix) Quantum Mechanics for $\kappa = -1(.1)1$ up to $J = 12$, where J is the quantum number associated with the total angular momentum. The values up to $J = 10$ have already been published by KING, HAINER & CROSS,[2] and it is stated that the values for $J = 11$ and 12 will be published shortly.

The present work aims at extending the calculation of $E(\kappa)$ to high values of J (up to about $J = 50$) by application of the Correspondence Principle. A diagram (Fig. 2) in the June report illustrates the relationship of the Correspondence Principle and New Quantum Theory levels for $J = 3$. For high J the C.P. values may be expected, in some regions of such a diagram, to lead to utilizable numerical information about the desired N.Q.T. values.

The C.P. levels are obtained from the Newtonian solution by subjecting the values of the usual cyclic integrals to quantum restrictions. It is well known that the Newtonian solution for an asymmetric rotor involves an elliptic integral of the third kind. Fortunately, only the complete integral (limits of integration 0 and $\frac{1}{2}\pi$) is required in the quantization, and this has been tabulated by HEUMAN[3] from a formula of LEGENDRE. The quantum problem requires a rather complicated inversion of the table, and it was found that this and other features rendered impracticable the use of punched-card methods with IBM equipment, which it was desired to use on account of the very large number of cases to be computed. Consequently it was decided to invert Heuman's table into the form required, so that direct interpolations could then be performed by machinery. It may prevent misconception to say that no punched-card technique is discussed in the particular pair of reports under review.

A few formulae will give some idea of the inversion which has to be made. A "reduced-energy ratio" is defined by $\eta = E(\kappa)_{J,K}/J(J + 1)$, and a "quantum-number ratio" by $\lambda = K/[J(J + 1)]^{\frac{1}{2}}$. Here J and K are quantum numbers, and $K = 0(1)J$, so that $0 \leq \lambda < 1$. The problem is to find η for given κ, J, K. There are different cases according to the values of κ and η. It has been found best to derive explicit formulae for the case $\eta \leq \kappa$. The result of the analysis is that

$$\lambda = \frac{2}{\pi}\int_0^{\frac{1}{2}\pi} \frac{\cos^2\alpha \sin\beta \cos\beta \sqrt{(1 - \cos^2\alpha \sin^2\beta)}}{\cos^2\alpha \cos^2\beta + \sin^2\alpha \cos^2\phi} \frac{d\phi}{\sqrt{(1 - \sin^2\alpha \sin^2\phi)}},$$

which is exactly the function tabulated as $\Lambda_0(\alpha, \beta)$ by Heuman,[3] provided that

$$\eta = \cos 2\beta, \qquad \kappa = \frac{1 - \sin^2\alpha \tan^2\beta}{1 + \sin^2\alpha \tan^2\beta}.$$

Heuman's table has to be inverted so as to give η as a function of κ and λ.

An interval .1 was chosen for κ and λ. Since the undersigned, working with the above formulae and Heuman's tables, at first failed entirely to reproduce a few tabulated values of η selected for test, and since it is because of the numerical tables that this review appears, it is believed that a description of the tables may usefully be prefaced by the following detailed considerations. Let us suppose the arguments κ to run horizontally along the top of the table, reading $-1(.1)1$ from left to right. Let us also suppose arguments λ to run vertically along the *right* side of the table (for reasons to appear), reading $0(.1)1$ from top to bottom. For each κ, a real solution of the above equations exists (η is unique and satisfies $\eta \leq \kappa$) only when λ is not less than a certain value (depending on κ). Thus from the above equations we can calculate a triangular array of values of η lying below a zigzag dividing line (determined by the tabular intervals through which the curve $\eta = \kappa$ passes), running from the bottom left corner to the top right corner. But for given λ, it is known that η is an odd function of κ, and the triangular table may be converted into a rectangular one, in which the values are antisymmetrical about the center of the array, by providing arguments λ running vertically along the *left* side, reading $0(.1)1$ from bottom to top, for use in reading values above the dividing line, and by inserting these upper values in accordance with $\eta(-\kappa) = -\eta(\kappa)$. The left and right argument scales apply respectively above and below the dividing line, across which the values of η, but not their second and higher derivatives, are continuous.

It is clearly sufficient to tabulate half the full rectangular array described above, and neater to tabulate the left or right halves of the rectangle than to tabulate the irregular triangles below or above the dividing line. Since the important paper of King, Hainer & Cross[2] employed the range $-1 \leq \kappa \leq 0$ (because in triatomic molecules κ, for reasons there given, is more often near the prolate value -1 than near the oblate value $+1$), Table I in the June report consists of the left half; it gives η to 6D for $\kappa = -1(.1)0$; a scale of $\lambda = 0(.1)1$, reading upwards, is given on the left, and no λ-scale appears on the right. Thus anyone wishing to use the above formulae, which as they stand apply only below the dividing line, to check a value in the table should remember (a) below the dividing line, to use 1 minus the indicated λ, (b) above the dividing line, to reverse the signs of both κ and η. Table I of the August report gives the same values, with two additions. In the first place, a right-hand scale of arguments λ is given, as far as is necessary for the half-table, for use below the dividing line; this simple little addition is very welcome, and makes caution (a) unnecessary. Secondly, values of $-\partial\eta/\partial\lambda$ and $\partial^2\eta/2\partial\lambda^2$, for use in interpolation, are given to 6D below every value of η (the signs are to be prefixed according to the instructions on the previous page).

There are two subsidiary tables. Table II (June) gives, for $\kappa = -1(.1)1$, coefficients in the expansion of η in powers of $1 - \lambda$, up to the coefficient of $(1 - \lambda)^6$, to 8–9D. The coefficients of the first and second powers of $1 - \lambda$ agree numerically (to 6D) with the values of $-\partial\eta/\partial\lambda$ and $\partial^2\eta/2\partial\lambda^2$ given in the top and bottom rows of triplets in Table I (August). The values in Table II were obtained by expanding an algebraic form of the integral for λ round $\eta = -1$, and inverting series. Table III (August) gives information about maximum errors in interpolating Table I; the greatest errors occur near the dividing line ($\eta = \kappa$), and are unimportant because in this region the Correspondence Principle completely ceases to be useful.

It is pleasant to see Heuman's fine table occupying such a vital place in an important quantitative investigation. It is impossible to say whether King's somewhat special inversion of it will find any application other than that for which it was made. Certainly, like the tables of King, Hainer & Cross, it is a purely mathematical table arising in a context which will interest many applied mathematicians, and it appears from the reports to have rendered good service in the analysis of several band spectra. A few misprints in mathematical formulae will doubtless disappear in the printed version.

ALAN FLETCHER

Department of Applied Mathematics
University of Liverpool

[1] B. S. RAY, "Über die Eigenwerte des asymmetrischen Kreisels," *Zeits. Physik*, v. 78, 1932, p. 74–91.
[2] G. W. KING, R. M. HAINER & PAUL C. CROSS, "The asymmetric rotor. I. Calculation and symmetry classification of energy levels," *Jn. Chem. Phys.*, v. 11, 1943, p. 27–42.
[3] C. HEUMAN, "Tables of complete elliptic integrals," *Jn. Math. Phys.*, v. 20, 1941, p. 127–206, 336.

468[M, P].—J. G. FREEMAN, "Mathematical theory of deflection of beam," *Phil. Mag.*, s. 7, v. 37, Dec. 1946 (publ. July 1947), p. 855–862. 17 × 25 cm.

The five 2D tables on p. 858 are, for $\alpha = 0(5°)180°$, of

$$P(\alpha) = \int_0^\alpha \sin^{\frac{1}{2}} t dt = 2\int_{\frac{1}{4}\pi-\frac{1}{2}\alpha}^{\frac{1}{4}\pi} (1 - 2\sin^2 u)^{\frac{1}{2}} du,$$

$$Q(\alpha) = 2\cos\alpha \sin^{\frac{1}{2}}\alpha + \sin\alpha P(\alpha),\ S(\alpha) = 2\sin\alpha\sin^{\frac{1}{2}}\alpha - \cos\alpha P(\alpha),$$

$S(\alpha)/Q(\alpha)$, and $Q^2 \cos\alpha$. In the last two tables the values are also given for $\alpha = 171°, 172°, 176°, 177°$.

469[N].—JOH. HAGE, *Bond Value Tables*. The Hague, Nijhoff, 1946. xii, 242 p. + folding plate. 18.7 × 26.7 cm. 20 gulden, clothbound.

This book is an edition in English of a set of tables published in the Dutch language, copyrighted in 1939. As stated in the introductory general notes, the main problems to be treated by these tables are of two types: A. Computation of the present value (hereafter called the value) of a bond or loan at a given yield rate, B. Computation of the yield rate for a given value. Four types of loans are considered by the four main tables: I. Irredeemable loans, p. 1–2; II. Loans redeemed in a single payment, p. 3–67; III. Loans redeemed in a series of equal payments, p. 69–133; IV. Loans redeemed by annuities, p. 135–199. The presence of Tables III and IV is a distinguishing characteristic of this book and is sufficient to justify its usefulness.

These four tables have the following form. The present value at yield rate of the loan occurs in the body of the table on the basis of 100, being entered to 2D. The coupon rate, called the nominal rate and quoted on a yearly basis compounded semiannually, is entered across the top, ranging from $2\frac{1}{2}\%$ (3% T. II–III) to 7% at $\frac{1}{2}\%$ intervals. The yield rate, compounded annually, is listed down both sides of the page, ranging from 2% to 8% at intervals of $\frac{1}{8}\%$. Also on the right hand side of each of Tables II, III are two additional columns. One, headed PP, gives to 3D, the differences in value for a 1% change in the coupon rate for each yield rate listed. The second column, headed C, in T. II–III, gives for each yield rate listed an entry for computing the reduction in value caused by a 1% coupon tax. This column, as explained in the illustrative examples, is also convenient to use for computations involving loans redeemed at a premium. This latter use would seem to be the only reason for the presence of the C column in the present edition. In Tables II, III, IV each page contains the entries for a specified year of maturity of a loan, and the time interval is 1(1)60(5)80 years.

In addition to the four main tables there are three others. Table IVa, p. 200, provides the entries for calculation of the reduction in value due to a coupon tax on an annuity loan, supplanting column C of Tables II, and III. Table V, p. 201–202, gives entries for computing corresponding rates of interest. Table VI, folding plate, provides factors for the computation of accrued interest between dividend dates.

Throughout the book the typography is excellent. The table entries are legible and well-spaced. Table VI is folded in such a way that it may be drawn out for convenient reference while the other tables are in use.

A comprehensive set of examples is given on the use of the tables for problems in which the solution is not read directly from the tables. These include for each of the four cases the following computations: of value if the yield rate is not tabulated, of value if the coupon rate is not tabulated, of yield rate with coupon tax considered, of yield rate for redemption at a premium (not for Table I), of yield rate if value is not tabulated, of value at intermediate dates, of yield rate for fractional time intervals, and of parity value for two loans. Two general types of problems are illustrated in the computation of value for deferred amortization and of value for loans with irregular amortization.

A final section of the text gives a brief description of the formulae used in the computation of the table entries. It is stated here that Tables I to IV have been calculated twice and in particular Tables II and III by two different formulae. Furthermore all four tables have been checked by a summation procedure. A similar check was also used to verify the proof sheets from the manuscript. This would seem to guarantee a very high degree of accuracy for the table entries.

An objection to the form of the tables might be in the use of yield rates on an annual basis rather than a semiannual basis as is customary. Also it would have been entirely feasible and an added convenience to have at least one of the columns of yield rates in decimal form for use in interpolation. A few minor typographical errors in the textual material may be noted; for example: on p. vii, last line, *for* 250, *read* 205; on p. 206, 1.–4, the spacing is omitted in "coupon dates"; p. 214, 1.–2, an extra *t* is inserted; p. 239, 1.–5, "an summation" should read "a summation." Some of the terms used, especially in the section describing the formulae for computation of the table entries, do not follow standard usage closely and make it a little difficult to follow the development. In particular two items are out of place in an edition for use in the United States. In the first place, since the coupon tax is not in use here, the emphasis on computations involving such a tax is not needed. In the second place, the fractional form given on page 242 for the computation of time between dates is based on European usage in which, for example, April 23, 1947, is written $19\frac{23}{\mathrm{IV}}47$, and it is not as familiar or convenient as the usual year, month, day form of subtraction for dates. The illustrative examples are clear and informative and do much to increase the possible range of usefulness of the book.

Of the four tables appearing in the book, the material of Table II is most available in other references. Such standard references are (a) Financial Publishing Co., *Monthly Bond Values*, second ed., Boston, 1941 (see *MTAC*, v. 1, p. 114–115), which contains entries to six decimal places for time intervals of one month and (b) D. C. Johnson, C. Stone, M. C. Cross, & E. A. Kircher, *Yields of Bonds and Stocks*, New York, Prentice Hall, 1923, which gives yield rates in the body of the table to three decimal places and values to the nearest half unit on the basis of 100. For example, for a 10 year 5% semi-annual bond to yield 6% semi-annually, the *Monthly Bond Values* shows a value of 92.561263. To find the value in the present tables it is first necessary to convert to an equivalent yearly rate of 6.090% and then to interpolate, obtaining 92.56. For amounts in excess of $1000 two decimal places would not be sufficient to give values to the nearest cent. In comparison with the second reference above, Table II of the present book would give more accurate values and equally accurate yield rates.

Tables III and IV are not duplicated in frequently used references and can be very useful for particular problems. Serial bonds fall naturally into the category treated by Table III. It is of interest to note that the annuity loans treated by Table IV are of the type wherein "the portion of the annuity relating to the repayment of capital is payable yearly and the portion relating to interest is payable half-yearly."

Donald W. Western

Brown University

470[Q].—JULIUS EBSEN, *Azimut-Tabellen enthaltend die wahren Richtungen der Sonne, des Mondes und anderer Gestirne deren Declination 29° Nord oder Süd nicht überschreitet für Intervalle von 10 Zeitminuten zwischen den Breitenparallelen von 0° bis 30° Nord oder Süd. (30° bis 72° Nord oder Süd.)* Eighth edition, Hamburg, Eckardt & Messtorff, 1940. Two v.: iii, 124 p. and v, 172 p. (numbered 120–291). 19.5 × 27.5 cm.

These two volumes are a reprint of a table which has gone through many editions. It will be noticed that the pagination is continuous, with an overlap of four pages to cover the repetition of latitude 30°. The fifth edition was published in 1914, the fourth in 1909, the third in 1903, and the original edition dates from 1899 (xiii, 291 p.). The tables are not listed in FMR, *Index* (but neither are H.O. 71 and 120) and do not seem to be widely known outside Germany.

The tables give azimuth, to 0°.1, for the three arguments: latitude 0(1°)72°, declination $\pm$ 0(1°)29° and hour angle $0^h00^m(10^m)8^h00^m$ or some smaller limit depending on the time of rising or setting. Azimuths are not tabulated when the body concerned is within about 13° of the zenith (here interpolation ceases to be linear, for the azimuth to have much meaning) or below the horizon. The limitation to within 8^h of the meridian is very odd and can only have been made to simplify the pagination scheme. At present four pages are devoted to each latitude, with 49 lines on each page covering 8 hours of hour angle; with 15 columns on each page, the first opening is devoted to declinations of the "same" name and the second to declinations of "opposite" name. At the top of each column, for declinations of 24° or less, are given the hour angle of rising and setting and the corresponding azimuths. One curious feature is that the hour angle of rising is measured from the antimeridian, whereas, in common with modern tables of this nature, the hour angle used as argument is measured from the meridian. This is undoubtedly a relic from earlier editions in which a double argument of hour angle "a.m. or p.m." was used.

Although the tables have certainly been reset since the earlier editions, the format, type and style remain the same. The printing, paper and binding are excellent, but the type used is not the most suitable for tables. Between the fifth and the eighth editions, the rules for the sense of the azimuth (printed on each page) have been reworded to accord with the new argument. The many pages of illustrations and examples in the fifth edition have been dropped in favour of one page of notation and examples, and one page of compass courses.

The tables cover a range which is already catered for by many other well-known tables; no detailed comparison has been made, but it is unlikely that there are many errors.

J. B. PARKER & D. H. SADLER

H. M. Nautical Almanac Office
Bath, England

471[Q].—GERMANY, DEUTSCHE SEEWARTE, *Azimutdiagramme für alle Breiten, Deklinationen und Stundenwinkel*, prepared for the Oberkommando der Kriegsmarine. Second edition, Hamburg, 1944, vii, 331 p. (some blank) + flap. 27.5 × 28.3 cm., bound.

As the title announces, this book of diagrams is designed to give azimuths of celestial bodies for all combinations of latitude, declination and hour angle. The diagrams take the form of declination curves drawn against a rectangular grid of hour angle and azimuth; separate diagrams are given for each degree of latitude up to 80° and thereafter for 82°, 84°, 86°, and 88°. The hour angle scale is such that the six hours arranged vertically on each page occupy 8.4 inches; with 1.4 inches to the hour, one minute is rather more than 0.02 inches and the scales on each side of the page are actually subdivided to minutes. The horizontal azimuth scale covers 90° on each page in 7.9 inches (exactly 20 cm.) giving nearly a tenth of an inch to a degree. The rectangular grid consists of pecked lines for every five minutes of hour angle (just greater than 0.1 inches) and every degree of azimuth; every half-hour and each 10° is drawn continuously, while the 5° lines are emphasized. Printed scale divisions are provided in clear type on both sides and on the top and bottom.

The declination curves, drawn for each degree, stand out prominently against this background; every fifth curve is labelled and is heavier than the others, with the 10° curves still more emphasized. The distance between successive declination curves varies considerably with azimuth and latitude; curves for individual degrees are dropped when the density becomes too high—for instance near hour angles of 0^h and 12^h.

The curves are bounded above by a pecked curve labelled "Höhe = 80°" (altitude = 80°) and below by a similar curve corresponding to rising or setting, i.e., to altitude 0°. The two sets of curves on either side of the zero declination curve are clearly marked as being of the same name or of opposite name to the latitude; all declinations from 90° "same name" to the limit 90° − latitude "opposite name" are given.

The upper bounding curve passes through the points 0^h, 0° and 0^h, 180°; the lower curve passes through the points 12^h, 0°; 6^h, 90°; 0^h, 180°. It is thus seen that the diagrams for each latitude can always be put on three pages, though in high latitudes two of these can be combined.

The preface suggests an accuracy of reading to 0°.1 or 0°.2, and this can doubtless be achieved with care in some areas. Interpolation for latitude is performed numerically, the variations for a degree of latitude being printed in red at frequent intervals on the declination curves; an auxiliary table (on the flap) caters for proportional parts up to a difference of 5°.7, the correction so formed being applied to the azimuth read from the diagram. Instructions are given on each page for converting diagram azimuths to true azimuths.

The diagrams essentially give the complete relationship between four consecutive parts of a spherical triangle, and thus can be used, within their obvious limitations, for the solutions of all problems involving the expression of one of the four parts in terms of the other three. Advantage is taken of this to apply the diagrams to the solution of a whole range of navigational problems, and elaborate rules for these are tabulated and exemplified. In particular, it is suggested that the diagrams can be used for determining altitude, after the azimuth has been found; to assist in this, and other problems, detachable auxiliary scales are provided for converting the azimuth scale to time, and the hour angle scale to arc.

The whole volume has been prepared with much care; the drawing and reproduction of the diagrams is in keeping with the finest German technical standards, while the paper and binding are excellent. Although a second edition, it is thought that the earlier, 1941, edition was confined to latitudes 54°–65°.

According to the preface, the values of the azimuth were taken from published tables for latitude and declinations less than 70° and computed thereafter.

What are the advantages of graphical presentation over tables, and how do these diagrams compare with H.O. 71 and 120, BURDWOOD, DAVIES, CUGLE, EBSEN and other well known azimuth tables? In the reviewers' opinion the relative advantages are nicely balanced: the diagrams are less bulky and interpolation is easier for hour angle and declination; the chances of error in using the tables are less especially in conditions of poor light. The diagrams are excellent of their kind, whereas none of the tables is first class; poorly produced diagrams would rapidly lose their advantage, even over poorly produced tables. In astronomical navigation both numerical and graphical methods must be used, and it is largely a matter of personal preference which method is used for the azimuth.

J. B. PARKER & D. H. SADLER

472[U].—GERMANY, REICHSLUFTFAHRTMINISTERIUM, *Höhentafeln nach Sternzeit für die Breiten 50° bis 56° N 1944–1945*. Berlin, 1944, 200 p. 19 × 26 cm. Not available.

These tables were designed for the reduction of star sights in the German Air Force. The altitudes and azimuths of twelve selected stars are given for four integral latitudes, N. 50°, N. 52°, N. 54°, and N. 56°, the argument being hours and minutes of sidereal time. The excellent principle of arrangement is similar to that first proposed by Commander C.

H. Hutchings, U.S.N., in 1942 and subsequently used experimentally in the *Experimental Astronomical Navigation Tables*[1] produced for the Royal Air Force in 1943. The same principle is used in *Star Tables for Air Navigation* (H.O. 249; see RMT **473**).

As in all German Air Force publications, sidereal time is here defined with reference to the transit of the first point of Aries across the anti-meridian; it thus differs by 12^h from sidereal time as normally defined. Discussion of the merits of this definition, which brings the measurement of sidereal time into line with mean time, is outside the scope of this review.

The tables are arranged in sections according to latitude, and there would appear no reason why more, or less, than four latitudes should not be bound together. Each opening contains the tabulations for an hour of sidereal time, at an interval of one minute; excellent thumb indexes for latitude and hours enable the appropriate opening to be found readily. At each opening are given the altitude, to the nearest minute, and the azimuth, to the nearest degree, of twelve stars, together with the "Pole Star Correction" and the azimuth of *Polaris* (to the nearest half degree). Six stars are assigned to each page, the *Polaris* data being given on the inside page margins; changes of stars are made only at integral hours.

Although the layout of the tables is good, various points indicate lack of experience of the presentation of tabular matter. The general appearance is marred by irritating details, such as the senseless repetition of degree and minute signs for each entry, the use of a vertical rule to indicate that the azimuth takes the same value for several consecutive entries, and too many horizontal and vertical rules; the 61 lines on each page are divided by heavy rules on each side of the multiples of 10° and by light rules on each side of the odd multiples of 5°. Modern style figures (without heads or tails) are used, to the undoubted detriment of easy legibility—of the first order of importance in the air. This feature is emphasized by the use of bright yellow paper (used also in the German Air Almanacs I and II) which does in fact slightly help legibility under conditions of poor illumination. The printing is good and the binding adequate. No errors have been found in a casual examination.

A statement on each page indicates that the tables are valid for the two years 1944 and 1945. If the star positions used were for the beginning of 1945, as is indicated by independent computation, the maximum error due to the combined effects of precession, nutation, and aberration will not exceed 1′ in the period concerned; the tables could be used for a further year without serious loss of accuracy. Since no annual corrections are given, it is clearly intended to reprint the table every two years. It is not known whether earlier or later editions exist.

The actual arrangement of the stars is curious. Each of the two groups of six is arranged alphabetically in order of star names, but the reason for allocating one star to the right-hand page, and another to the left, is not clear. On the whole, the "best" stars (from the point of view of altitude) appear to be given on the left-hand page, but this is not always the case. In some instances there is difficulty in selecting twelve suitable navigational stars, and some very low altitudes are tabulated: *Procyon* is listed for N. 50°, 1^h00–2^h00 S.T., even though its altitude range is 9° 58′ to 0° 28′. No star magnitudes are given.

The minimum explanation and instructions are given with the tables—and no illustrations of their use. Three small auxiliary tables are printed on the first page: the first gives mean refraction for various heights, the second the Coriolis correction to bubble sextant readings for the mean latitude, and the third is a conversion table from minutes of arc to kilometers. The refraction table confirms that refraction has not been included in the computed altitude. The explanation suggests that the tables were primarily intended for use with a timepiece keeping Greenwich sidereal time ($+12^h$); failing this, provision seems to have been made, in the shape of canvas pockets, for the insertion of conversion tables from mean to sidereal time.

No interpolation table for altitude (maximum difference 15′) is provided, as an assumed position, making the local sidereal time an integral minute, can always be used.

It is tempting to go beyond the scope of this review and to assess the value of these tables for air navigation. It will suffice here, however, to say that their chief disadvantage is that the method cannot be used for Sun, Moon and planets, and that therefore some other method must co-exist; whether the saving on the stars outweighs the disadvantages of learning two methods and having two sets of tables, is not within our province.

J. B. PARKER & D. H. SADLER

[1] The full title of this eight-page pamphlet published by H. M. Nautical Almanac Office in 1943, "for official use only", is *Experimental Astronomical Navigation Tables for Latitude North 53° (for use between N. 52°30′ and N. 53°30′) and Time Scale Setting Data for October–November 1943*, 16.8 × 24.5 cm. The fundamental principle underlying these tables was conceived by Wing Commander E. W. ANDERSON and developed by Squadron Leader A. POTTER.

EDITORIAL COMMENT: Professor C. H. SMILEY has contributed the following review: This leaflet was prepared so that the method represented by it could be compared with other methods in common use. For each five minutes during the night in England and Western Europe, altitudes to the nearest minute of arc and azimuths to the nearest degree are given for six of the following twelve stars: Aldebaran, Alpheratz, Altair, Arcturus, Capella, Deneb, Dubhe, Pollux, Procyon, Regulus, Sirius and Vega. Altitudes as low as 6°14′ and as high as 75°17′ are given in order to have the available stars well distributed in azimuth.

The tabulated altitudes include a correction for atmospheric refraction at 20,000 ft. elevation. So long as the plane remains above 5,000 ft. elevation and the celestial body has an altitude greater than 12°, the additional correction required to take complete account of refraction is less than a minute of arc. Opposite certain sequences of altitudes of particular stars appears a heavy black line, warning the navigator that the star is near the meridian and the altitude can no longer be considered to change in a linear fashion with time; in such a case the observed altitude can be used as the argument instead of the time of observation.

Beside the altitudes and azimuths of six stars, for each five minutes of the night, the azimuth of Polaris is given as well as the Q-correction which, added to the observed altitude of Polaris, gives the latitude of the observer.

The argument in the tables is $(T - t)$, where T is an auxiliary scale-time which, over-printed on the plotting charts, replaces the longitude scale. This device allows the use of a mean-time chronometer without the usual transformation from mean time to sidereal time.

Beside the principal table of altitudes and azimuths, there is one giving scale-time settings and time-difference corrections allowing the table to be used on any night during the two months of October and November 1943.

The method appears to be one of local value, depending on the availability of the necessary over-printed charts. It would be an expensive system to maintain continuously for all latitudes and longitudes. It has the advantage that the navigator can look ahead and see what stars are going to be available at a particular time and place.

473[U].—U. S. HYDROGRAPHIC OFFICE, *Star Tables for Air Navigation. Computed Altitude and true Azimuth for all Latitudes. Preliminary edition.* (H.O. no. 249.) Washington, D. C., U. S. Government Printing Office, 1947. viii, 322 p. 23.7 × 30.2 cm. For sale by the Hydrographic Office and by the Superintendent of Documents, Washington, D. C. $2.00; foreign price, postage extra.

The preface of this volume specifically credits Commander C. H. HUTCHINGS, U. S. Navy, with having "conceived and designed" these tables, "as a rapid method of determining computed altitudes and azimuths of prominent fixed navigational stars." Tables of this character were independently suggested at about the same time by two Americans, GEORGE G. HOEHNE and C. H. Hutchings (see RMT **313**, and below).

These tables give the altitude to the nearest minute of arc and the true azimuth to the nearest degree for six stars for each integral degree of the Local Hour Angle of the March Equinox (LHA ARIES or LHA ♈) for integral latitudes 69°S to 69°N. For integral latitudes in the polar regions, 70°N–89°N and 70°S–89°S, similar information is given except the interval of the argument, LHA ♈ is 2° rather than 1°. The six stars are chosen from a list of 38, half of which are brighter than the second magnitude and the remainder are of the second magnitude. At intervals of 15° of LHA ♈ (30° in the polar regions), the table is interrupted and new stars are introduced to replace those which have moved too high or too low for good observing. The stars are listed in order of increasing azimuth, left to right, even though this means that a star must be moved to the right or left as a 15° break is crossed.

The altitudes ordinarily range from about 20° to 70°, although in some cases, altitudes down to about 10° and up to 85° are included. For example, for latitudes 15°N and 41°N, Aldebaran and Capella respectively are given down to altitudes 10°34′ and 10°41′, and for latitude 39°N, an altitude of 85°11′ is given for Vega. The altitudes have been corrected for atmospheric refraction at 5000 feet above sea level and an auxiliary table, I, provides corrections for other elevations at 5000 ft. intervals up to 40,000 ft.

Mean star places for 1948 have been used, and *average* corrections for precession have been provided in Table II for the years 1947–1951. It is precession which will make this type of table expensive; after a brief interval the table will have to be recomputed and reprinted, or elaborate tables provided to take care of the changes due to precession.

Other auxiliary tables included are: Table III, for the conversion of arc to time; Table IV, lists of the 38 stars, alphabetically arranged and in order of declination; Table V, corrections to be added to sextant altitudes of Polaris to obtain latitudes; Tables VIa and VIb, corrections for the motion of the observer for ground speeds up to 450 knots; Tables VIIa and VIIb, Coriolis corrections for a similar range of speeds.

In the principal table, all of the data for a single latitude in the interval, 69°S–69°N, are contained on two facing pages. Each page has two vertical sections of six columns each, covering an interval of 90° in LHA ♈. Each column is broken into six segments of 15°, with a star name at the head of each segment. For the polar latitudes, all of the data for a given latitude are on a single page and the blocks cover 30° of LHA ♈. The convenience and practicality of this arrangement, showing which stars will be available for observation at a given time and place, and their altitudes and azimuths, can hardly be exaggerated.

The Air Almanac may be dispensed with, providing a sidereal chronometer reading in degrees and minutes of arc is available. The advantages of such a chronometer were pointed out by AQUINO in 1933. To date, only a limited number have been made and they are quite expensive.

In the choice of stars, great emphasis appears to have been placed on having the stars well distributed in azimuth and considerably less emphasis placed on continuity. If an observation of a star happens to fall just outside of the time interval for which altitudes and azimuths are given, the navigator will be obliged to discard the observation or to resort to the uncertain process of extrapolation. For this reason, "ends" might reasonably have been kept to a minimum.

The data for latitude 42°N have been examined in this connection; it is found that 25 of the 38 stars appear on the two pages. Nunki and Marfak each appear in only a single 15° block, introducing altogether 8 ends. Rasalague and Rigel each appear in two separate single blocks. By removing one block for Vega (LHA ♈ 285°–299°) in which the altitudes range from 74°18′ to 84°15′, and replacing it by one for Rasalague, one would have three consecutive blocks instead of two isolated ones, and four fewer ends. Likewise if the blocks for Rigel (LHA ♈ 30°–44° and 90°–104°) were replaced by similar blocks for Aldebaran and Betelgeuse respectively, six ends would be eliminated. It is appreciated, however, that no two people would make precisely the same choice of stars. Undoubtedly a great deal of time and thought went into the choosing of stars and unquestionably good arguments could be offered for the choices made.

The tabular values in this volume should establish a high standard of accuracy since they were computed by experts at the NBSCL. Altitudes of Vega near the meridian for latitudes 65°N to 74°N appear to be out by 1′, probably due to an erroneous correction for refraction.

Although the paper and the plastic binding are good, the printing is not up to the excellent quality usually provided by the Hydrographic Office. The material is too crowded and the type too small. Even if the volume has to be expanded into two, one for the northern hemisphere and another for the southern, an effort should be made to improve the legibility of the material. The printing appears to have been done by photo-offset or some similar process from sheets turned out by the automatic machines of the NBSCL. Even so, an occasional digit fails to appear in print, for example, the terminal digit of the azimuth of Fomalhaut near the meridian in 25° south latitude.

It is true that an aeroplane can many times rise above the clouds and observe stars at altitudes 20° and greater. However, no provision has been made for the case where the only stars available are at altitudes less than 10°. Although altitudes of 10° or less may be uncertain due to refraction, the corresponding azimuths are reliable. With the rapidly increasing speeds of planes, one may well look ahead to the time when it will again be feasible to "steer by a star"; stars of low altitude will be very useful for this purpose.

To return briefly to the development of these tables, Hutchings' specific recommendations were published in 1942[1] and comments by AGETON,[2] WEEMS,[3] and AQUINO[4] appeared in the next two years. In addition, other individuals upon request sent their comments on the proposed tables directly to the Hydrographic Office. It is interesting to note that in Hutchings' original proposal, the interruption of a given column to introduce a new star might be made at any point; altitudes were to be given to the nearest 0′.1 and azimuths to the nearest 0°.1. In a description by Commander W. J. CATLETT in 1945 of the forthcoming tables,[5] the 15° break had been introduced, altitudes were given to the nearest minute of arc and azimuths to the nearest degree.

Although several other tables of this general design have appeared earlier, this volume is apparently the only one in which *all* latitudes have been covered in a single volume. One may reasonably hope that another volume (or volumes) of tables of modern design allowing one to use observations of the sun, moon and planets may also be provided in the near future, perhaps as H.O. no. 250.

CHARLES H. SMILEY

[1] C. H. HUTCHINGS, U. S. Naval Inst., *Proc.*, v. 68, 1942, p. 1279–1284.
[2] A. A. AGETON, U. S. Naval Inst., *Proc.*, v. 68, 1942, p. 1303.
[3] P. V. H. WEEMS, U. S. Naval Inst., *Proc.*, v. 68, 1942, p. 1760–1761.
[4] F. R. DE AQUINO, U. S. Naval Inst., *Proc.*, v. 70, 1944, p. 315–318.
[5] Institute of Navigation, *Minutes of New England Regional Meeting* . . . 27 Aug. 1945, offset print, p. 7.

474[V].—HOWARD W. EMMONS, *Gas Dynamics Tables for Air.* New York, Dover Publications, 1947. 46 p. 15.4 × 26.5 cm. $1.75. "The author wishes to acknowledge his indebtedness to Mr. J. ARTHUR GREENWOOD, who very ably carried out all the numerical computations required to produce these tables" (p. 16; Tables I–IV, p. 17–36).

"*Introduction.* Recent trends in the development of high-speed aircraft render increasingly erroneous calculations based upon the assumption of an incompressible fluid. There is no need, however, to look to the future for justification of tables such as these, for ballistics already deals with velocities several times that of sound."

"The speed of sound that enters as the significant reference velocity in all discussions of compressibility effects on the flow of fluids is for air a function of temperature given in Table IV. At the standard sea level temperature of 59F the speed of sound is 1116.4 ft/sec, 761.2 miles/hour or 340.3 meters/second."

T. I: *Isentropic Gas Dynamics Functions for Air*, p. 17–30. For the MACH number, $M = V/a$, V (velocity), a (speed of sound), $M = 0(.001).05(.01).8(.001)1.2(.01)2(.1)5(1)25$, the values are given, 3 to 5S mostly, for the following 8 functions:

$T/T_0 = [1 + \frac{1}{2}(\gamma - 1)M^2]^{-1}$, $\gamma = 1.4$ (isentropic exponent), T (temperature);
$p/p_0 = (T/T_0)^{\gamma/(\gamma-1)}$, p (pressure); $\rho/\rho_0 = (T/T_0)^{1/(\gamma-1)}$, ρ (mass density);
$a/a_0 = (T/T_0)^{\frac{1}{2}}$; $V/a_0 = aM/a_0$; $\rho V/\rho_0 a_0 = (\rho/\rho_0)(V/a_0)$;
$\rho V^2/2p_0 = \frac{1}{2}\gamma p M^2/p_0$; $A/A^* = (\rho^*/\rho_0)(a^*/a_0)/(\rho/\rho_0)(V/a_0)$,
$\rho^*/\rho_0 = [2/(\gamma + 1)]^{1/(\gamma-1)} = .63394$, $a^*/a_0 = .91287$; * indicates the critical condition; i.e., the condition in which the fluid velocity equals the local speed of sound. Graphs p. 37–40.

T. II: *Gas Dynamics Functions for Normal Shocks*, p. 31–33. For M_1 = [1(.01)2(.1)5(1)25; 5S] are given the values of the following 8 functions:

$p_2/p_1 = 2\gamma M_1^2/(\gamma+1) - (\gamma-1)/(\gamma+1)$; $V_2/V_1 = 2(\gamma+1)^{-1}(V_1/a_0)^{-2}$;
$\rho_2/\rho_1 = V_1/V_2$; $T_2/T_1 = (p_2/p_1)/(\rho_2/\rho_1)$;
$M_2 = [2/(\gamma+1)]^{\frac{1}{2}}/[(p_2/p_1)(T_2/T_0)]^{\frac{1}{2}}$; $P_{20}/p_1 = (P_{20}/p_2)(p_2/p_1)$;
$p_{2_0}/p_{1_0} = (p_{2_0}/p_1)(p_1/p_{1_0})$, 1_0 indicating isentropic stagnation condition immediately before a shock wave, 2_0 indicates isentropic stagnation condition immediately behind a shock wave. Graphs p. 41–44.

T. III: *Characteristics Table*, p. 34–35.
$M = \csc\alpha$ or $\alpha = \sin^{-1}(1/M)$,
$\omega = [(\gamma+1)/(\gamma-1)]^{\frac{1}{2}} \tan^{-1}\{[(\gamma-1)/(\gamma+1)]^{\frac{1}{2}} \cot\alpha\}$;
for $\nu = \omega + \alpha - \frac{1}{2}\pi = 0(.5)130$ are given the values of α, 2D, and of M, 3–5S; graphs p. 45.

T. IV: *Acoustic Velocity-Temperature Table*, p. 36.
For T = 300(2)698 are given the values of $a = 49.019T^{\frac{1}{2}}$ ft./sec., 4–5S; graph, p. 46.

Extracts from text

475[V].—MASSACHUSETTS INSTITUTE OF TECHNOLOGY, Department of Electrical Engineering, Center of Analysis, Technical Report no. 1, work performed under the direction of ZDENĚK KOPAL, under NOrd Contract No. 9169: *Tables of Supersonic Flow around Cones*. Cambridge, Mass., 1947, xviii, 558 p. + 9 folding plates. 20.4 × 26.8 cm. For sale by Library of Congress, Washington, D. C., $3.50.

The general equations of fluid mechanics have been known for over a century. These equations, including the continuity equation, the NAVIER-STOKES momentum equation, and some form of energy equation, are adequate to solve problems of flow of fluids with viscosity through channels of arbitrary shape or around bodies of arbitrary shape. Many special solutions are known. Most of them, however, are for the motion of a perfect fluid (that is, one without viscosity or heat conduction), around some simple geometric shape. The majority of exact solutions available to date concern the flow of an incompressible fluid, that is, a fluid whose density is constant. For the motion of air, such incompressible fluid solutions are adequate, provided the fluid velocities are so low that no appreciable changes of density occur. The number of engineering problems in which the fluid velocity became high enough to include the effects of compressibility were limited to a few applications in steam turbine design and projectile design, until the speed of aircraft became so great that the compressibility problems became significant in that field. During the 1930's, the speeds of aircraft became so high that serious compressibility effects became apparent and thus necessitated the search for additional solutions to the problems of the flow of air about bodies of various shapes. The development of rockets and the probable development in the near future of supersonic aircraft have extended the need for solutions to the equations of fluid mechanics and to the range of MACH numbers greater than unity.

The problem of the supersonic flow about a cone is a most significant one because of the fact that a sharp, pointed fusilage is an appropriate one for supersonic aircraft and rockets, just as it is appropriate for projectiles. Fortunately, the exact solution for the flow about cones is reducible to a fairly simple numerical process and the present tables are designed to supply such solutions. The fact that the infinite cone is the appropriate "simple" geometric form to consider in supersonic flow (rather than the sphere as in subsonic flow), was pointed out in a qualitative way by BUSEMANN[1] and BOURQUARD.[2] It remained for TAYLOR & MACCOLL[3] to work out the detailed equations and to obtain numerical solutions for a number of important values (mentioned later), and to check these solutions against experimentally observed flows. At the present time, the calculated flow around cones has two principal uses, (1) the predicted wave angles check so well with experiment that the cone is now frequently used as a means of calibrating an air stream of unknown velocity; (2) many projectiles, rockets, and supersonic planes have a conical nose where these solutions can be used directly to predict the air forces encountered. Even in the case that an ogival nose is used, the computation of the air forces must start with the conical solution at the vertex. In mathematical terms, this means that the type of singularity in the solution to the supersonic flow of air over a pointed body of revolution is the same as that for the flow about a cone.

The present work presents an extensive set of tables giving a complete solution in 5D tables for the flow of air around cones at zero angle of attack. These solutions are complete in the sense that not only do they give the cone angle, wave angle, and Mach number relations, but also the velocity components and speed of sound in the region between the shock wave and the cone surface. Such results are adequate for the solution of other problems of flow about bodies of revolution when it is pertinent to start their solution with the corresponding cone solution. In a twelve-page introduction to the Tables, there is a review of the nature of the problem and the general aspects of its solution, as well as a derivation of the necessary equations. Essentially the Tables are a solution of the differential equation:

$$\frac{d^2u}{d\theta^2} + u = \frac{a^2(u + v\cot\theta)}{v^2 - a^2} \tag{1}$$

where u is the radial velocity component
v is the tangential velocity component
θ is the colatitude
a is the velocity of sound in the gas.

The velocity of sound is given by:

$$a^2 = \tfrac{1}{2}(\gamma - 1)(c^2 - u^2 - v^2)$$

where c is the (constant) velocity of expansion into a vacuum
γ is the (constant) ratio of specific heats for the gas.

Equation (1) is subject to boundary conditions $u = u_s$ and $v = 0$ for cone semi-vertex angles $\theta = \theta_s$. For each u_s, a solution is constructed numerically toward increasing θ until the shock wave equation $\tan\theta = -\dfrac{\gamma - 1}{\gamma + 1}\dfrac{c^2 - u^2}{uv}$ is satisfied.

This value of $\theta = \theta_w$ gives the position of the conical shock wave attached to the nose of the cone. All of the remaining physical properties of the solution now follow immediately from well-known gas dynamics relations.

As is well known, there are in general two solutions for each initial value of the cone angle θ_s. In the present work, both solutions are given. The introduction includes an adequate description of the reason for obtaining these two solutions and notes that only the weak shock or "first" solution is observed in experiments. It is also noted that solutions exist only for cone angles: $\theta_s \leq 57°.5253\ldots$

The only previously published calculations of the supersonic flow of air about cones and, in fact, the original work presenting the theory and its experimental verification was that of Taylor and Maccoll.[3] This work, although it contains very limited tables, is the classic work on this problem and has given its name, The Taylor & Maccoll Flow, to the phenomena of flow about cones with attached shock. The tables contained in this early publication are for $\theta_s = [10°(10°)30°; 3D]$, over a range of some ten values of u_s. No complete solutions are given, however, and if these were desired they would have to be reconstructed. Hence the present work is noteworthy not only because of its more extensive character and greater accuracy but also because it gives the complete solution.

A few typographical errors are worth noting because of the incorrect meaning implied as written. (1) On page xiv of the introduction, the fourth and fifth lines from the top now read, "In doing so we find, however, that U/a_1 is *not* a single-valued function of u_s/c." This sentence should have U/a_1 and u_s/c interchanged, because in fact two values of u_s/c correspond to each value of U/a_1. (2) On page xv, on the fourth and ninth lines from the top, there appear the words "downstream." These should read "upstream."

The Tables presented are as follows:

I. Tables of Supersonic Flow of Air ($\gamma = 1.405$), p. 1–468. Values of the square of the speed of sound and the radial and tangential velocity components are given in terms of the (constant) velocity that would be attained by the air in front of the shock wave if it expanded into a vacuum. These solutions are given for values of $\theta_s = 5°(2°.5)25°(5°)50°$, for 32 values of u_s ranging from .175 to .99551 (with increments ranging from .00051 to .05). For each complete solution, that is, for each pair of θ_s, u_s values, the corresponding wave angle and Mach number ahead of the shock are given. The large number of pages used in this table could have been reduced by two-thirds through using a finer print and closer spacing. 5D values are given which, for most of the table, are rounded from computations to 6D. Those parts which were carried out to 5D only are indicated by an *. The strong shock solutions are indicated by (S) following the Mach number.

II. In this table, p. 469–475, the physical properties of major interest from the preceding solutions are tabulated. The tabulation includes 5S values of θ_s, M, T_s/T_w, p_s/p_w, ρ_s/ρ_w, T_w/T_1, p_w/p_1, ρ_w/ρ_1, K_D, as functions of θ_s, u_s for the cases presented in Table I.

III. To facilitate the use of Table I, values of u_s are given, p. 477–479, as function of the semi-apex angle of the cone $\theta_s = 5°(2°.5)25°(5°)50°$ and the Mach number $M = 1.05(.05)4.00$.

IV. Table, p. 481–483, giving, for the same range, the semi-apex angle of the shock wave, θ_w, in terms of M and θ_s. This table is the one that has been the object of former computations and is the one that would be most frequently used by those using cones for supersonic flow measurements.

V. In a small group of solutions, the velocity immediately behind the shock wave is supersonic but changes with decreasing θ and becomes subsonic at the cone surface. This table, p. 485–487, gives values relating to these solutions as well as to the division between strong and weak shock solutions. Specifically, the individual columns indicate:

(1a) Minimum Mach number for which solutions are possible.
(1b) The value of u_s corresponding to the division between 'first' and 'second' waves.
(2a) Maximum Mach number for which the stream between the cone and the shock wave is completely subsonic.
(2b) The corresponding value of u_s.
(3a) Minimum Mach number for which the stream between the cone and the shock wave remain supersonic.
(3b) The corresponding value of u_s.

VI. Besides the physically important properties possessed by the solutions to equation (1), there are the mathematical second-order discontinuities, where $v^2 = a^2$. One such discontinuity appears beyond the strong shock; the other appears inside of the cone surface. The location of these two discontinuities are listed, p. 489–491.

VII. This table, p. 493–548, is identical with Table I except that $\gamma = 1\frac{1}{3}$. This additional table was computed to show the effect of changing γ, and this particular value was chosen because steam and other commercially important gases have values of γ in this neighborhood. This table is much less extensive than Table I. $\theta = 10°(5°)40°$, while u_s has an increment of .1, except at the highest values.

VIII. This table, p. 549–551, summarizes the results given in Table VII. The quantities listed are the same as in Table II.

IX. In this Table, p. 553–555, the change of the adiabatic constant, K, and entropy, s, across the shock wave as a function of shock strength, p_w/p_1, is given.

X. Here are nine folding plates with graphs, giving the physically important properties of the solution in a form which will be readily usable in practical work. The graphs are good to 3D at the most, and are valuable in those places where high accuracy is not required.

HOWARD W. EMMONS

Dept. Engin. Sciences and
Applied Physics, Harvard University

[1] VON A. BUSEMANN, "Drücke auf kegelförmige Spitzen bei Bewegung mit Überschallgeschwindigkeit." *Z. f. angew. Math.*, v. 9, 1929, p. 496–498.
[2] F. BOURQUART, "Aerodynamique—Ondes balistiques planes obliques et ondes coniques application à l'étude de la résistance de l'air." Acad. d. Sci., Paris, *C.R.*, v. 194, 1932, p. 846–848. Also *Mém. d'Artill. Franç.*, v. 11, 1932, p. 135f.
[3] G. I. TAYLOR & J. W. MACCOLL, "The air pressure on a cone moving at high speeds," R. Soc. London, *Proc.*, v. 139A, 1933, p. 278–311.

MATHEMATICAL TABLES—ERRATA

References have been made to Errata in the article "A New Approximation to π (conclusion)"; RMT **451** (Schulze), **452** (Müller, Rajna & Gabba), **453** (Prokeš), **463** (Ziaud-Din, Kerawala & Hanafi), **464** (NBSCL), **465** (Col. Univ. Press), **466** (Reiz), **469** (Hage), **473** (U. S., H. O.), **475** (M. I. T.); UMT **63** (Hayashi, Brandenburg).

118. C. GUDERMANN, "Theorie der potenzial- oder cyklisch-hyperbolischen Functionen," *Jn. f. d. reine u. angew. Math.*, v. 8–9, 1832.

In the course of reading proofs of the new Chambers' 6-figure tables the logarithmic values of sinh, cosh and tanh for the range $k = 2(.001)3(.01)6$ were compared with the first six decimals of Gudermann. The following errors were noted:

	Page	k	Function	*For*	*Read*
v. 8,	195	2.018	sinh	2345	6345
		2.036	tanh	1940	1949
	196	2.063	sinh diff	4445	4485
		2.081	sinh	9189	9188
	198	2.169	tanh diff	266	226
	199	2.248	cosh diff	33	23
	200	2.258	cosh diff	4279	4249
		2.258	tanh	50 6	5036
		2.284	tanh diff	980	180
	201	2.301	sinh	6.693	0.693
	202	2.353	cosh diff	4205	4265
		2.377	sinh	529	5293
	203	2.414	sinh	69	59
		2.415	tanh	4628	0628
		2.445	tanh diff	310	130
	204	2.489	cosh	9190	9100
		2.498	Argument	489	498
	205	2.506	tanh	2272	2172
	209	2.701	sinh	6.870	0.870
	210	2.759	sinh diff	4477	4377
	212	2.854	cosh	999	939
		2.882	cosh	9679	9676
		2.893	sinh diff	4399	4369
		2.898	Argument	889	898
		2.898	sinh diff	4379	4369
	304	3.061	cosh	.029	1.029
	306	3.159	sinh diff	4458	4358
	307	3.202	sinh diff	4375	4357
	308	3.251	sinh diff	4356	4355
		3.298	Argument	289	298
	313	3.506	cosh	1.122	1.221
	314	3.598	Argument	589	598
	316	3.659	sinh diff	4448	4348